UNIVERSITY COLLEGE DUBLIN

National University of Ireland

An Coláiste Ollscoile, Baile Átha Cliath
Ollscoil na hÉireann, Baile Átha Cliath

AUTUMN EXAMINATIONS, 2004

SCBDF0001, SCBDF0015

FIRST SCIENCE EXAMINATION

B.Sc. COMPUTER SCIENCE, Year 1

MATH 1200: PASS MATHEMATICS Paper 2

Professor C.-H. Chu
Professor S.J. Gardiner
Dr K. Hutchinson*
Dr R. Quinlan.*

Instructions for Candidates

Attempt questions 1 and 7 (each worth 16% of the marks) and an additional three questions from *each* of sections A and B.

Please use separate answer books for sections A and B

Time allowed: three hours

Notes for Invigilators

Non-programmable calculators may be used during this test.
Programmable calculators, mathematical tables and graph paper may *not* be used.

SECTION A: ALGEBRA

1. (i) If $\vec{u} = [2, -3], \vec{v} = [1, 1]$ and $\vec{w} = [-5, 2]$ in $\mathbb{R}^2$, find $||2\vec{u} - 4\vec{v} - \vec{w}||$.

   (ii) Find the cosine of the angle between the vectors $\vec{u} = [4, 1]$ and $\vec{v} = [3, -1]$ in $\mathbb{R}^2$.

   (iii) Give an example of a non-zero vector in $\mathbb{R}^3$ that is orthogonal to $[1, 1, 1]$.

   (iv) In $\mathbb{R}^3$, find a vector of length 2 having the same direction as $\vec{u} = [2, -2, 1]$.

   (v) In $\mathbb{R}^3$, compute the cross product $\vec{u} \times \vec{v}$ where $\vec{u} = [2, -2, -1]$ and $\vec{v} = [4, 3, -1]$.

   (vi) In $\mathbb{R}^3$, find the intersection point of the line

   $$L : \vec{r}(t) = [8, -5, -5] + t[-2, 1, 2], \ t \in \mathbb{R}$$

   and the plane $P : 2x - 3y + z = 11$.

   (vii) Show that $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ is an eigenvector of the matrix $\begin{pmatrix} 1 & 6 \\ 8 & 3 \end{pmatrix}$, and find the corresponding eigenvalue.

   (viii) Write the complex number $1 + i$ in polar form.

2. (a) In $\mathbb{R}^2$, let $\vec{u}$ denote the vector $[4, -1]$ and let L be the line with equation $2x - y = 5$. Find vectors $\vec{u}_1$ and $\vec{u}_2$ for which $\vec{u}_1$ is parallel to L, $\vec{u}_2$ is perpendicular to L, and $\vec{u} = \vec{u}_1 + \vec{u}_2$.

   (b) Find the distance from the point $(2, -3)$ to the line L of part (a) above.

   (c) Let $\vec{u} = [x_1, y_1]$ and $\vec{v} = [x_2, y_2]$ be non-zero vectors in $\mathbb{R}^2$. Show that $\vec{u}$ and $\vec{v}$ are orthogonal to each other if and only if

   $$x_1 x_2 + y_1 y_2 = 0.$$

3. (a) In $\mathbb{R}^3$, find the equation of the plane P that contains the points $A(5, 3, 1)$, $B(-2, -9, 0)$ and $C(1, -1, -1)$.

   (b) Find the area of the parallelogram that has the vectors $\vec{AB}$ and $\vec{AC}$ as two of its sides.

   (c) If D is the point $(1, 4, -1)$, find the volume of the parallelepiped that has $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ as three of its edges.

4. (a) Let L_1 and L_2 be lines in $\mathbb{R}^3$ with vector equations

$$\begin{array}{lclcll} L_1 & : & \vec{r}(t) & = & [5,7,-4] & + \; t[-2,-2,1], & t \in \mathbb{R} \\ L_2 & : & \vec{m}(s) & = & [-2,15,4] & + \; s[-1,4,2], & s \in \mathbb{R}. \end{array}$$

Find the intersection point A of L_1 and L_2.

(b) Find parametric equations for the line L that contains both the point A and the point $B(5,4,1)$.

(c) Find a normal vector for the plane P that contains both the lines L_1 and L_2. Find also the equation of the plane Q that is parallel to P and contains the point B.

5. (a) Let A be a $n \times n$ matrix. What is meant by an *eigenvector* of A?

(b) Find the characteristic polynomial of the matrix $A = \begin{pmatrix} -1 & -3 & -1 \\ 6 & 5 & 0 \\ -6 & -9 & -2 \end{pmatrix}$.

(c) Find all the eigenvalues of the matrix A above, and find an eigenvector of A corresponding to the eigenvalue $\lambda = -1$.

6. (a) Write the complex number $\dfrac{1+2i}{3-4i}$ in the form $a + bi$, for real numbers a and b.

(b) Find all complex cube roots of the complex number -27.

(c) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, for $\theta \in \mathbb{R}$.

SECTION B: CALCULUS

7. (i) Differentiate $f(x) = \sin(x/2)$ with respect to x.

(ii) Sketch the graph of $y = \cot x$, indicating at least three points on the curve.

(iii) Find the two critical points of the cubic $f(x) = x^3 - 12x + 7$.

(iv) What are the vertical asymptotes of the rational function

$$f(x) = \frac{7x^3 + 11x + 2}{x^2 - x} \; ?$$

(v) Find

$$\int 20x^9 \, dx.$$

(vi) Evaluate the definite integral

$$\int_1^2 3x^2 + 6x \, dx.$$

(vii) Find the area under the curve $y = e^x$ from $x = 0$ to $x = 2$.

(viii) Write down all solutions of the differential equation

$$\frac{dy}{dx} = x^2.$$

8. Consider the polynomial

$$f(x) = 2x^3 - 9x^2 + 12x.$$

Find the following information about f: the critical points, the local maxima and minima, the intervals on which f is increasing or decreasing and the asymptotes (if any). Sketch the graph of f. What are the x-intercepts of the graph?

9. (i) Let b be a positive real number. Explain what it means to say that *the logarithm of* c *to the base* b *is* a; i.e,

$$\log_b(c) = a.$$

Show that $\log_b(c_1 c_2) = \log_b(c_1) + \log_b(c_2)$.

(ii) A bacteria culture is growing exponentially. It is measured at the same time on two successive days. The amount of the bacteria on the second day is 1.02 times the amount on the first day. Write down a formula for the amount of bacteria after t days (in terms of the initial amount, A_0).

Use logarithms to determine the doubling time of the culture.

10. (i) A box with square base and open top is to have a capacity (i.e. volume) of 4 cubic metres.

Let x be the length of a side of the base and y the height of the box. Express the volume of the box in terms of x and y. Use the information given to express y in terms of x.

Express the amount, A, of material required to make the box in terms of x and y. Express A in terms of x alone. Using calculus, determine for which value of x the least amount of material is required.

continued overleaf

(ii) Find the area under $y = x\sqrt{2x^2+1}$ from $x = 0$ to $x = 2$

11. (i) Differentiate the following expressions with respect to x:

$$\sin(x^3), \qquad \cos(e^x), \qquad 4^x$$

(ii) Find the following integrals

$$\int \frac{x}{x^2-1}\,dx, \qquad \int \ln x\,dx$$

12. (i) Sketch the region in the plane bounded by the parabola $y = x^2 - 4x + 4$ and the line $y = x$. Find the area of this region.

(ii) Solve the initial value problem

$$\frac{dy}{dx} + y = e^x, \quad y(0) = 1.$$

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