George Gabriel Stokes's Fundamental Contributions to Fluid Dynamics

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IHoM V, Maynooth, 2 August 2019

Outline

Outline

George Gabriel Stokes

New Book on Stokes

Navier-Stokes Equations

Campbell-Stokes Sunshine Recorder

Stokes and the Royal Society

Weather Forecasting Today

Finis

George Gabriel Stokes, 1819–1903

George Gabriel Stokes,
founder of modern hydrodynamics.

Childhood and Education

George Gabriel Stokes was born in Skreen,
Co. Sligo on 13 August 1819 [200 years ago].

He was the youngest of seven children of Rev. Gabriel Stokes, Rector of the Church of Ireland.

From an early age, Stokes showed signs of brilliance:

His school-teacher wrote that
"Master George was working out new ways of doing sums, far better than those given in the book."

The ‘Old’ Rectory at Skreen

Descendants of Gabriel Stokes (1682–1768), Great-grandfather of GGS

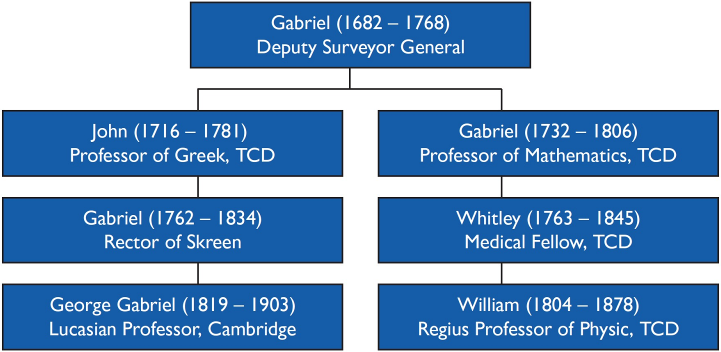

Fig. 1.2 The academic descendants of Gabriel Stokes (1682–1768).

Childhood and Education

- Educated in Skreen, Dublin and Bristol.
- 1837: Pembroke College in Cambridge.
- 1841: Graduated as Senior Wrangler.
  First place in Mathematical Tripos.
  Winner of the Smith's Prize.

Childhood and Education

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Success in Tripos a passport to a great career:

A relative wrote that Stokes had only
"…to decide whether he would be Prime Minister, Lord Chancellor or Archbishop of Canterbury."

Stokes as Senior Wrangler (1841)

Career

1841: Elected a Fellow of Pembroke College.

1849: Appointed Lucasian Professor of Mathematics.

Stokes held this chair for over fifty years.

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The Lucasian Chair of Mathematics

Table of Lucasian Professors of Mathematics

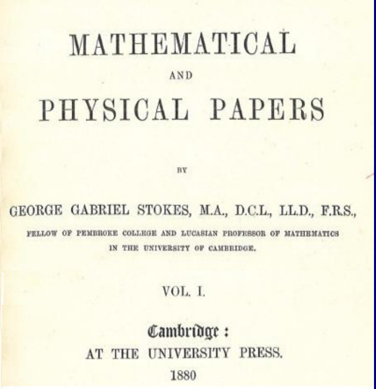
Isaac Barrow	1663–9	Charles Babbage	1828–39
Isaac Newton	1669–1702	Joshua King	1839–49
William Whiston	1702–10	**George Gabriel Stokes**	**1849–1903**
Nicholas Saunderson	1711–39	Joseph Larmor	1903–32
John Colson	1739–60	Paul Dirac	1932–69
Edward Waring	1760–98	James Lighthill	1969–80
Isaac Milner	1798–1820	Stephen Hawking	1980–2009
Robert Woodhouse	1820–2	Brian Green	2009–15
Thomas Turton	1822–6	Michael Cates	2015–
George Airy	1826–8		

Stokes's Wife Mary

In 1859 Stokes married Mary Susannah, daughter of Thomas Romney Robinson, astronomer at Armagh Observatory.

George and Mary had five children.

Mathematical & Physical Papers



MATHEMATICAL

AND

PHYSICAL PAPERS

BY

GEORGE GABRIEL STOKES, M.A., D.C.L., LL.D., F.R.S.,

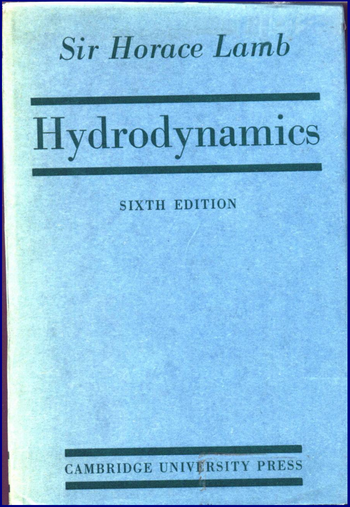
FELLOW OF PEMBROKE COLLEGE AND LUCASIAN PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. I.

Cambridge:
AT THE UNIVERSITY PRESS.
1880

Stokes's Collected Works, in 5 volumes, include some 140 papers.

A Crude but indicative Metric



In his book *Hydrodynamics*, (6th edition), Horace Lamb has more than 50 page references to Stokes.

A meaningful H-Factor?

Some Contributions of Stokes

- Stokes' Theorem
- Stokes Drag
- Stokes' Law
- Stokes Drift
- Stokes Waves
- Stokes Parameters
- Stokes Phenomenon
- Campbell-Stokes Sunshine Recorder
- The Navier-Stokes Equations

Stokes' Theorem

$$\oint_{\Gamma} \mathbf{V}\cdot d\mathbf{l} = \iint_{\Sigma} \nabla \times \mathbf{V} \cdot \mathbf{n}\, da\,.$$

Stokes' Theorem was actually discovered by Kelvin in 1854. It's of central importance in fluid dynamics.

It played a rôle in the development of V. Bjerknes' Circulation Theorem:

$$\frac{dC}{dt} = -\iint_{\Sigma} \nabla\frac{1}{\rho} \times \nabla p\cdot d\mathbf{a} = -\oint_{\Gamma} \frac{dp}{\rho}\,,$$

which generalized Kelvin's Circulation Theorem to baroclinic fluids (ρ varying independently of p), and ushered in the study of Geophysical Fluid Dynamics.

Stokes Drag and Stokes' Law

A Child's Query:

Son: **Daddy, why don't clouds fall down?**

Dad: **Clouds do fall, but very slowly!**

Stokes Drag and Stokes' Law

A Child's Query:

Son: **Daddy, why don't clouds fall down?**

Dad: **Clouds do fall, but very slowly!**

Stokes formulated the drag law for small particles:

$$F = 6\pi\mu r v$$

This leads an expression for the terminal velocity:

$$v_s = \frac{2r^2\rho g}{9\mu}$$

A droplet of radius 5 microns falls with a terminal speed of about 3 mm/s (about four days for 1 km).

Stokes Flow

Stokes Flow is steady flow in which there is a balance between the viscous and pressure gradient forces:

$$\nu\nabla^2\mathbf{V} = \frac{1}{\rho}\nabla p\,.$$

This balance may be valid for small Reynolds Number.

This balance leads to **Stokes' Paradox**: Such flow is not possible everywhere. The effect of an obstacle is felt at large distances: inertial terms are important.

Hydrodynamics: A study in Logic, Fact and Similitude. By Garrett Birkhoff

Chapter 1 of the book is entitled

HYDRODYNAMICAL PARADOXES.

By a *Paradox*, we mean a plausible argument that yields conclusions at variance with observations.

In fluid systems paradoxes often arise because:

- Arbitrarily small causes can produce finite effects
- An apparent symmetry of causes is not necessarily preserved in the effects.

Some Paradoxes in Hydrodynamics

- D'Alembert's Paradox
- The Reversibility Paradox
- Paradoxes of Airfoil Theory
- The Rayleigh Paradox
- Von Neumann's Paradox
- Kopal's Paradox
- The Eiffel Paradox
- The Rising Bubble Paradox
- The Magnus Effect Paradox
- Stokes' Paradox

Euler's Equations

Leonhard Euler, born on 15 April, 1707 in Basel. Died on 18 September, 1783 in St Petersburg.

Euler formulated the equations for incompressible, inviscid fluid flow:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \mathbf{g} \,.$$

$$\nabla \cdot \mathbf{V} = 0$$

Jean Le Rond d'Alembert

A body moving at constant speed through a gas or a fluid **does not experience any resistance** (d'Al. 1752).

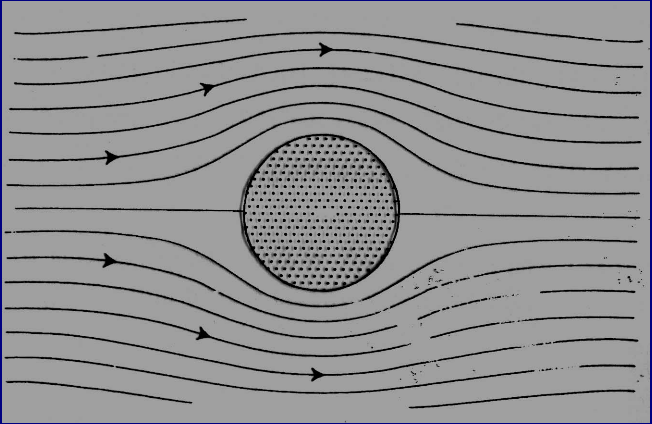
Jean Le Rond d'Alembert

D'Alembert expressed his concerns thus:

"I do not see how one can satisfactorily explain, by theory, the resistance of fluids."

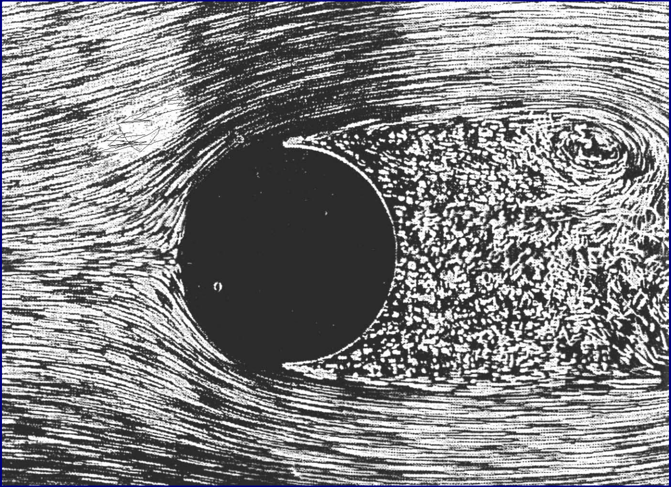
He remarked that the theory leads to "a singular paradox which I leave to future geometers for elucidation."

Hypothetical Fluid Flow



Purely Inviscid Flow.
Upstream-downstream symmetry.

Actual Fluid Flow



Viscous Flow.

Strong upstream-downstream assymmetry.

Resolution of d'Alembert's Paradox

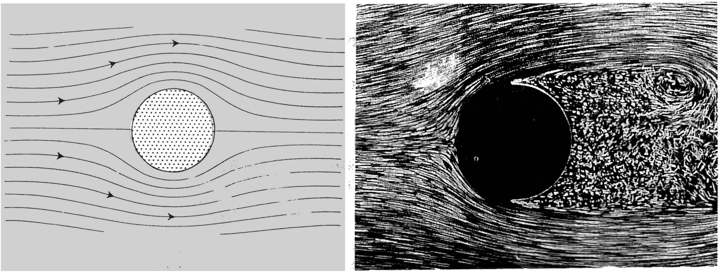


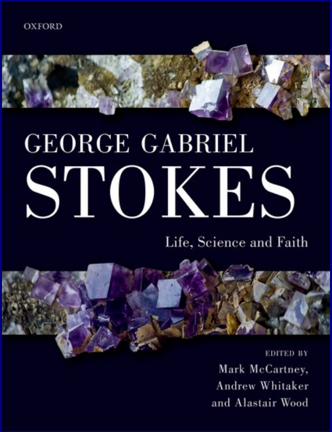
Fig. 9.1 Flow past a circular cylinder for (a) a hypothetical fluid with zero viscosity, (b) a real fluid with very small viscosity μ (from van Dyke 1982).

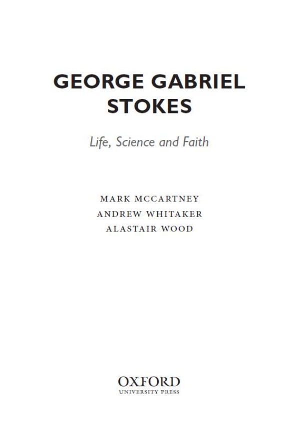
The minutest amount of viscosity has a profound qualitative impact on the character of the solution.

The N-S equations incorporate the effect of viscosity.

Outline

Stokes: Life, Science and Faith




GEORGE GABRIEL STOKES

Life, Science and Faith

MARK MCCARTNEY
ANDREW WHITAKER
ALASTAIR WOOD

OXFORD
UNIVERSITY PRESS

George Gabriel Stokes: Life, Science and Faith.
Eds. Mark McCartney, Andrew Whitaker, and Alastair Wood,
Oxford University Press (2019). ISBN: 978-0-1988-2286-8

Stokes: Life, Science and Faith

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C. L. M. H. Navier, 1785–1836

Claude Louis Marie Henri Navier

See *Notices of the American Mathematical Society*, Vol 50, 7– 13 (Jan. 2003).

Article on Navier's collapsing bridge.

Basic Publications and Review

Navier, C. L. M. H., 1822:
Mémoire sur les lois du mouvement des fluides.
Mém. Acad. Sci. Inst. France, **Vol. 6, 389–440.**

Stokes, G. G., 1845:
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Between Hydrodynamics and Elasticity Theory:
The First Five Births of the Navier-Stokes Equation.
Olivier Darrigol, 2002: ***Arch. Hist. Exact Sci.*****,**
Vol. 56, (2), 95–150.

The Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} .$$

The Navier-Stokes Equations describe how the change of velocity (the acceleration) is determined by the pressure gradient force and frictional force.

The Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} \,.$$

The Navier-Stokes Equations describe how the change of velocity (the acceleration) is determined by the pressure gradient force and frictional force.

For motion relative to the rotating earth, we include gravity and the Coriolis force:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\boldsymbol{\Omega} \times \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g} \,.$$

Outline

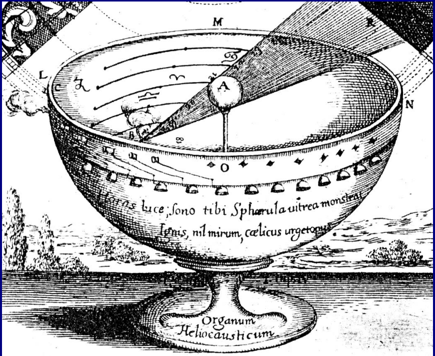
An Early Sunshine Recorder

Athanasius Kircher was Professor of Mathematics and Hebrew at the *Collegio Romano*.

Around 1646 he devised a recording sundial called the Horologium Helio-causticum.

The Horologium Helio-causticum

A Sundial is drawn in the shell, “together with things for burning and making sounds.”



“With light and sound the glassy sphere shows thee the hours; truly, it is the work of the heavenly fire.”

Campbell’s Sunshine Recorder.

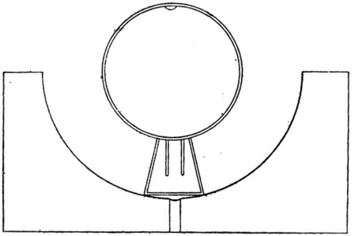


Fig. 1.—Section of Mr. Campbell’s original Sunshine Recorder.

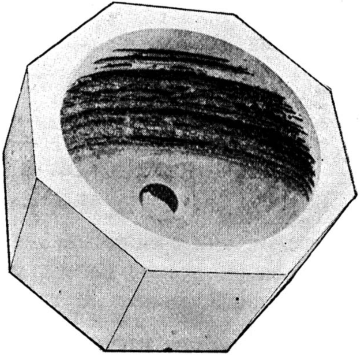
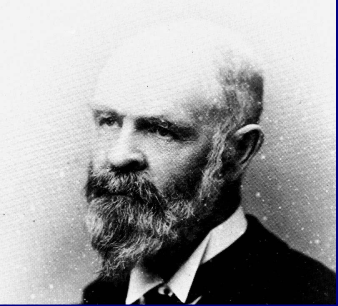


Fig. 2.—Wooden Sunshine Bowl.

The “self-registering sundial” of J. F. Campbell (c. 1853).

Robert Henry Scott (1833–1916)



Robert Scott, born in Dublin, was founder of Valentia Observatory and first Director of the British Met Office.

Scott proposed some improvements to Campbell's sunshine recorder.

The detailed design was due to G. G. Stokes.

Stokes' Quarterly Journal Paper

Description of the Card Supporter for Sunshine Recorders adopted at the Meteorological Office

George Gabriel Stokes

Quarterly Journal of the Royal Meteorological Society, **Vol. 6 (1880) 83–94.**

"The method of recording sunshine by the burning of an object placed in the focus of a glass sphere freely exposed to the rays of the sun, which was devised by Mr. Campbell, commends itself by its simplicity, and seems likely to come into pretty general use."

Stokes' Quarterly Journal Paper

Description of the Card Supporter for Sunshine Recorders adopted at the Meteorological Office

George Gabriel Stokes

Quarterly Journal of the Royal Meteorological Society, **Vol. 6 (1880) 83–94.**

In the discussion following the reading of the paper, a Mr. Mawley remarked:

"The fact of this sunshine-recorder being in all respects an English invention, adds much to its interest."

Campbell-Stokes Sunshine Recorder

FIGURE 138
Campbell-Stokes sunshine recorder.

No moving parts.

Campbell-Stokes Sunshine Recorder

FIGURE 138
Campbell-Stokes sunshine recorder.

No moving parts.

One moving part!
(In Biblical Coordinates)

Outline

Royal Society

1851: Stokes elected a Fellow of the Royal Society.

(Along with William Thomson, Thomas H. Huxley and John Tyndall.)

1854–1884: Stokes Secretary of the Royal Society

President from 1885 to 1890.

"I am naturally of rather a retiring character,
and should feel not a little out of my element
in being brought so prominently forward."

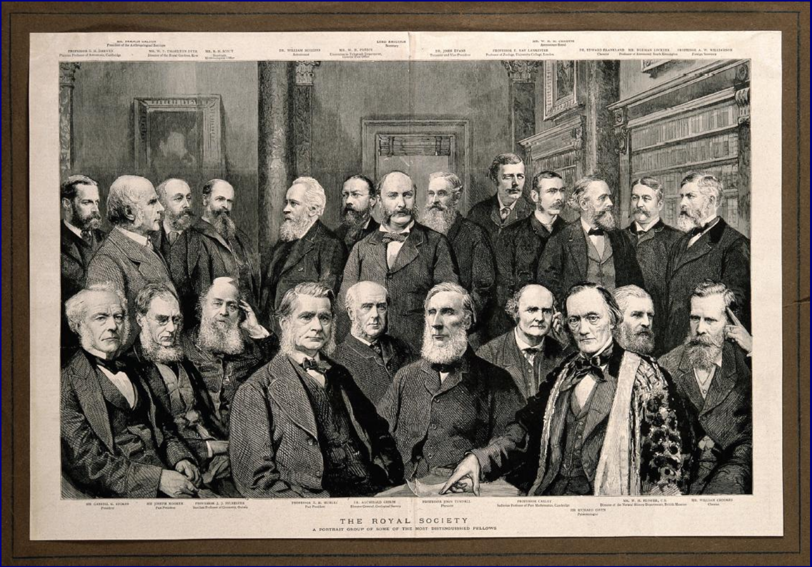
Stokes to Th. R. Robinson.

**T. H. Huxley criticised Stokes for his "ultra-
"conservative and theological viewpoint."**

Stokes as President of Royal Society

President of Royal Society

Prominent Members of the Royal Society



Royal Society

William Thomson followed Stokes as PRS.
Stokes awarded Copley Medal in 1893.

Fig. 8.4 The Royal Society Copley Medal awarded to Stokes in 1893. Image courtesy Nick Lefebvre.

Outline

George Gabriel Stokes

New Book on Stokes

Navier-Stokes Equations

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Weather Forecasting Today

Finis

Equations of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)

Relates the pressure, temperature and density

CONTINUITY EQUATION

Conservation of mass; air not created or distroyed

WATER CONTINUITY EQUATION

Conservation of water (liquid, solid and gas)

EQUATIONS OF MOTION: Navier-Stokes Equations

Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction

THERMODYNAMIC EQUATION

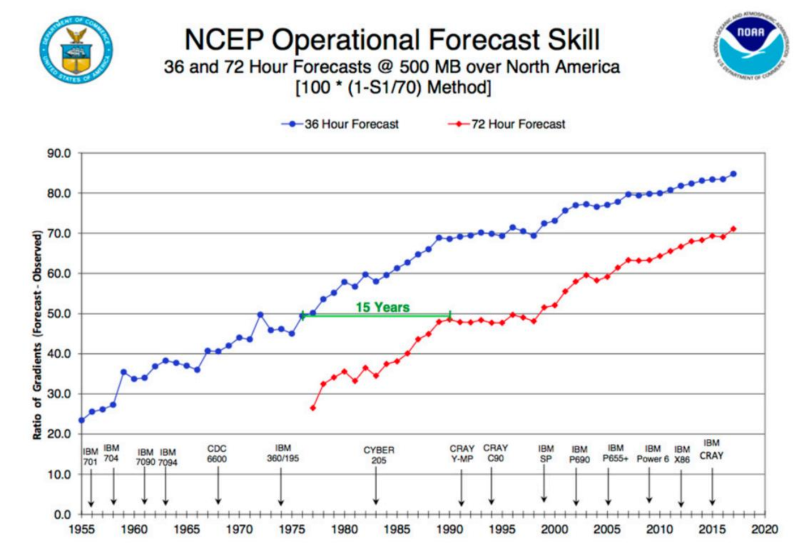
Changes of temperature due to heating, cooling, compression, rarifaction, etc.

Seven equations; seven variables (u, v, w, ρ, p, T, q).

Weather Forecasting in a Nut-Shell

- The atmosphere is a physical system
- Its behaviour is governed by the laws of physics
- These laws are expressed quantitatively in the form of mathematical equations
- Using observations, we can specify the atmospheric state: "Today's Weather"
- Using the equations, we can calculate how this state changes with time: "Tomorrow's Weather"

Long-term Skill Growth



Forecast of Hurricane Sandy

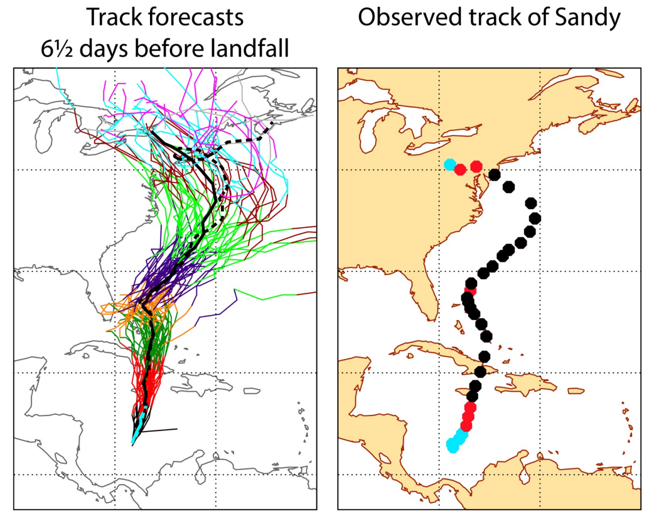


Figure : Landfall, New Jersey, 30 October 2012

Outline

Summary

Stokes never forgot his origins in Skreen.

He returned to Sligo and elsewhere in Ireland regularly for summer vacations.

In one of his heavily mathematical papers he wrote of

"the surf that breaks upon the western coasts as a result of storms out in the Atlantic",

recalling the majestic rollers thundering in as he strolled as a boy along Dunmoran Strand.

Thank you