First Arts Modular Degree

Mathematical Studies 2004–2005

Combinatorics and Number Theory Problem Sheet 3

1. Find the coefficient of x^{11} in the expansion of

$$\left(x^2 - \frac{3}{x}\right)^{10}.$$

2. Find the coefficient of x^3 in the expansion of $(x+2)^3(x-2)^5$.

3. Find the coefficient of x^2 in the expansion of $(1+2x+2x^2)^5$.

4. Find the coefficient of x in the sum

$$1 + (1+x) + (1+x)^2 + (1+x)^3 + \cdots + (1+x)^n.$$

Hint: the sum above is a geometric progression. Use the formula for the sum of a geometric progression.

5. Find the greatest common divisor c of 69 and 117, and find integers s and t such that $69s + 117t = c$.

6. Do the same as in question 7 for 312 and 1084.

7. Do the same as in question 7 for 594 and 781.

8. Explain why the number $111,111$ is not a prime. Is the number $11,111$ a prime?

9. Suppose that c is an integer and 3 divides $c-1$. Prove that c^2+c+1 is divisible by 3 but not by 9.

10. Let c be an even integer. Prove that $c+1$ and c^2+1 are relatively prime.