# The Enduring Wonders of Enormous Numbers

**Peter Lynch**

**School of Mathematics & Statistics**
**University College Dublin**

***India-Ireland Friendship Lecture Series***

**Indian Embassy, Dublin, 3 November 2022**

# Outline

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# Overview

Mathematics has played an important role in Indian culture for millennia.

For more than three thousand years there has been a tradition of pursuing mathematical quests.

Sometimes for practical reasons and sometimes for pure intellectual delight.

Examples are trigonometry for solving problems in astronomy, and the contemplation of infinities for philosophical or theological ends.

# Outline

# A Simple Demonstration

I will begin with a simple demonstration of
something that has never been done before.

Take a pack of fifty-two playing cards,
and mix them up by shuffling them well.

**Claim:** the resulting arrangement of cards has
never occurred before and never will again,
during the entire life of our Universe!

# How Many Possible Arrangements?

The first card may be chosen in any of 52 ways.

The next one can be any of the remaining 51 cards.

For the third, there are 50 choices, and
so on, until just one card remains.

# How Many Possible Arrangements?

The first card may be chosen in any of 52 ways.

The next one can be any of the remaining 51 cards.

For the third, there are 50 choices, and so on, until just one card remains.

Therefore, the total number of possibilities is

$$52! \equiv 52 \times 51 \times 50 \times \cdots \times 3 \times 2 \times 1 \, .$$

This number is called factorial 52.

# Factorial 52 is Enormous

**To call 52! "large" is a gross understatement.**

$$52! = 80658175170943878571660636856403766\ldots$$
$$\ldots 697528950544088327782400000000000000$$

**To a single figure of accuracy, this is $10^{68}$.**
**That is, 1 followed by 68 zeros.**

**Note: We use superscripts to indicate large numbers:**

$$1000 = 10^3 \qquad 1{,}000{,}000 = 10^6 \qquad 1{,}000{,}000{,}000 = 10^9 \cdots$$

# Uniqueness of the Card Deck

The Universe is $4 \times 10^{17}$ seconds old.

Choose a random arrangement of cards every second. Only a tiny fraction of all orderings would be selected during the life of the Universe.

Even if a billion arrangements were chosen every second, there would be no chance of a duplicate.

So, the arrangement is unique:
it has never been seen before,
and it will never again be seen.

# Bang! Bang! Bang!


Typical Comic-book ‘bang’ mark [Image from `vectorstock.com`].

# Bang! Bang! Bang!

There are many ways to generate enormous numbers. One of the simplest is by using repeated factorials.

We can read the number $52!$ aloud as

"Fifty-two Bang".

By taking factorials of factorials, we encounter numbers large beyond our wildest imaginings. The double factorial of 52 is utterly vast:

52!! or "Fifty-two Bang Bang".

# Bang! Bang! Bang!

**Let us consider a small beginning:**

**"3 Bang" is** $3 \times 2 \times 1 = 6$ **.**
**"3 Bang Bang" is** $6! = 720$ **.**
**"3 Bang Bang Bang" is** $720!$ **.**

**But** $720!$ **is ineffable: we have already seen the vastness of 52!, and 720! is incomparably larger:**

$$3!!! = 6!! = 720! \approx 2.6 \times 10^{1746}$$

# Bang! Bang! Bang!

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**A curiosity:**

$$\begin{aligned} 4! &= 24 \\ 4!! = 24! &= 6.2 \times 10^{23} \\ \textbf{Avogadro's Number} &= 6.02 \times 10^{23} \end{aligned}$$

# Title Slide in English

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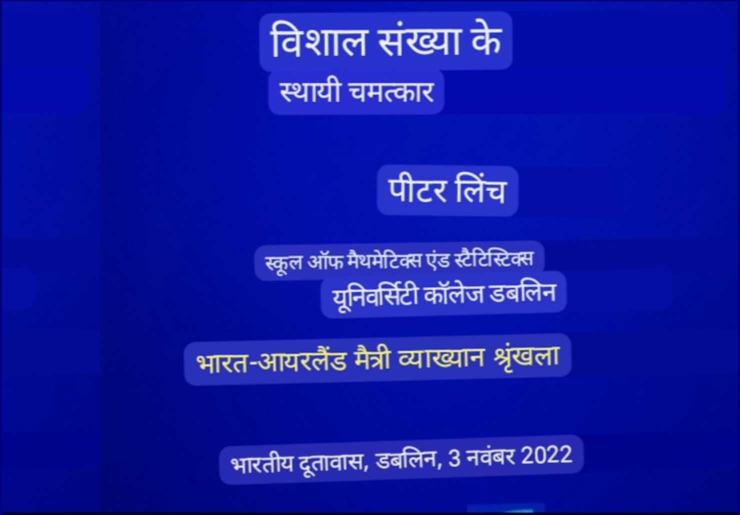
# Title Slide in Hindi

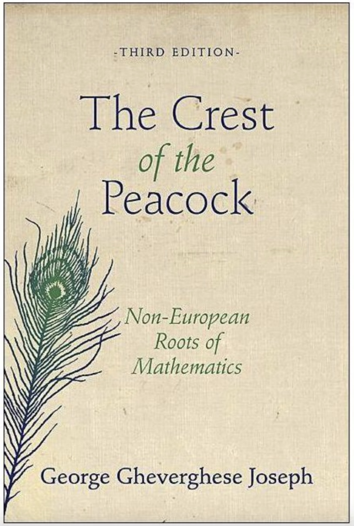


# Outline

# Mathematics in India

- Harappa Civilization.
- Vedic & Jaina Maths. Large Numbers.
- Classical Period.
- The Number System. Zero.
- Astronomy. Trigonometry.
- The Kerala School.
- Srinivasa Ramanujan.

# The Crest of the Peacock

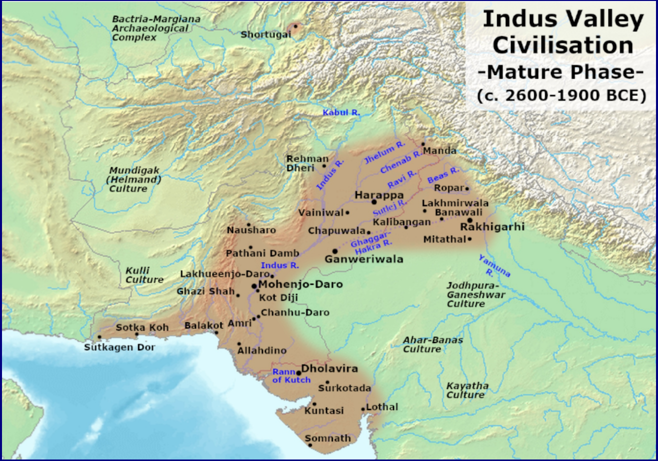



**George G Joseph is an Indian-born mathematician and historian of maths.**

**His main focus is on the Kerala School and the transmission of mathematics from India to Europe.**

# The Indus Valley

## Mathematics was used for engineering and architecture in the Harappan Civilization.



# The Indus Valley

Earliest mathematical traces in
the Indus valley around 3000 BC.

Structured systems of weights and measures.

Samples of decimal-based numeration.

Large numbers of bricks with regular shapes.
Regular proportions in the ratio $1 : 2 : 4$.

Language family of the Harappans unknown.
No “Rosetta stone” to decrypt it.

# Vedic Mathematics

Vedas are amongst oldest literary works of mankind.

Written in the form of sutras, or aphorisms.
Details of rituals, hymns and prayers.

Geometric instructions for the constructing
altars (vedis) and sacred fireplaces (agnis).

The shape, size and orientation of altars
are prescribed in great detail.

# Vedic Mathematics

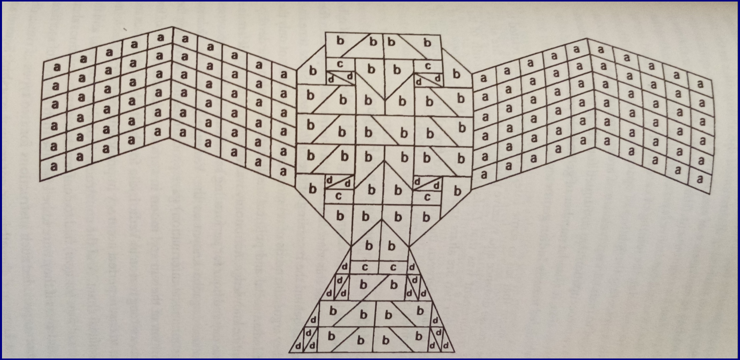


Diagram of a brick altar in the form of a falcon [Image G G Joseph].

# Sulbasutras

The Sulbasutras are supplements to the Vedas. They contain many mathematical passages.

Geometric techniques for laying out temples and measurement and construction of altars.

Vedic writers knew the Theorem of Pythagoras.

Rules for the construction of right angles using “Pythagorean triads” e.g. (3, 4, 5).

Methods for calculating square roots, for the duplication of a square and for constructing a circle with area equal to a square.

# Mathematics of the Jains

The *Suryaprajnapti Sutra* gives the approximation $\sqrt{10}$ for $\pi$, which is within 1% of the true value.

Later, Virasena (8th century) gave the approximation

$$\pi \approx \frac{355}{113}$$

which is accurate to seven decimal places.

The Jains were interested in cosmology, not in rituals or altar building.

Their cosmological studies led them to consider orders of infinity, anticipating Georg Cantor by millennia.

**Suryaprajnapti Sutra, c. 400 BC (MS 5297, c. AD 1500)**

# Astronomy and the *Siddhantas*

Mathematical astronomy has been a continuous interest for several thousand years.

The modern trigonometric function known as the sine of an angle emerged from India.

The description of the sine function is one of the chief contributions of the Siddhantas to mathematics.

The first major figure in India is *Aryabhata* (AD 476—550), generally regarded as the founder of scientific astronomy in India.

# The *Aryabhatiya*

The Aryabhatiya, written in AD 499, contains

- A table of values of the sine ratio.
- Methods for calculating square and cube roots.
- Several approximations for $\pi$.
- Formulas for various areas and volumes.
- Some important trigonometric results.

# Outline

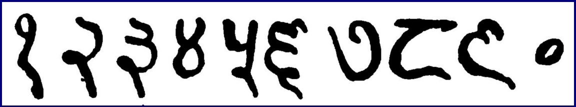
# Positional Decimal System of Numbers

**The decimal place-value system — and zero — are great contributions of India to mathematics.**

**In the positional system, the symbol "7" means different things in different positions:**

**7,707,747 .**

**This is an enormously powerful and flexible idea.**



**The Devanagari numerals, essentially the system used today.**

# Positional Decimal System of Numbers

Pierre-Simon Laplace

"It is India that gave us the ingenious method of expressing all numbers by means of ten symbols . . .

". . . [an idea] that escaped the genius of Archimedes and Apollonius."

# Zero

The number zero emerged in the first millennium AD.

In the seventh century work of Brahmagupta,
— the Brahma-sphuta-siddhanta — there is an
arithmetic treatment of zero as a number.

Brahmagupta also discusses negative numbers.

In Europe, zero and negative numbers did not
gain acceptance for another eight centuries.

# Zero

**The earliest known inscription of the symbol “0” to represent zero is on a tablet at Chaturbhuj, a Hindu temple in Gwalior Fort ( c. AD 875).**

**We see the numerals “270” as we write them today.**

# Outline

# Mathematicians of the Classical Age

- Aryabhata (c. 500)
- Brahmagupta (c. 630)
- Bhaskara I (c. 660)
- Shridhara (c. 800)
- Mahavira (c. 850)
- Aryabhata II (c. 950)
- Bhaskaracharya (c. 1150)

# Brahmagupta (c. AD 598—668)

Brahmagupta lived in Central India.

Perhaps his most beautiful result is a formula to find the area of a quadrilateral:

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}\,,$$

where $a$, $b$, $c$, $d$ are the sides and $s = \frac{1}{2}(a+b+c+d)$ (The formula is correct only for a cyclic quadrilateral).

His contributions to algebra were substantial.

The arithmetic of negative numbers and zero is first found in his work.

# Bhaskaracharya (c. AD 1114–1185)

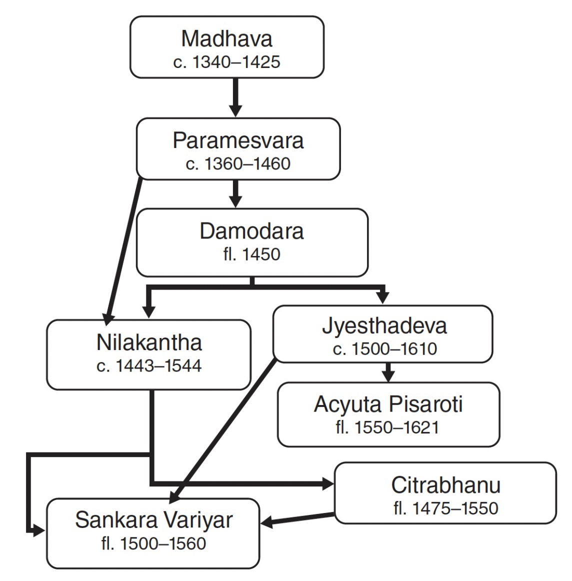
Bhaskara II was the leading mathematician of the twelfth century.

His best-known treatise is the Lilavati. It contains numerous problems on

- Linear and quadratic equations,
- Simple mensuration . . . measuring areas,
- Arithmetic and geometric progressions,
- Irrational numbers and
- Pythagorean triads.

# Outline

# Kerala Mathematicians for 200 Years



# “The Founder of Mathematical Analysis”

**Madhava** (c. 1340–1425) developed series expansions for the sine, cosine and inverse tangent functions.

In modern notation

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

For the specific value $x = 1$ this gives a series for $\pi$

$$\frac{\pi}{4} = \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) .$$

# Computing $\pi$ in Prose

Add 4 to 100, multiply by 8, and add 62,000. The result is approximately the circumference of a circle whose diameter is 20,000. (Verse 10)

From the *Aryabhatiya*

**This recipe gives the value $\pi = 3.1416$.**

**Evaluating the inverse tangent with $x = 30°$, Madhava obtained a rapidly converging series.**

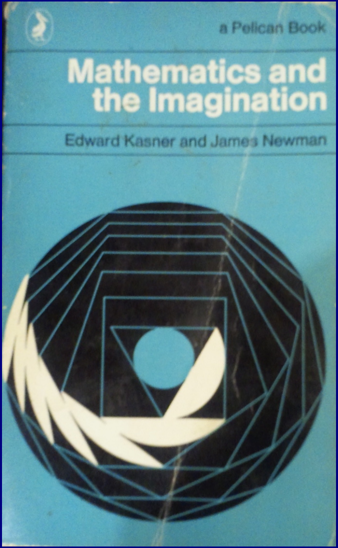
**He then evaluated $\pi$ to eleven decimal places.**

# Ideas Originating in India

| *Topic* | *First appears in* | *Repeated or used by* |
|---|---|---|
| Arithmetic involving zero | Brahmagupta 7th century AD | Arab mathematics 9th century, European mathematics 13th century |
| Elements of the calculus including derivatives of sine and cosine | Bhaskara II 12th century AD | Renaissance mathematics 17th–18th centuries |
| Continued fractions formulas | Bhaskara II 12th century AD | John Wallis 1655 |
| Infinite series for $\pi$ | Madhava 15th century AD | Leibniz, James Gregory, 17th century |
| Infinite series for sine and cosine functions | Madhava 15th century AD | Newton 17th century |

# Outline

# A Googol




In 1938, a 9 year old boy coined the name googol for an enormous number.

Edward Kasner and James Newman popularised the name in this book, published in 1940.

The boy called the number a googol, defining it as 1 followed by a hundred zeros:

10 000 000 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000 000 000

This is long-winded to write. There is a better way:

$$\textbf{1 googol} = 10^{100}\,.$$

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000 000 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000 000 000

This is long-winded to write. There is a better way:

$$1 \text{ googol} = 10^{100} .$$

The company name Google is derived from googol.

# A Googolplex

Kasner and Newman also defined a googolplex:
That is 1 followed by a googol zeros.

We could never write it without superscript notation.

$$1 \text{ googolplex} = 10^{\text{googol}} = 10^{(10^{100})}$$

This is a vast quantity, far beyond any of the constants appearing in the physical sciences.

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The corporate headquarters of Google Inc. in California is called the Googleplex.

The search for huge numbers is called Googology.

# Indian Tradition of Googology

From the earliest times there has been a passion in Indian culture with large numbers.

This is found in the Vedic, Jaina and Buddhist traditions. The Yajurveda names powers up to $10^{12}$.

Much larger numbers are found in the *Ramayana.*

# Large Numbers in the *Ramayana*

| Name | Indian notation | Power notation | Indian system |
|---|---|---|---|
| एक (*ēka*) | 1 | $10^0$ | One |
| दश (*daśa*) | 10 | $10^1$ | Ten |
| शत (*śata*) | 100 | $10^2$ | One hundred |
| सहस्र (*sahasra*) | 1,000 | $10^3$ | One thousand |
| लक्ष (*lakṣa*) | 1,00,000 | $10^5$ | One lakh |
| कोटि (*kōṭi*) | 1,00,000 *śata* | $10^7$ | One crore |
| शङ्कु (*śaṅku*) | 1,00,000 *koṭi* | $10^{12}$ | Ten kharab or One lakh crore |
| महाशङ्कु (*mahāśaṅku*) | 1,00,000 *śaṅku* | $10^{17}$ | One shankh or One thousand crore crore |
| वृन्द (*vṛnda*) | 1,00,000 *mahāśaṅku* | $10^{22}$ | |

# Large Numbers in the *Ramayana*

| | | | |
|---|---|---|---|
| महावृन्द (*mahāvṛnda*) | 1,00,000 *vṛnda* | $10^{27}$ | |
| पद्म (*padma*) | 1,00,000 *mahāvṛnda* | $10^{32}$ | |
| महापद्म (*mahāpadma*) | 1,00,000 *padma* | $10^{37}$ | |
| खर्व (*kharva*) | 1,00,000 *mahāpadma* | $10^{42}$ | |
| महाखर्व (*mahākharva*) | 1,00,000 *kharva* | $10^{47}$ | |
| समुद्र (*samudra*) | 1,00,000 *mahākharva* | $10^{52}$ | |
| ओघ (*ogha*) | 1,00,000 *samudra* | $10^{57}$ | |
| महौघ (*mahaugha*) | 1,00,000 *ogha* | $10^{62}$ | |

# A Legend from the *Lalitavistara*

The fascination of Indian scholars with large large numbers is illustrated by the Lalitavistara.

The Mahayana Lalitavistara Sutra tells the story of the Buddha, Siddhartha Gautama, up to the time of his first sermon at Sarnath.

He was renowned for his numerical prowess.

When asked to name the numerical magnitudes, young Siddhartha listed powers of ten up to $10^{421}$.

# More Gigantic Numbers

**The Lalitavistara Sutra gave a list of numbers going up to $10^{421}$.**

**Later Hindu and Buddhist texts extended the range.**

**The Buddhist Avatamsaka Sutra ("Flower Garland") has an even more extensive list, the largest being the "untold":** ***nirabhilapya nirabhilapya parivarta***

$$10^{7\times 2^{122}} \approx 10^{3.7*10^{37}}$$

**for which the number of digits is approximately 100 million million million million million million.**

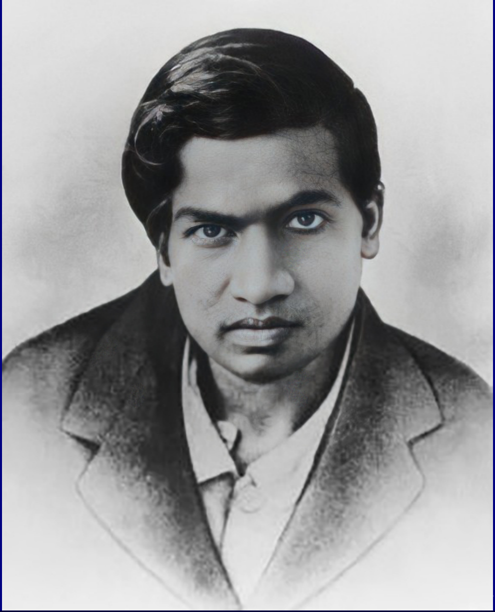
# A Summary of Enormous Numbers

**A Million** = $10^6$
**Avogadro's Number** = $6.02 \times 10^{23}$
**Particles in the Universe** = $10^{85}$
**A Googol** = $10^{100}$
**A Centillion** = $10^{303}$
**The "Untold"** = $10^{3.7*10^{37}}$
**A Googolplex** = $10^{10^{100}}$
**Skewes Number** = $10^{10^{10^{34}}}$
**Poincaré's Recurrence Time** = $10^{10^{10^{10^{1.1}}}}$

$\infty$

# Outline

# Srinivasa Ramanujan (1887–1920)



# Srinivasa Ramanujan (1887–1920)

Ramanujan was a self-taught mathematical prodigy from South India, born in 1887 in the town of Erode.

He grew up in Kumbakonam in Tamil Nadu, about 200 km from Madras, now Chennai.

His family was very poor, of the Brahmin caste, strict vegetarians, who treasured learning and valued philosophical and intellectual inquiry.

Ramanujan's parents supported his mathematical pursuits, even though they brought no material benefits to the family.

# Kumbakonam, Tamil Nadu


# Ramanujan’s House, Kumbakonam

# Sarangapani Temple

# Kumbakonam, Tamil Nadu

Kumbakonam was noted for its metal work, in copper, silver and brass.

It is also renowned for production of high-quality silk saris.

Silk Loom, Kumbakonam

# Namagiri of Namakkal

Ramanujan was intensely religious, and devoted to the family Goddess, Namagiri of Namakkal.

Ramanujan believed that Namagiri inspired him, and filled his dreams with mathematical insights.

"An equation for me has no meaning unless it expresses a thought of God."

# Numbers were his Personal Friends

**Every positive integer was one of Ramanujan's personal friends.**

**Visiting Ramanujan, Hardy remarked that he came a taxi numbered 1729, "a rather dull number".**

**"O no", said Ramanujan, "it is very interesting: it is the smallest number expressible as the sum of two cubes in two different ways."**

**Ramanujan had realized immediately that**

$$\begin{aligned} 1729 &= (12^3 + 1^3) \\ &= (9^3 + 10^3)\,. \end{aligned}$$

# To Europe

Ramanujan's academic record was atrocious. He failed exams in everything except maths.

There were few in India who could appreciate the true value of Ramanujan's mathematics.

Knowledgeable mentors convinced him that, to gain recognition, he should go to England.

Ramanujan wrote letters to several famous mathematicians in Cambridge. One replied!

# The Letter

"DEAR SIR,

"I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. [*He was actually 25—Ed.*] I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling'. . . .

"I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

"I remain, Dear Sir, Yours truly,

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# Formulae 1 to 4

$$(1.1)\quad 1-\frac{3!}{(1!2!)^3}x^2+\frac{6!}{(2!4!)^3}x^4-\dots$$

$$=\left(1+\frac{x}{(1!)^3}+\frac{x^2}{(2!)^3}+\dots\right)\left(1-\frac{x}{(1!)^3}+\frac{x^2}{(2!)^3}-\dots\right).$$

$$(1.2)\quad 1-5\left(\frac{1}{2}\right)^3+9\left(\frac{1.3}{2.4}\right)^3-13\left(\frac{1.3.5}{2.4.6}\right)^3+\dots=\frac{2}{\pi}.$$

$$(1.3)\quad 1+9\left(\frac{1}{4}\right)^4+17\left(\frac{1.5}{4.8}\right)^4+25\left(\frac{1.5.9}{4.8.12}\right)^4+\dots=\frac{2^{3/2}}{\pi^{1/2}\{\Gamma(3/4)\}^2}.$$

$$(1.4)\quad 1-5\left(\frac{1}{2}\right)^5+9\left(\frac{1.3}{2.4}\right)^5-13\left(\frac{1.3.5}{2.4.6}\right)^5+\dots=\frac{2}{\{\Gamma(3/4)\}^4}.$$

# Formulae 5 and 6

$$(1.5) \quad \int_0^\infty \frac{1+\left(\dfrac{x}{b+1}\right)^2}{1+\left(\dfrac{x}{a}\right)^2} \cdot \frac{1+\left(\dfrac{x}{b+2}\right)^2}{1+\left(\dfrac{x}{a+1}\right)^2} \cdots dx$$

$$= \tfrac{1}{2}\pi^{1/2} \frac{\Gamma(a+\frac{1}{2})\,\Gamma(b+1)\,\Gamma(b-a+\frac{1}{2})}{\Gamma(a)\,\Gamma(b+\frac{1}{2})\,\Gamma(b-a+1)}.$$

$$(1.6) \quad \int_0^\infty \frac{dx}{(1+x^2)(1+r^2x^2)(1+r^4x^2)\ldots}$$

$$= \frac{\pi}{2(1+r+r^3+r^6+r^{10}+\ldots)}.$$

# Formulae 7 to 10

(1.7) If $\alpha\beta = \pi^2$, then

$$\alpha^{-1/4}\left(1 + 4\alpha\int_0^\infty \frac{xe^{-\alpha x^2}}{e^{2\pi x}-1}\,dx\right) = \beta^{-1/4}\left(1 + 4\beta\int_0^\infty \frac{xe^{-\beta x^2}}{e^{2\pi x}-1}\,dx\right).$$

(1.8)
$$\int_0^a e^{-x^2}dx = \tfrac{1}{2}\pi^{1/2} - \frac{e^{-a^2}}{2a+}\,\frac{1}{a+}\,\frac{2}{2a+}\,\frac{3}{a+}\,\frac{4}{2a+\ldots}.$$

(1.9)
$$4\int_0^\infty \frac{xe^{-x\sqrt{5}}}{\cosh x}\,dx = \frac{1}{1+}\,\frac{1^2}{1+}\,\frac{1^2}{1+}\,\frac{2^2}{1+}\,\frac{2^2}{1+}\,\frac{3^2}{1+}\,\frac{3^2}{1+\ldots}.$$

(1.10) If
$$u = \frac{x}{1+}\,\frac{x^5}{1+}\,\frac{x^{10}}{1+}\,\frac{x^{15}}{1+\ldots},\quad v = \frac{x^{1/5}}{1+}\,\frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+\ldots},$$

then
$$v^5 = u\,\frac{1-2u+4u^2-3u^3+u^4}{1+3u+4u^2+2u^3+u^4}.$$

# Formulae 11 to 13

(1.11) $$\frac{1}{1+}\,\frac{e^{-2\pi}}{1+}\,\frac{e^{-4\pi}}{1+\dots} = \left\{\sqrt{\left(\frac{5+\sqrt{5}}{2}\right)} - \frac{\sqrt{5}+1}{2}\right\} e^{\frac{2}{5}\pi}.$$

(1.12) $$\frac{1}{1+}\,\frac{e^{-2\pi\sqrt{5}}}{1+}\,\frac{e^{-4\pi\sqrt{5}}}{1+\dots} = \left[\frac{\sqrt{5}}{1+\sqrt[5]{\left\{5^{\frac{3}{4}}\left(\frac{\sqrt{5}-1}{2}\right)^{\frac{5}{2}}-1\right\}}} - \frac{\sqrt{5}+1}{2}\right] e^{2\pi/\sqrt{5}}.$$

(1.13) If $F(k) = 1 + \left(\frac{1}{2}\right)^2 k + \left(\frac{1.3}{2.4}\right)^2 k^2 + \dots$ and

$$F(1-k) = \sqrt{(210)}\,F(k), \text{ then}$$

$$k = (\sqrt{2}-1)^4(2-\sqrt{3})^2(\sqrt{7}-\sqrt{6})^4(8-3\sqrt{7})^2(\sqrt{10}-3)^4 \times (4-\sqrt{15})^4(\sqrt{15}-\sqrt{14})^2(6-\sqrt{35})^2.$$

# Formulae 14 and 15

(1.14) The coefficient of $x^n$ in $(1 - 2x + 2x^4 - 2x^9 + \ldots)^{-1}$ is the integer nearest to

$$\frac{1}{4n}\left(\cosh \pi\sqrt{n} - \frac{\sinh \pi\sqrt{n}}{\pi\sqrt{n}}\right).$$

(1.15) The number of numbers between $A$ and $x$ which are either squares or sums of two squares is

$$K\int_A^x \frac{dt}{\sqrt{(\log t)}} + \theta(x),$$

where $K = 0{\cdot}764\ldots$ and $\theta(x)$ is very small compared with the previous integral.

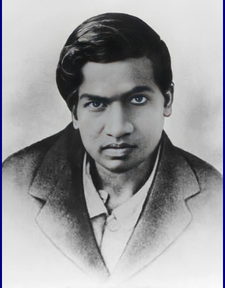
# G H Hardy

Godfrey Harold Hardy was one of the leading British mathematicians.

Hardy was convinced that the formulae were the work of a genius.

He arranged for Ramanujan to come to Cambridge.

# Hardy and Ramanujan



**Hardy and Ramanujan collaborated fruitfully for five years, producing some remarkable mathematics.**

**Ramanujan was elected Fellow of the Royal Society and Fellow of Trinity College Cambridge.**

# Ramanujan's Legacy

Over recent decades, there has been growing appreciation of Ramanujan's brilliance.

His work is of enormous influence, and pervades many areas of modern mathematics.

The modern theory of modular forms springs directly from the work of Ramanujan.

# Indian Postage Stamps


**Stamps picturing Ramanujan were issued by the Government of India in 1962, 2011, 2012 and 2016.**

Bust of Srinivasa Ramanujan at the

Birla Industrial & Technological Museum, Kolkata.

# Outline

# Quo Vadimus?

Knowledge of the history of Indian mathematics is fragmentary . . . great opportunities for scholars.

Our understanding of the importance of India for the history of maths has changed markedly over recent decades.

Current research is providing us with greater insight on the contributions of India to mathematics.

Who knows where future research will lead us?

# Thank You

धन्यवाद