Richardson's Dream: The Emergence of Scientific Weather Forecasting

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Outline of this Lecture

- *The Pre-history of Numerical Weather Prediction*
- *Richardson's Magnificent Fiasco*
- *An Analysis of what went wrong*
- *The Development of NWP since 1950*
- *Computer Weather Forecasting Today*

Newton's Law of Motion

The rate of change of momentum of a body is equal to the sum of the forces acting on the body.

If $\mathbf{F}$ is the total applied force, Newton's Second Law gives

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}\,.$$

The acceleration $\mathbf{a}$ is the rate of change of velocity, that is, $\mathbf{a} = d\mathbf{V}/dt$. If the mass m is constant, we have

$$\mathbf{F} = m\mathbf{a}\,.$$

$$\textbf{Force} = \textbf{Mass} \times \textbf{Acceleration}\,.$$

Edmund Halley (1656–1742)

Edmund Halley was a contemporary of Isaac Newton and also a friend; *This was quite an achievement*: Newton didn't have too many friends!

Halley was largely responsible for persuading Newton to publish his *Principia Mathematica*.

Halley and his Comet

Halley's analysis of what is now called Halley's comet is an excellent example of the scientific method in action.

As we shall see, there is a *link* between Halley's Comet and Richardson's Forecast.

Vilhelm Bjerknes (1862–1951)

Bjerknes' 1904 Manifesto

Objective:

To establish a science of meteorology

Acid test:

To predict future states of the atmosphere.

Necessary and sufficient conditions for the solution of the forecasting problem:

1. **A knowledge of the initial state of the atmosphere**
2. **A knowledge of the physical laws which determine the evolution of the atmosphere.**

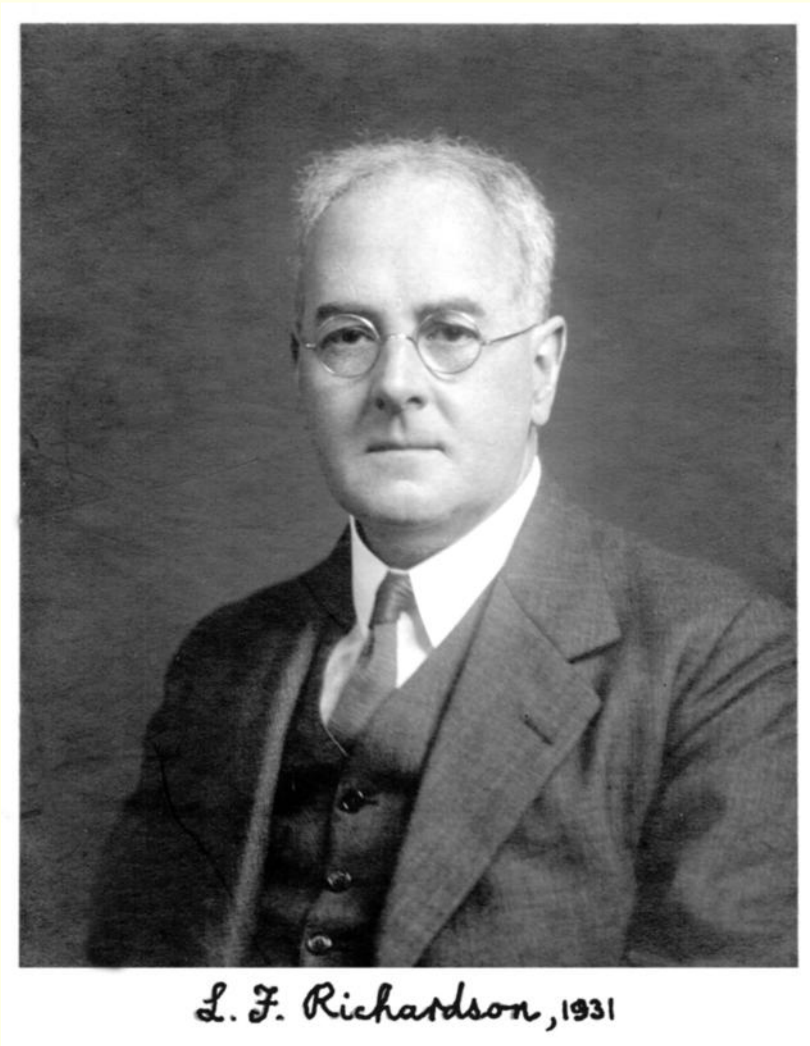
Step (1) is Diagnostic. **Step (2) is Prognostic.**

Scientific Weather Forecasting in a Nut-Shell

- **The atmosphere is a physical system**
- **Its behaviour is governed by the laws of physics**
- **These laws are expressed as mathematical equations**
- **Using observations, we determine the atmospheric state at a given initial time: "Today's Weather"**
- **Using the equations, we calculate how this state changes over time: "Tomorrow's Weather"**

- **The equations are very complicated (non-linear) and a powerful computer is required to do the calculations**
- **The accuracy decreases as the range increases; there is an inherent limit of predictibility.**

Lewis Fry Richardson, 1881–1953.



During WWI, Richardson computed by hand the pressure change at a single point.

It took him two years !

His 'forecast' was a catastrophic failure:

$$\Delta p = \mathbf{145\ hPa\ in\ 6\ hours}$$

His method was unimpeachable.

So, *what went wrong?*

Max Margules (1856–1920)

In 1904, Margules published a paper in the *Festschrift* marking the sixtieth birthday of his teacher Ludwig Boltzmann:

Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung.

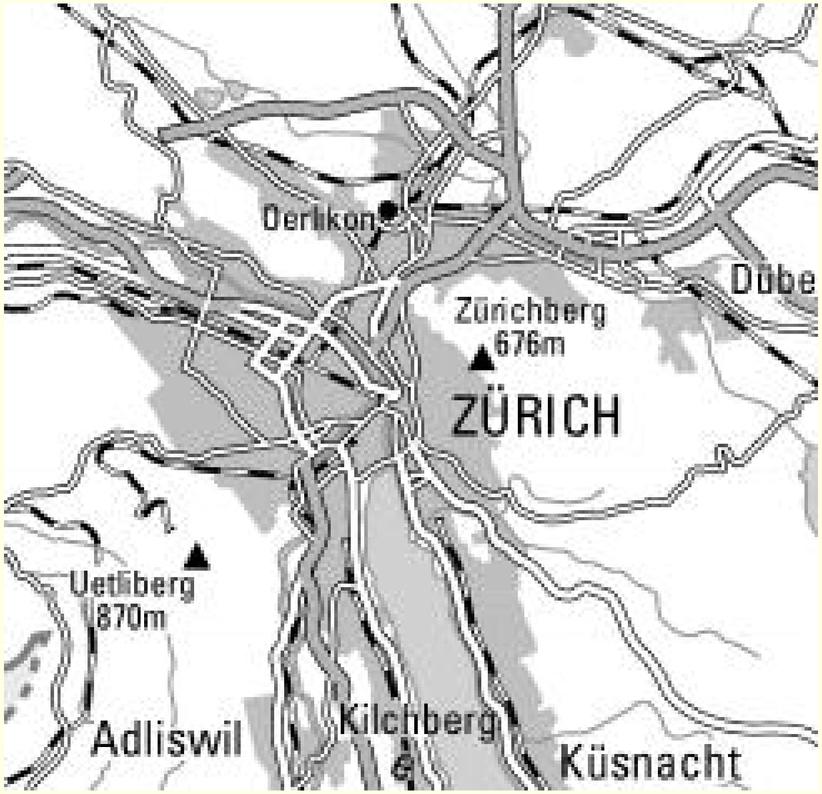
"On the Relationship between Barometric Variations and the Continuity Equation."

- ***Margules examined pressure changes predicted using the <u>continuity equation</u>.***
- ***He found that, to obtain a realistic pressure tendency, the winds must be known to an <u>unrealistic precision</u>.***
- ***He showed that synoptic forecasting by this means was <u>doomed to failure</u>.***

According to Fortak (2001), Margules was convinced that weather forecasting was:

"immoral and damaging to the character of a meteorologist."

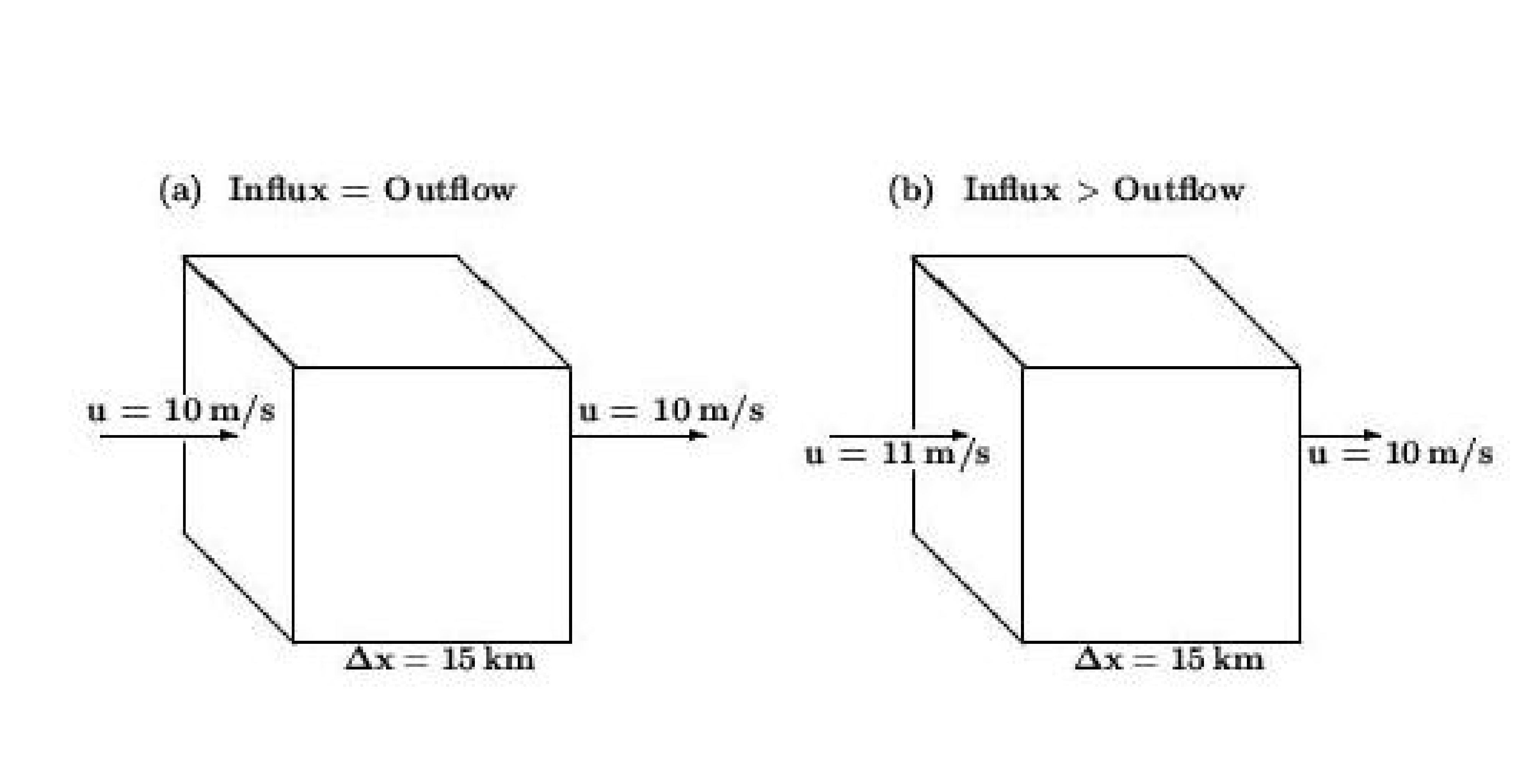
Tendency *via* Continuity Equation



The environs of Zürich:

- **A square of side ∼15km.**
- **Analogous to a cell of a finite difference model of the atmosphere.**

A Box of Air over Zürich



Influx equals Outflow:
Pressure remains unchanged.

Influx exceeds Outflow:
Pressure will rise.

Pressure Tendency

Assume a westerly wind over Zürich

$$u > 0\,, \qquad\qquad v = 0\,.$$

Assume also that the surface pressure is initially 1000 hPa.

Using Conservation of Mass, a simple *back-of-the-envelope* calculation yields the following amazing result:

- **If the speed on the western side *exceeds* that on the east by *just* 1 m/s, the pressure tendency is about 7 Pa/s.**
- **If this influx continues, the pressure will *double* in about 4 hours.**

Conclusion:

We must apply the Continuity Equation with great care.

Felix Maria Exner (1876–1930)

A first attempt at calculating synoptic changes using physical principles was made by Felix Exner, working in Vienna.

Exner followed a radically different line from Bjerknes.

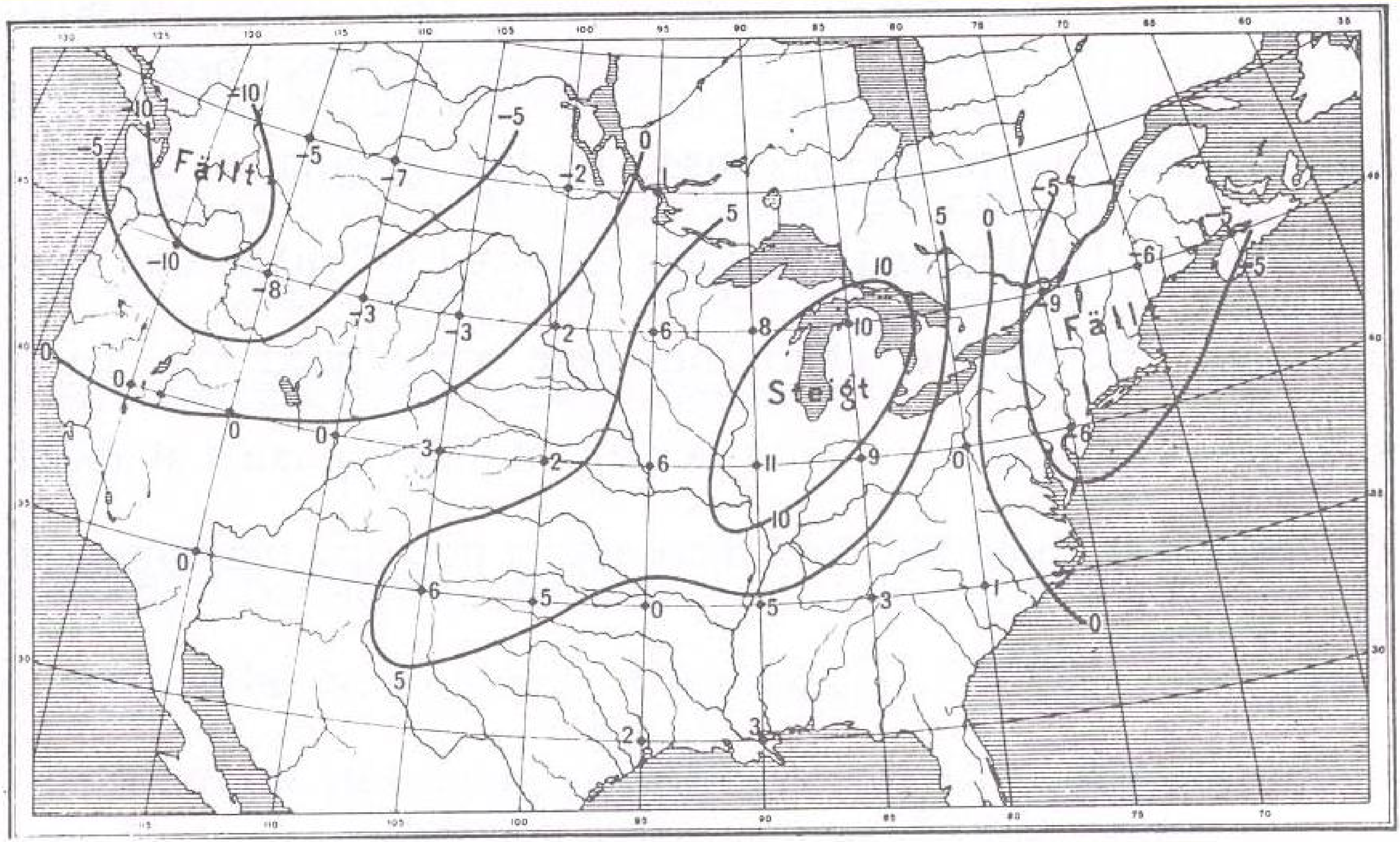
He did *not* make direct use of the continuity equation.

His method was based on a system reduced to the essentials.

Exner's Method

- ***Exner assumed geostrophic balance and thermal forcing constant in time.***
- ***He deduced a mean zonal wind from temperature observations.***
- ***He derived a prediction equation for advection of the pressure pattern with constant speed, modified by heating.***
- ***His method yielded a realistic forecast in the case illustrated in his paper.***

Exner's Forecast

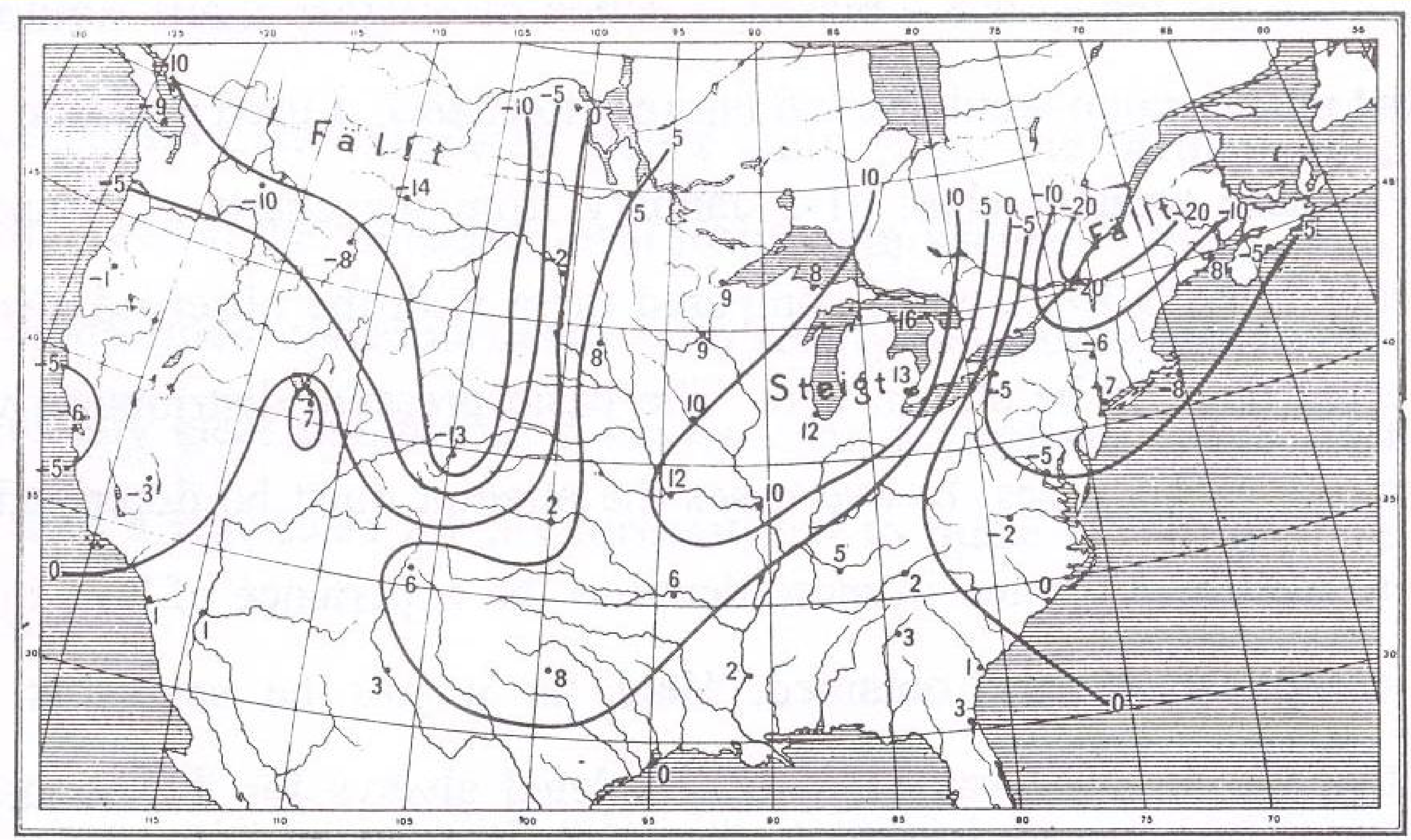


Calculated Pressure Change

between 8pm and 12pm on 3 January, 1895

Hundreths of an inch. [*Steigt*=rises; Fällt=falls].

Verification



Observed Pressure Change
between 8pm and 12pm on 3 January, 1895

Hundreths of an inch. [*Steigt*=rises; Fällt=falls].

Richardson's Reaction

Exner's work deserves attention as a first attempt at systematic, scientific weather forecasting.

The only reference by Richardson to the method was a single sentence in his book *Weather Prediction by Numerical Process* (p. 43):

"F. M. Exner has published a prognostic method based on the source of air supply."

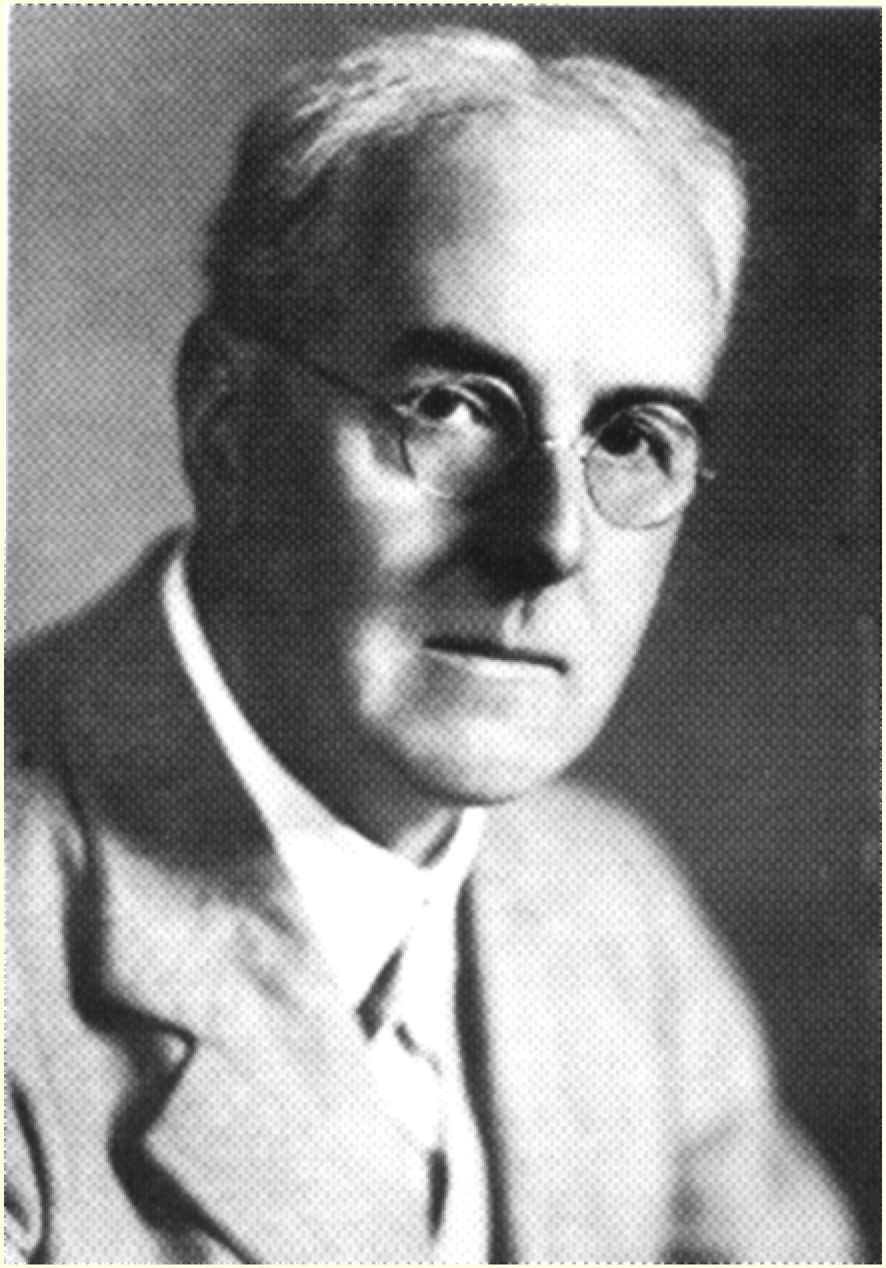
It would appear from this that Richardson was not particularly impressed by it!

⋆ ⋆ ⋆

It is noteworthy that

- **Exner's forecast was unspectacular but reasonable.**
- **Richardson's forecast was spectacularly unreasonable.**

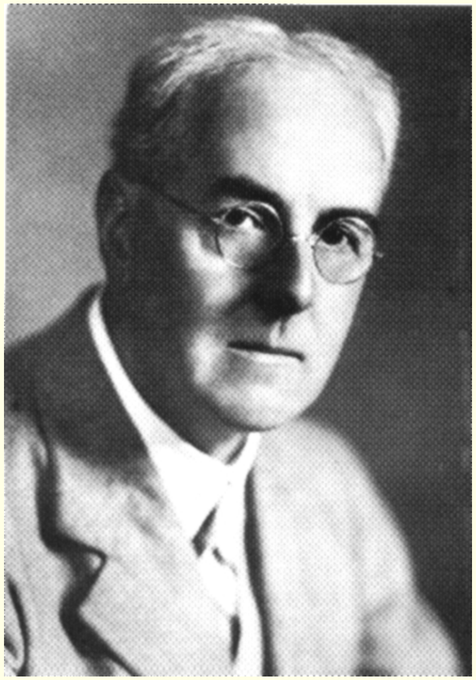
Lewis Fry Richardson, 1881–1953.



Bjerknes proposed graphical methods for the solution of the forecasting problem

Richardson was bolder — or perhaps more foolhardy — than Bjerknes.

He attempted a bulldozer approach, calculating changes from the full PDEs.



- Born, 11 October, 1881, Newcastle-upon-Tyne
- Family background: well-known quaker family
- 1900–1904: Kings College, Cambridge
- 1913–1916: Met. Office. Superintendent, Eskdalemuir Observatory
- Resigned from Met Office in May, 1916. Joined Friends' Ambulance Unit.
- 1919: Re-employed by Met. Office
- 1920: M.O. linked to the Air Ministry. LFR Resigned, on grounds of concience
- **1922:** *Weather Prediction by Numerical Process*
- 1926: Break with Meteorology. Worked on Psychometric Studies. Later on Mathematical causes of Warfare
- 1940: Resigned to pursue "peace studies"
- Died, September, 1953.

Richardson contributed to Meteorology, Numerical Analysis, Fractals, Psychology and Conflict Resolution.

Eskdalemuir Observatory in 1911

(where Richardson's dream began to take shape)

Chapters of Richardson's Book

Table 1.1. *Chapter titles of* Weather Prediction by Numerical Process.

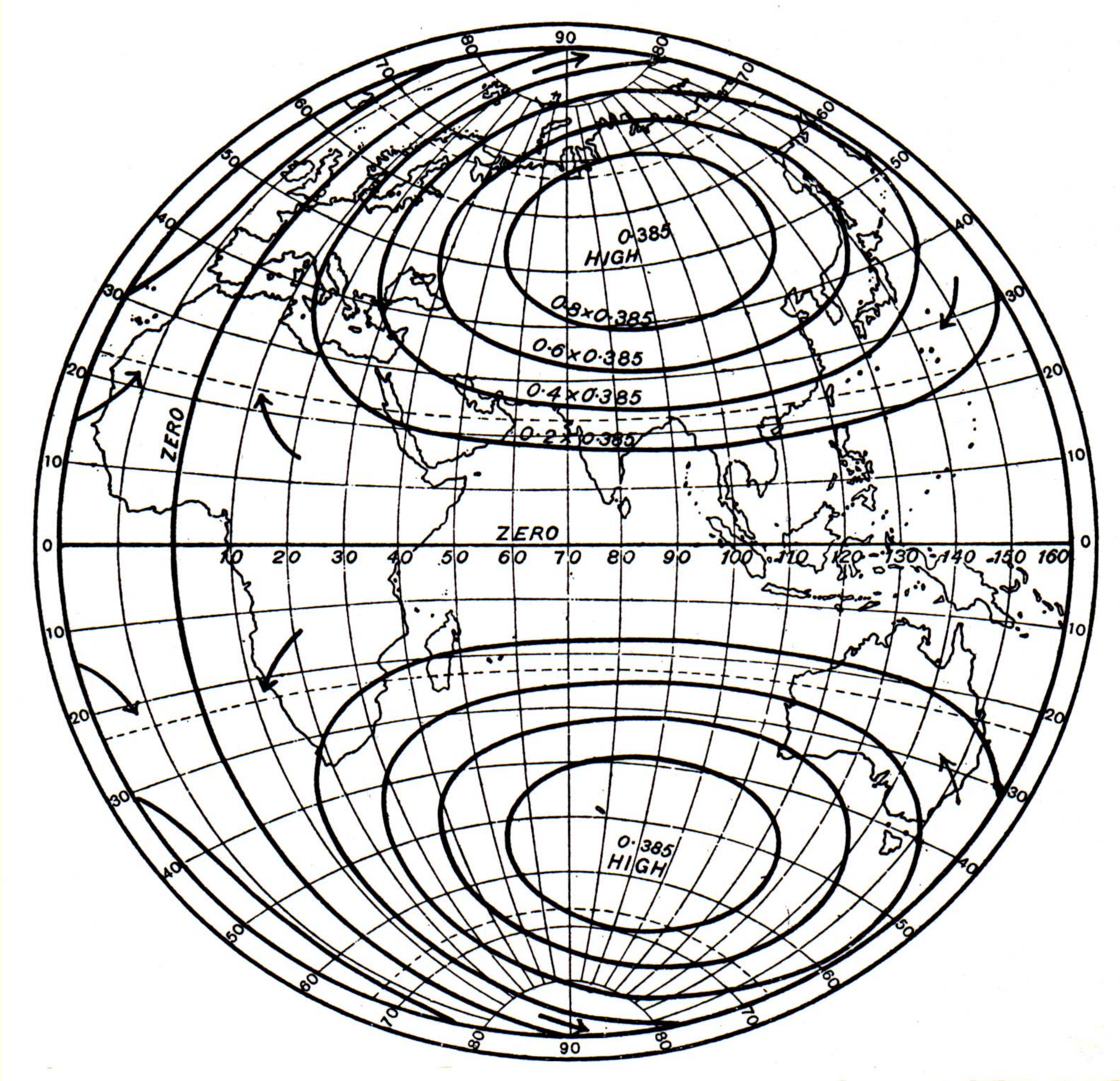
Chapter 1	Summary
Chapter 2	Introductory Example
Chapter 3	The Choice of Coordinate Differences
Chapter 4	The Fundamental Equations
Chapter 5	Finding The Vertical Velocity
Chapter 6	Special Treatment For The Stratosphere
Chapter 7	The Arrangement of Points and Instants
Chapter 8	Review of Operations in Sequence
Chapter 9	An Example Worked on Computing Forms
Chapter 10	Smoothing The Initial Data
Chapter 11	Some Remaining Problems
Chapter 12	Units and Notation

The barotropic forecast

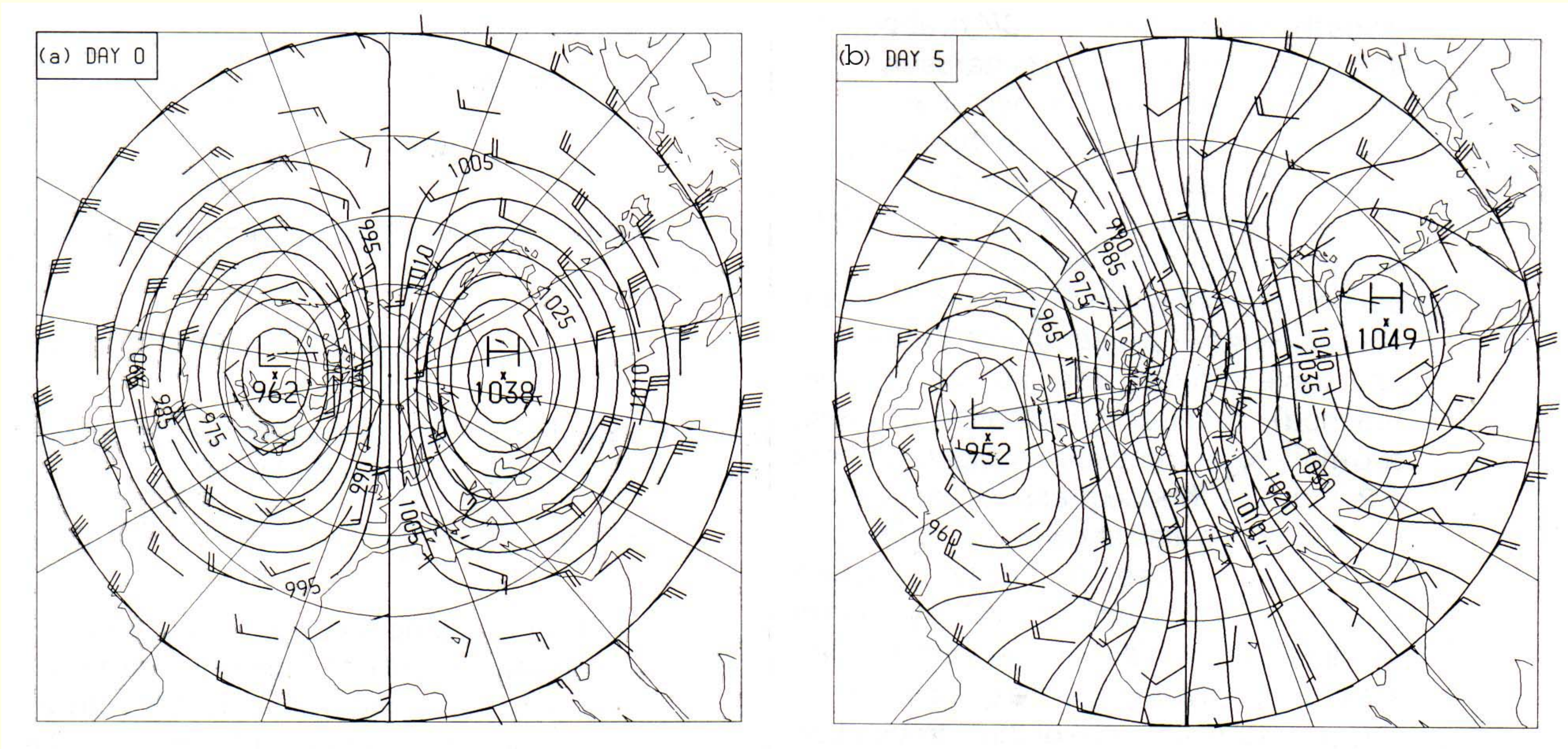
"Before attending to the complexities of the actual atmosphere . . . it may be well to exhibit the working of a much simplified case."

The linear shallow water equations:

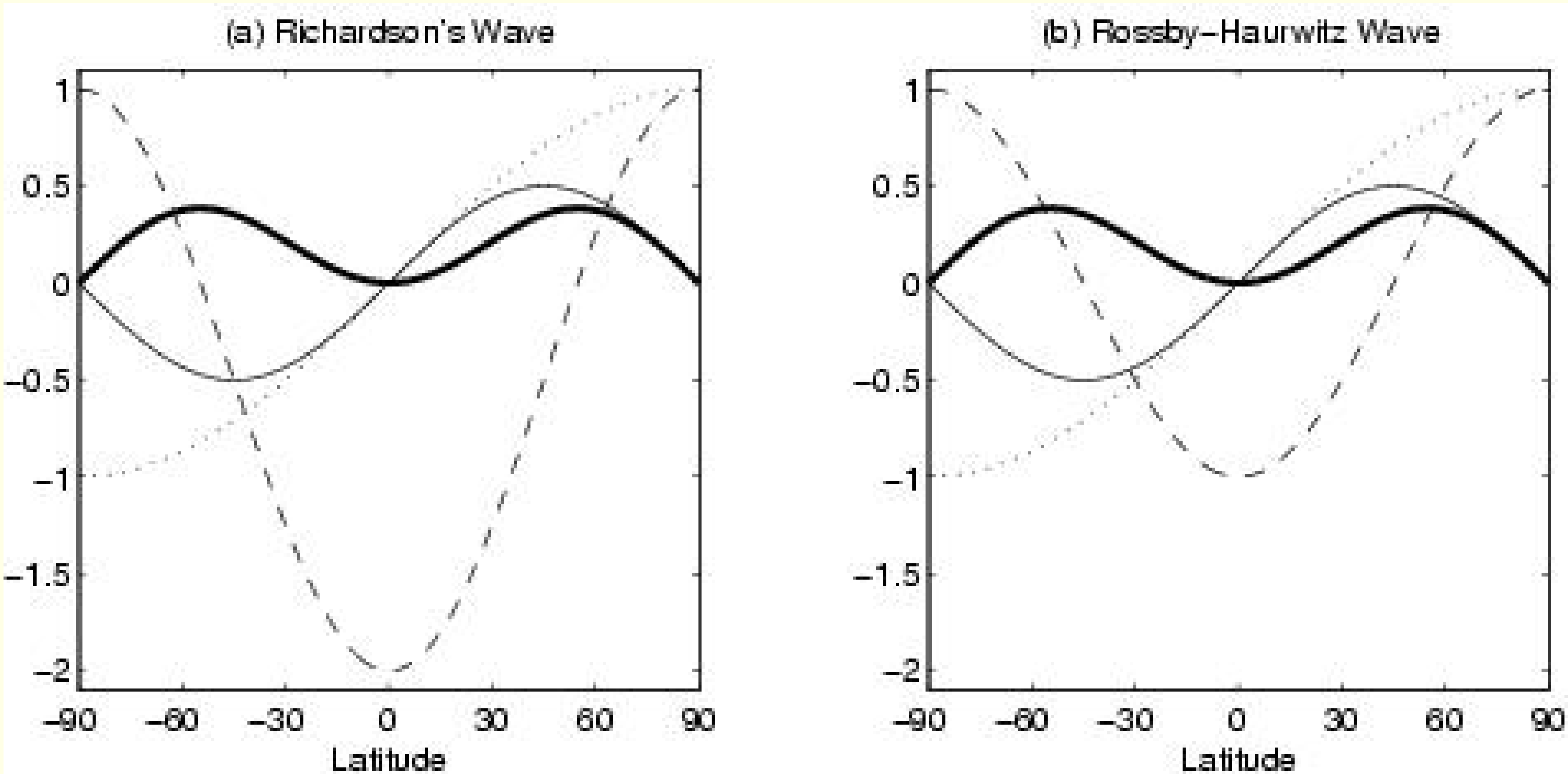
$$\begin{aligned}
\frac{\partial U}{\partial t} - fV + \frac{\partial P}{\partial x} &= 0 \\
\frac{\partial V}{\partial t} + fU + \frac{\partial P}{\partial y} &= 0 \\
\frac{\partial P}{\partial t} + gh\nabla \cdot \mathbf{V} &= 0\,.
\end{aligned}$$



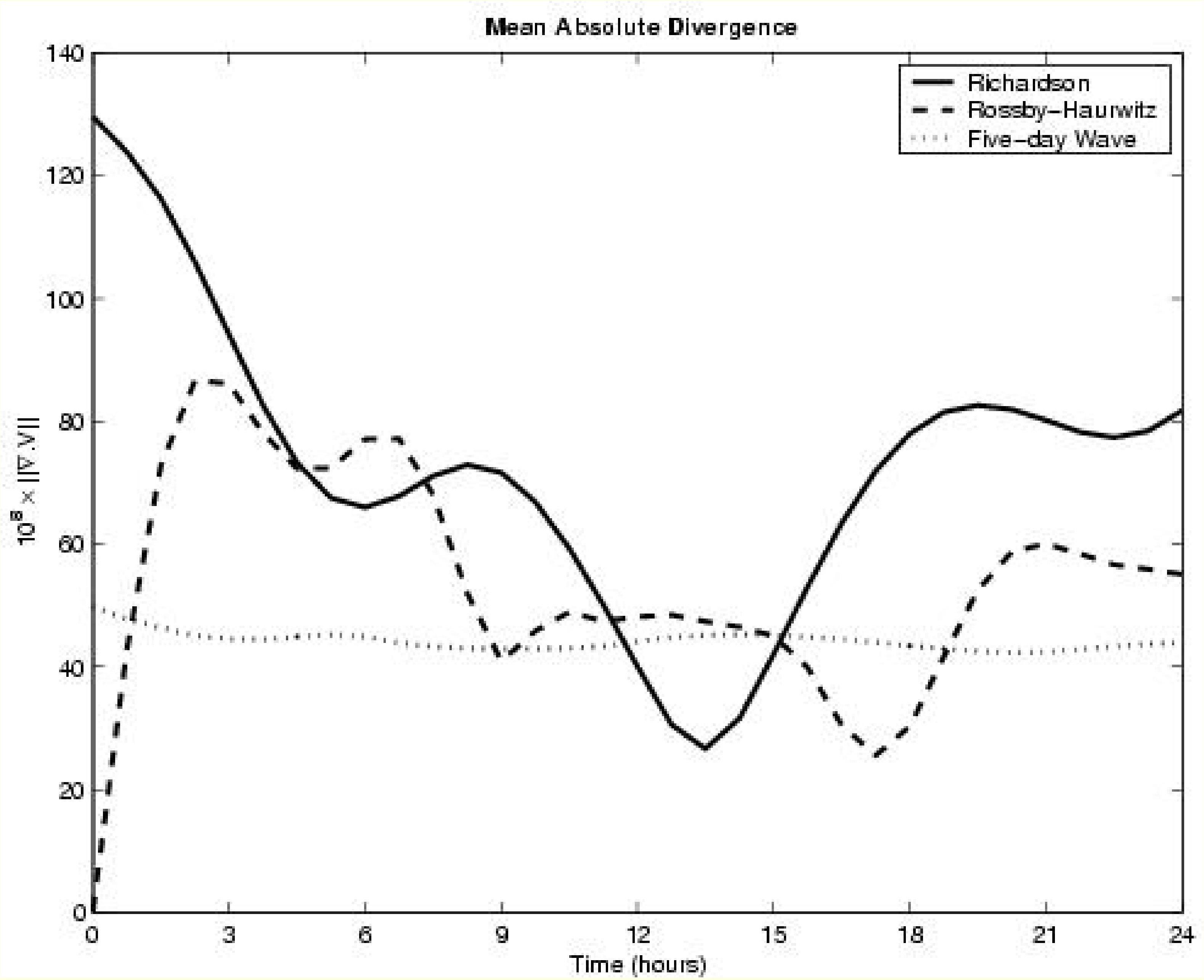
The initial pressure for Richardson's barotropic forecast.



(a) The initial pressure and wind fields in the Northern hemisphere as specified by Richardson.
(b) Forecast pressure and wind valid five days later.



(a) Richardson's Initial Conditions
(b) First symmetric Rossby-Haurwitz wave RH(1,2)
(only the zonal winds differ)



Mean absolute divergence for three 24-hour forecasts.

The Equations of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)

Relates the pressure, temperature and density

CONTINUITY EQUATION

Conservation of mass; air neither created nor distroyed

WATER CONTINUITY EQUATION

Conservation of water (liquid, solid and gas)

EQUATIONS OF MOTION: Navier-Stokes Equations

Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction

THERMODYNAMIC EQUATION

Determines changes of temperature due to heating or cooling, compression or rarifaction, etc.

Seven equations; seven variables (u, v, w, ρ, p, T, q)**.**

The Primitive Equations

$$\frac{du}{dt} - \left(f + \frac{u\tan\phi}{a}\right)v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$

$$\frac{dv}{dt} + \left(f + \frac{u\tan\phi}{a}\right)u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$

$$p = R\rho T$$

$$\frac{\partial p}{\partial y} + g\rho = 0$$

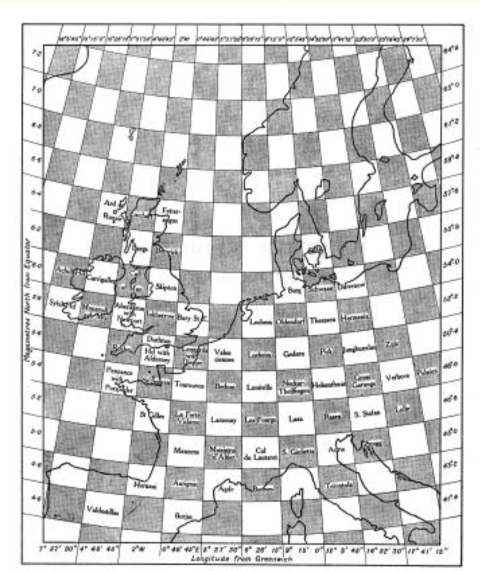
$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$

$$\frac{\partial \rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [\mathrm{Sources} - \mathrm{Sinks}]$$

Seven equations; seven variables $(u, v, w, p, T, \rho, \rho_w)$**.**

The Finite Difference Scheme



The globe is divided into cells, like the checkers of a chess-board.

Spatial derivatives are replaced by finite differences:

$$\frac{df}{dx} \rightarrow \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} .$$

Similarly for time derivatives:

$$\frac{dQ}{dt} \rightarrow \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

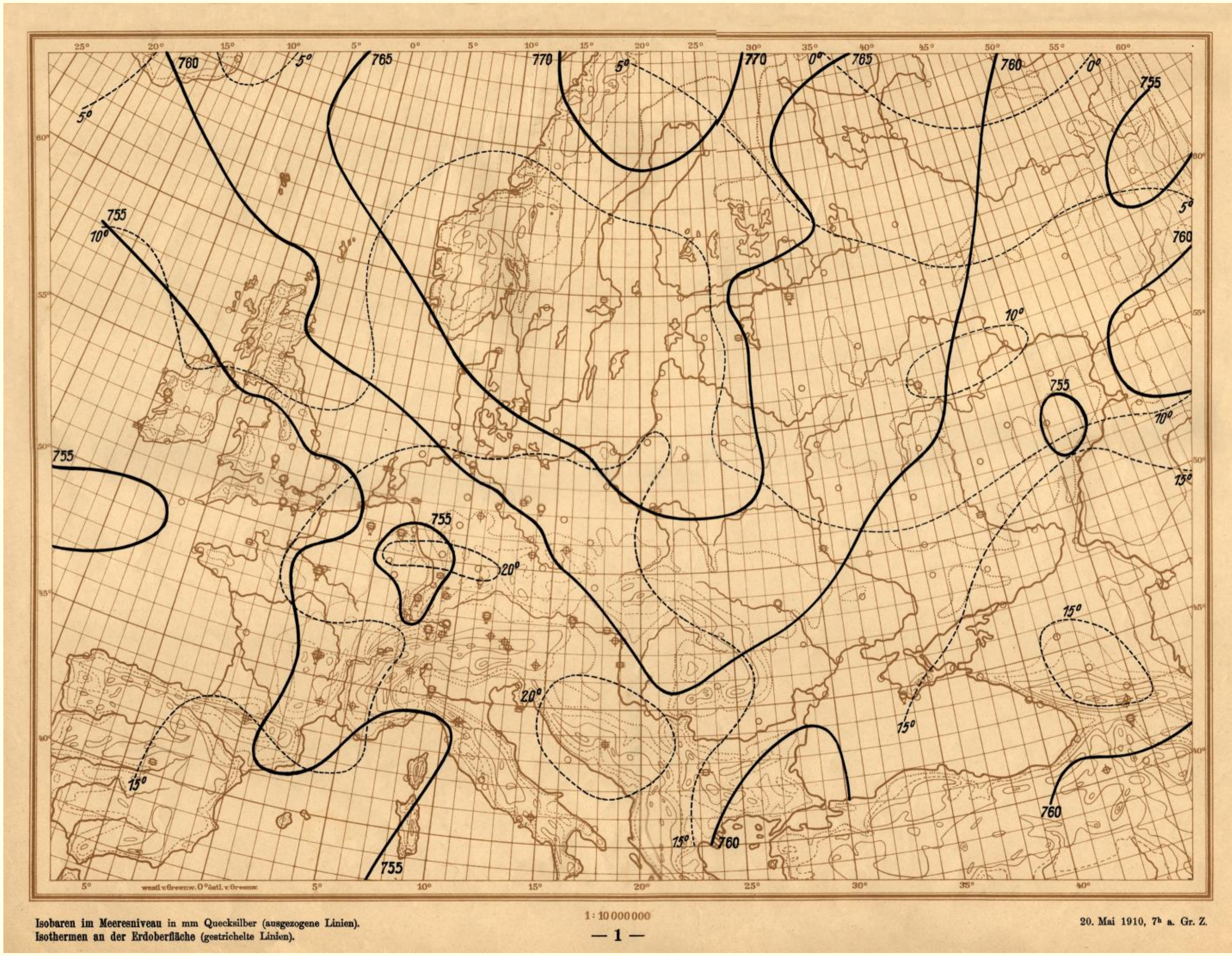
This can immediately be solved for Q^{n+1}:

$$Q^{n+1} = Q^{n-1} + 2\Delta t F^n .$$

By repeating the calculations for many time steps, we can get a forecast of any length.

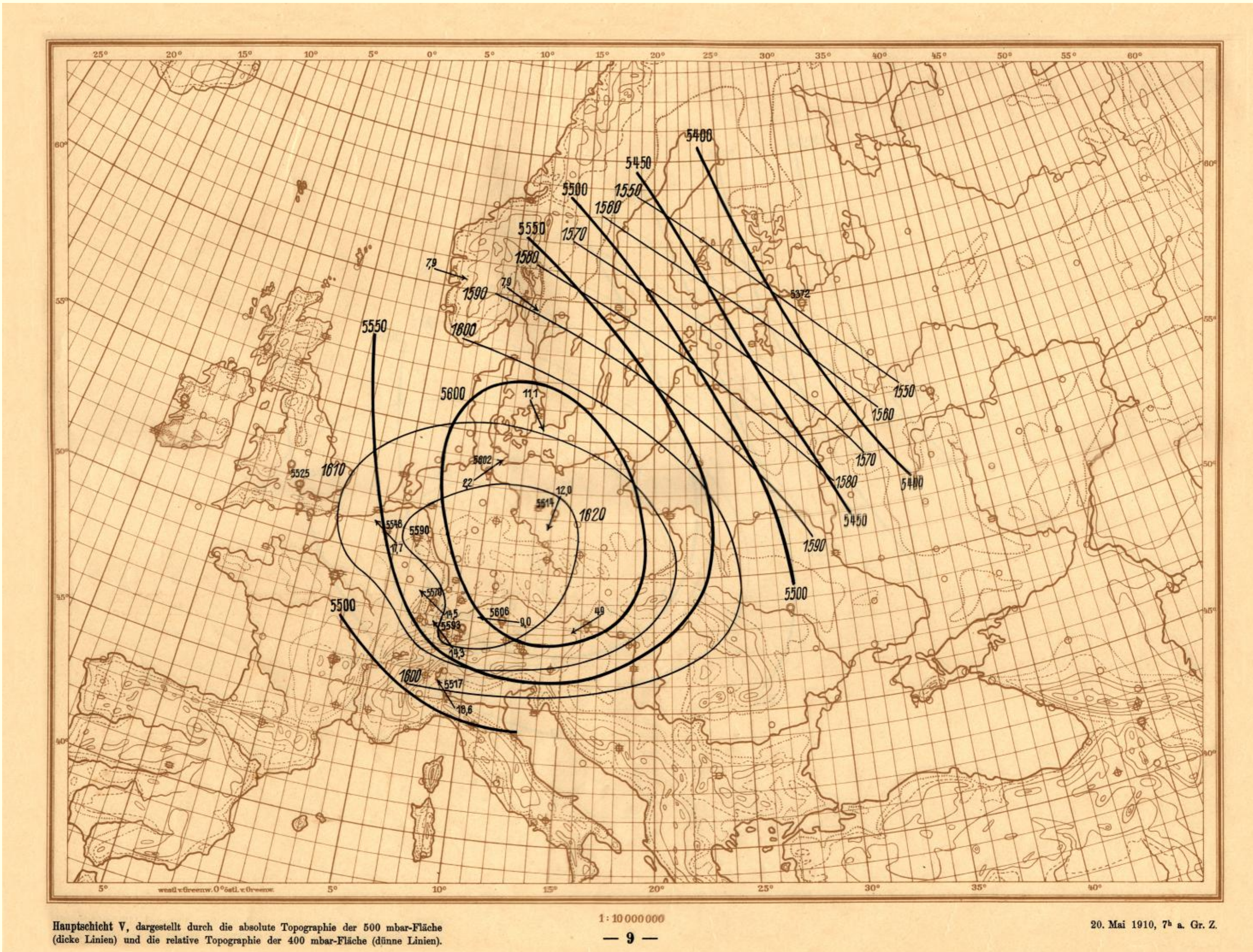
Richardson calculated only the initial rates of change.

The Leipzig Charts for 0700 UTC, May 20, 1910



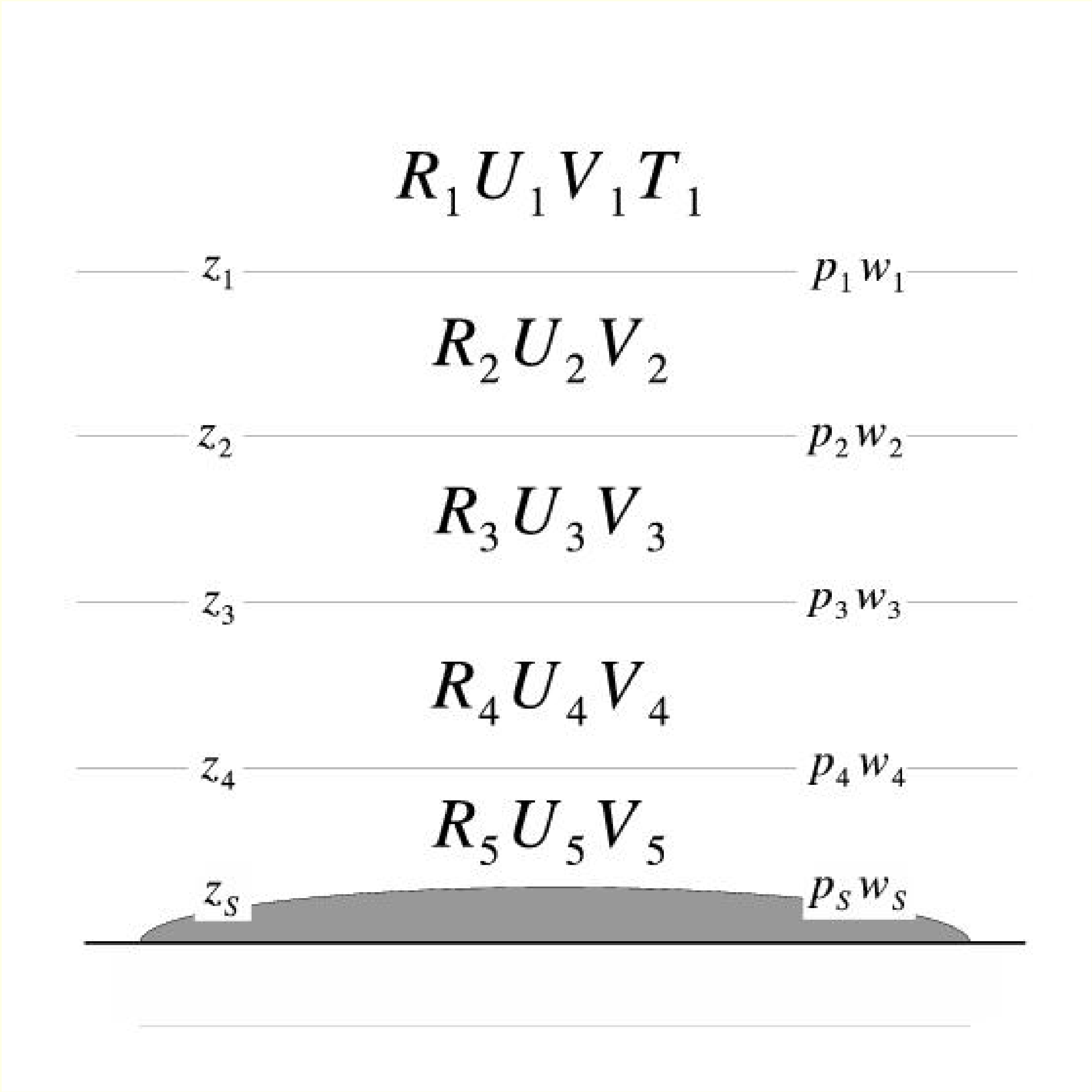
Bjerknes' sea level pressure analysis.

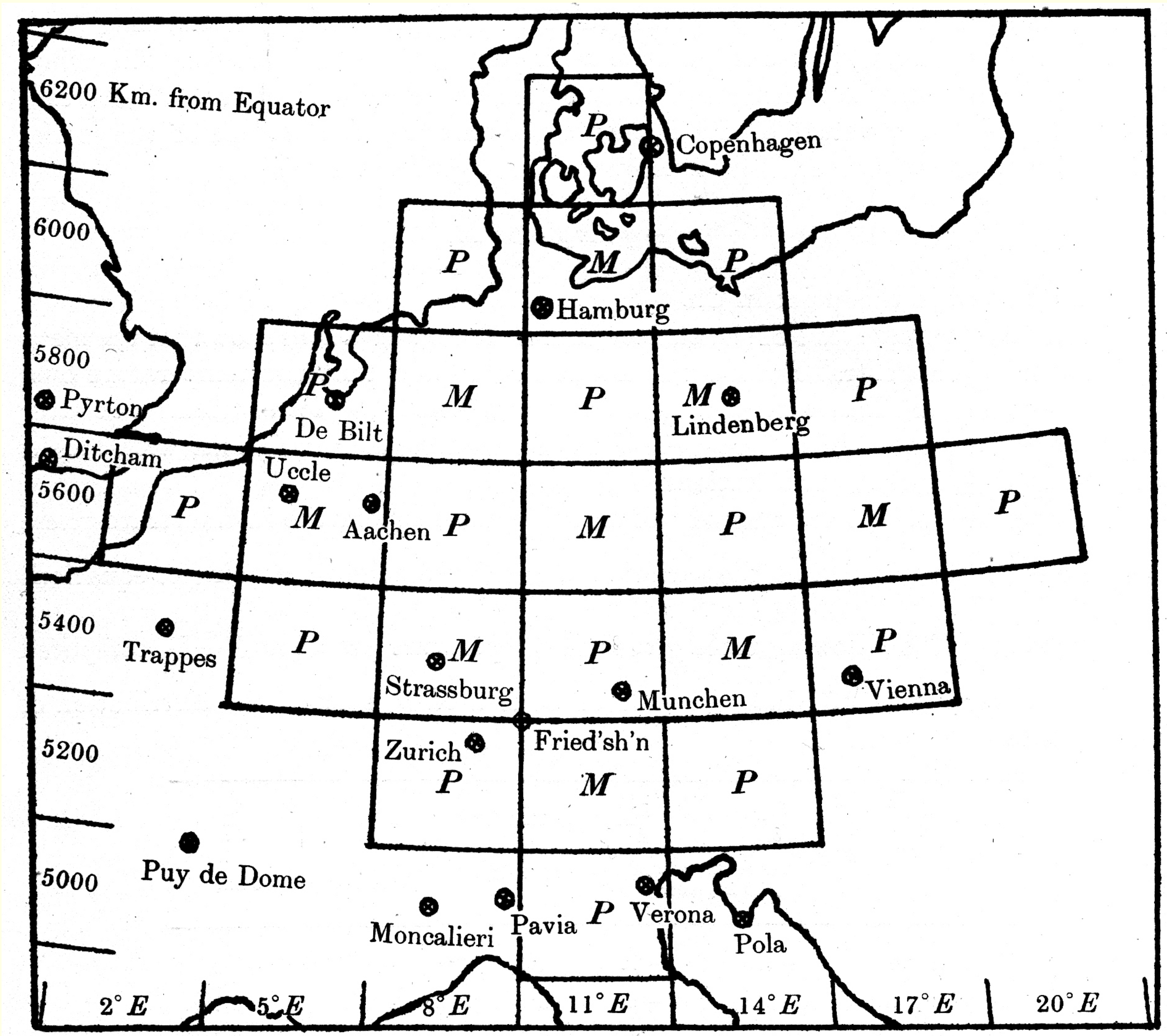
The Leipzig Charts for 0700 UTC, May 20, 1910



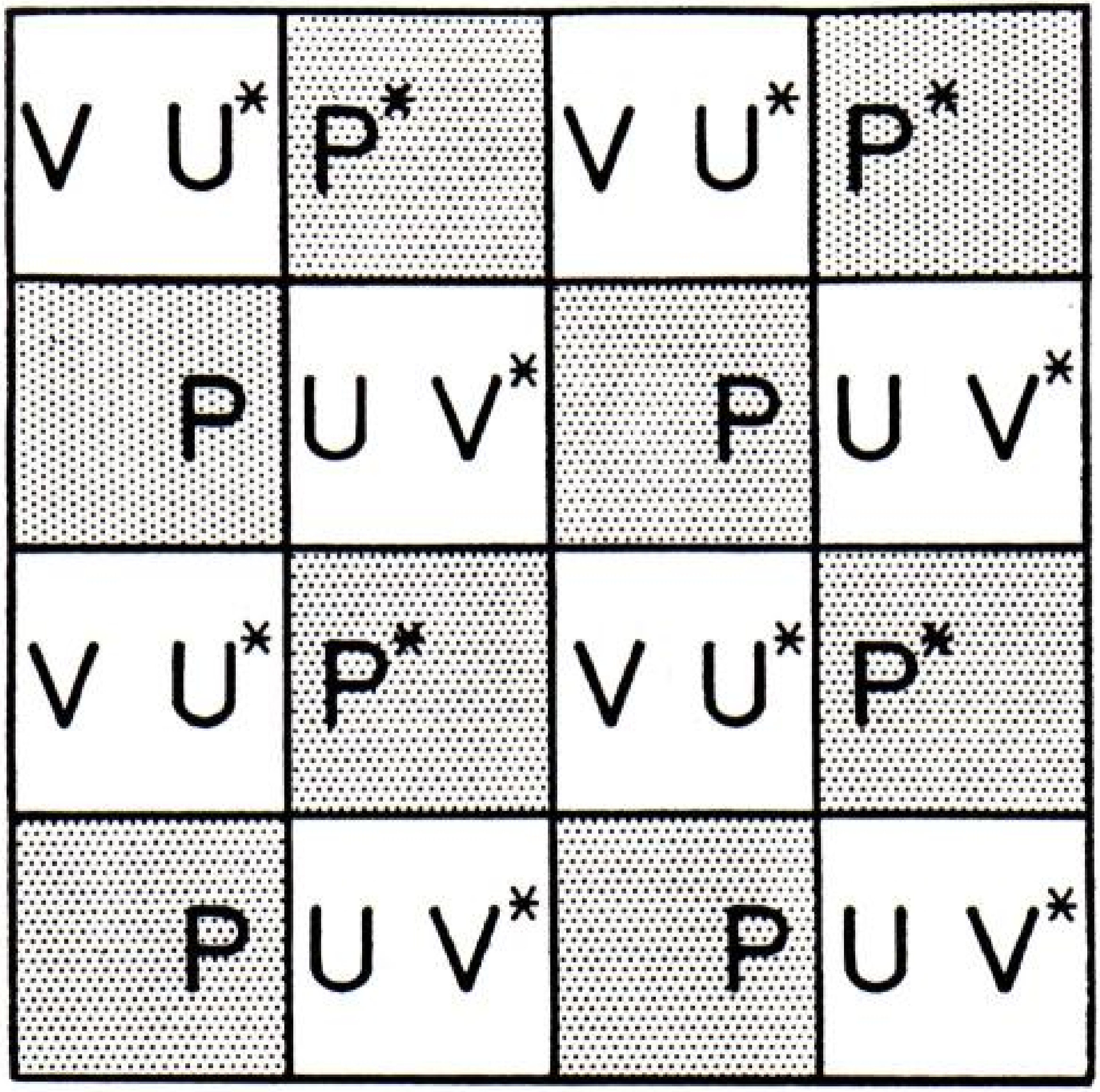
Bjerknes' 500 hPa height analysis.

Richardson's vertical stratification





Grid used by Richardson for his forecast.



Richardson Grid (also called an Arakawa E-grid)

Richardson's *Spread-sheet*

Computing Form P XIII. Divergence of horizontal momentum-per-area. Increase of pressure

The equation is typified by: $-\frac{\partial R_{86}}{\partial t} = \frac{\partial M_{E86}}{\partial e} + \frac{\partial M_{N86}}{\partial n} - M_{N86}\frac{\tan\phi}{a} + m_{H6} - m_{H8}{}^{*} + \frac{2}{a}M_{H86}$. (See Ch. 4/2 #5.)

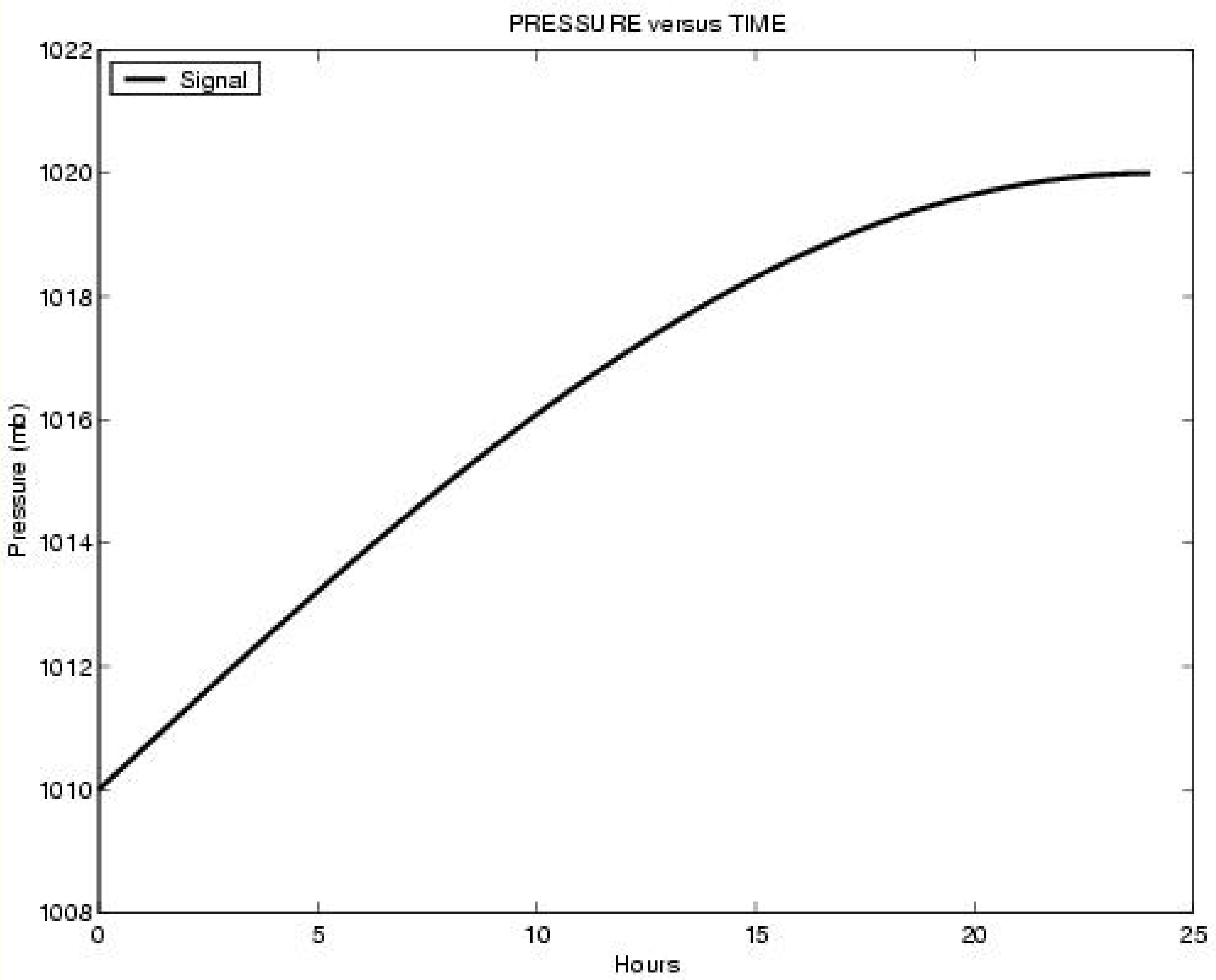
* In the equation for the lowest stratum the corresponding term $-m_{G8}$ does not appear

Longitude *11° East*, $\delta e = 441 \times 10^5$ — Latitude *5400 km North*, $\delta n = 400 \times 10^5$ — Instant *1910 May 20^d 7^h G.M.T.*, $a^{-1}.\tan\phi = 1{\cdot}78 \times 10^{-9}$ — Interval, δt *6 hours*, $a = 6{\cdot}36 \times 10^8$

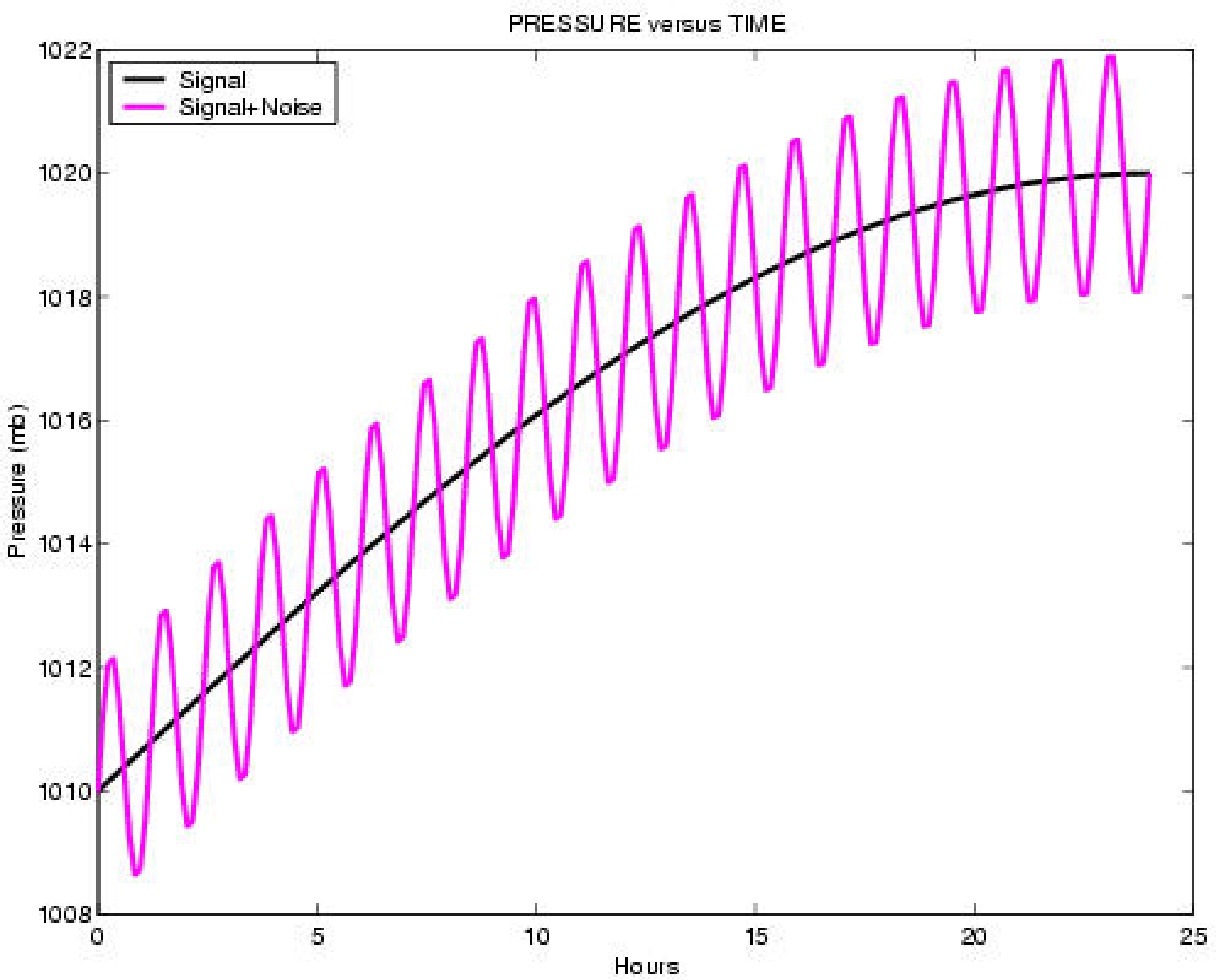
Ref.:—				previous 3 columns	previous column	Leave the subsequent columns to be filled up after the vertical velocity has been computed on Form P XVI	Form P XVI	Form P XVI	equation above	previous column	previous column	previous column
h	$\frac{\delta M_E}{\delta e}$	$\frac{\delta M_N}{\delta n}$	$-\frac{M_N \tan\phi}{a}$	$\mathrm{div}'_{EN}M$	$-g\delta t\,\mathrm{div}'_{EN}M$		m_H	$\frac{2M_H}{a}$	$-\frac{\partial R}{\partial t}$	$+\frac{\partial R}{\partial t}\delta t$	$g\frac{\partial R}{\partial t}\delta t$	$\frac{\partial p}{\partial t}\delta t$
	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$100\times$		$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$		$100\times$	$100\times$
h_0							0					0
	−61	−245	−6	−312	656				−229	49·5	483	
h_2							−83					483
	367	−257	2	112	−236			0·06	−136	29·4	287	
h_4							165					770
	93	−303	−16	−226	478			0·11	−124	26·8	262	
h_6							63					1032
	32	−55	−12	−35	74			0·07	−110	23·8	233	
h_8							138					1265
	−256	38	−8	−226	479			0·03	−88	19·0	186	
h_G												1451
	Note: $\mathrm{div}'_{EN}M$ is a contraction for $\frac{\delta M_E}{\delta e} + \frac{\delta M_N}{\delta n} - M_N\frac{\tan\phi}{a}$				SUM = 1451 $= \frac{\partial p_G}{\partial t}\delta t$							check by $\Sigma - g\delta t\,\mathrm{div}'_{EN}M$

Richardson's Computing Form $\mathrm{P_{XIII}}$
The figure in the bottom right corner is the forecast change in surface pressure: 145 mb in six hours!

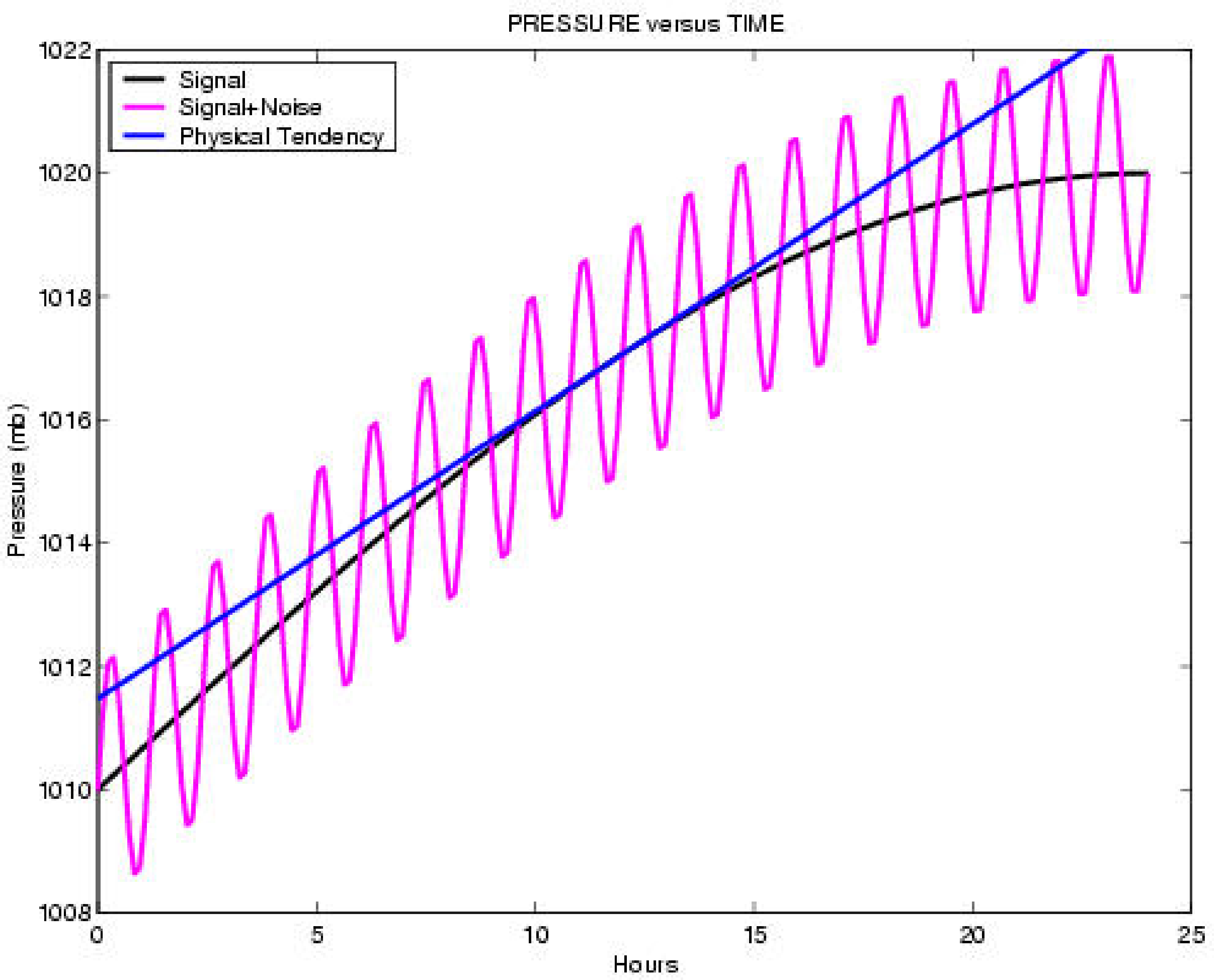
Smooth Evolution of Pressure



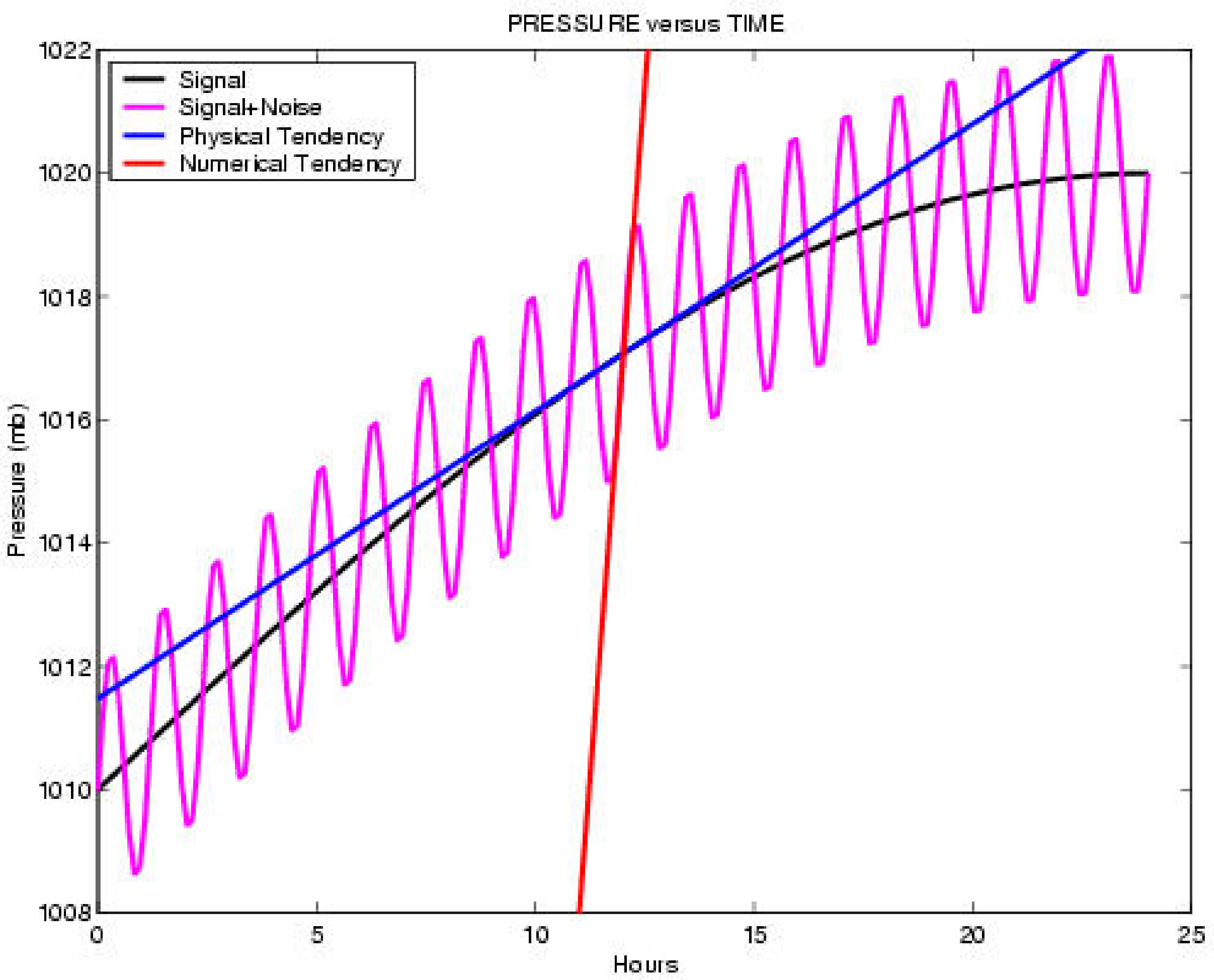
Noisy Evolution of Pressure

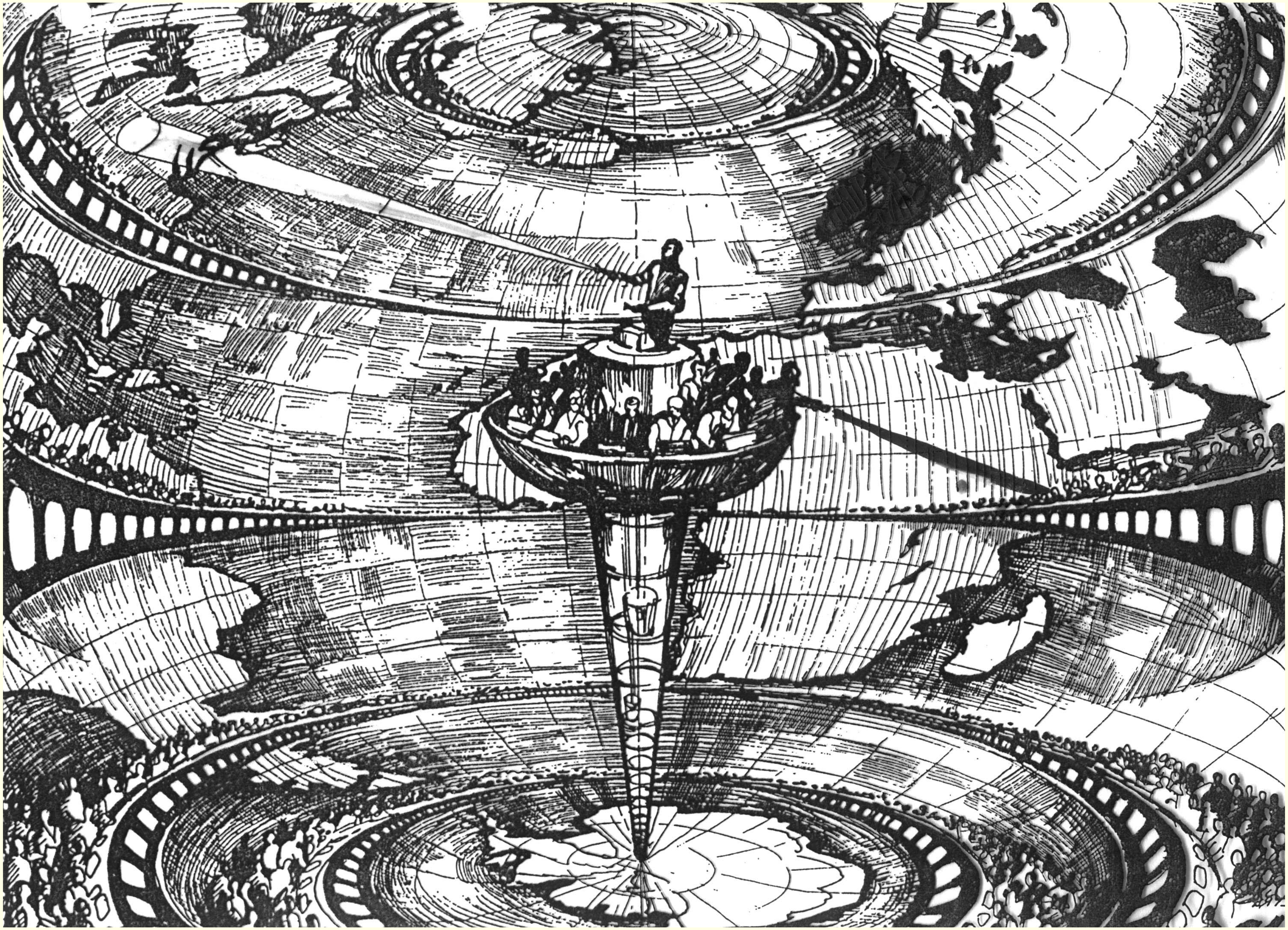


Tendency of a Smooth Signal



Tendency of a Noisy Signal





Richardson's Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984

64,000 Computers: The first Massively Parallel Processor

What went wrong?

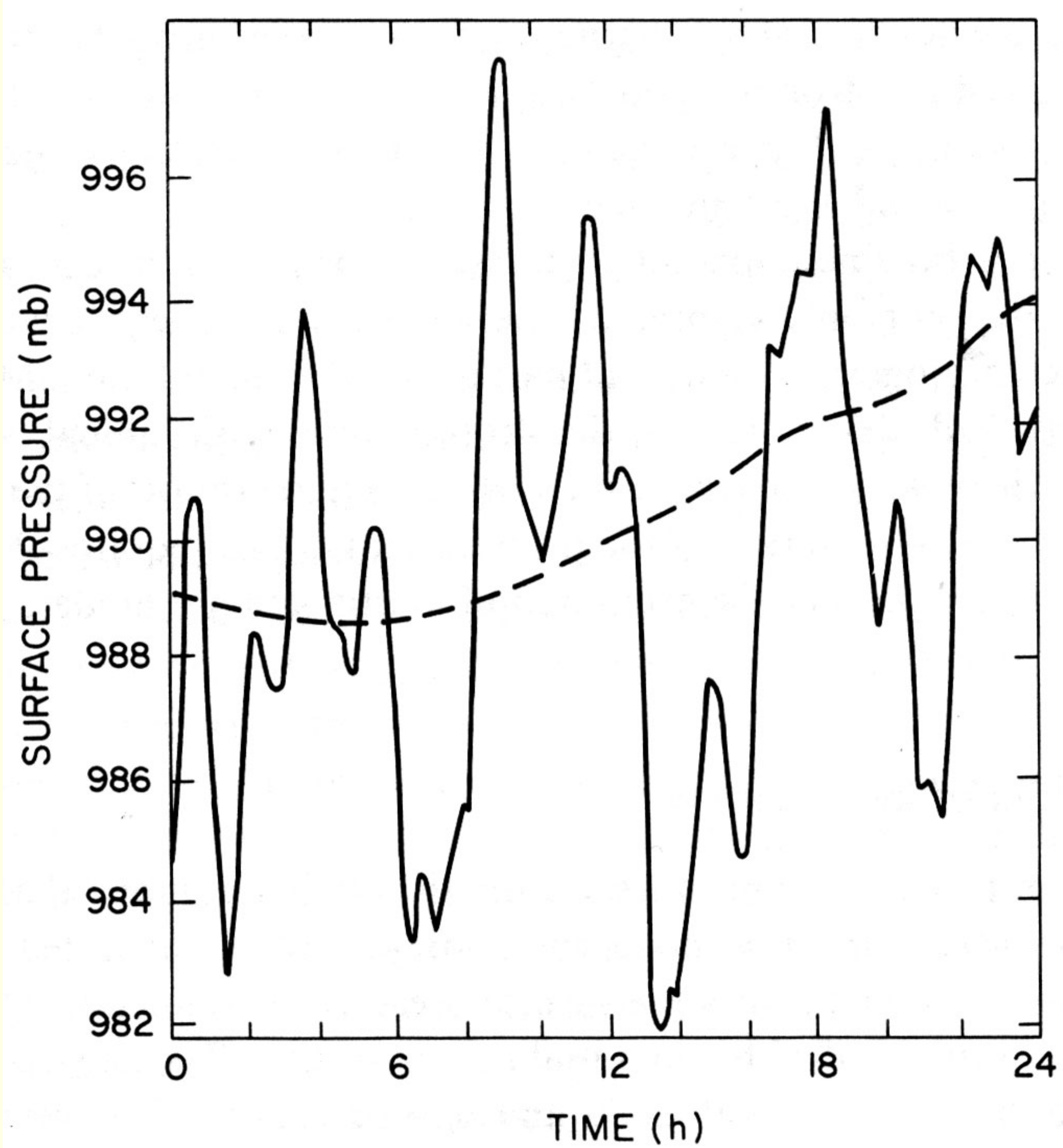
- **Richardson extrapolated *instantaneous* pressure change, assuming it to remain constant over a long time period.**
- **This ignores the propensity of the atmosphere to respond rapidly to changes.**
- **An increase of pressure causes an immediate pressure gradient which acts to resist further change.**
- **The resulting gravity wave oscillations act in such a way as to restore balance.**
- **They result in pressure changes which may be large but which oscillate rapidly in time.**

(Margules, 1893, was the first comprehensive study of gravity wave dynamics)

The ineluctable conclusion is that . . .

the instantaneous rate of change is not a reliable indicator of the long-term variation in pressure.

- ***To obtain an accurate prediction, the <u>time step has to be short enough</u> to allow the adjustment to take place.***
- ***Gravity-wave oscillations <u>need not spoil</u> the forecast.***
- ***They may be regarded as noise superimposed on the synoptic evolution.***
- ***They may also be effectively removed by an adjustment of the data, known as initialization.***



Evolution of surface pressure before and after NNMI.

(Williamson and Temperton, 1981)

Richardson on Smoothing

"The scheme of numerical forecasting has been developed so far that it is reasonable to expect that when the smoothing . . . has been arranged, it may give forecasts agreeing with the actual smoothed weather."

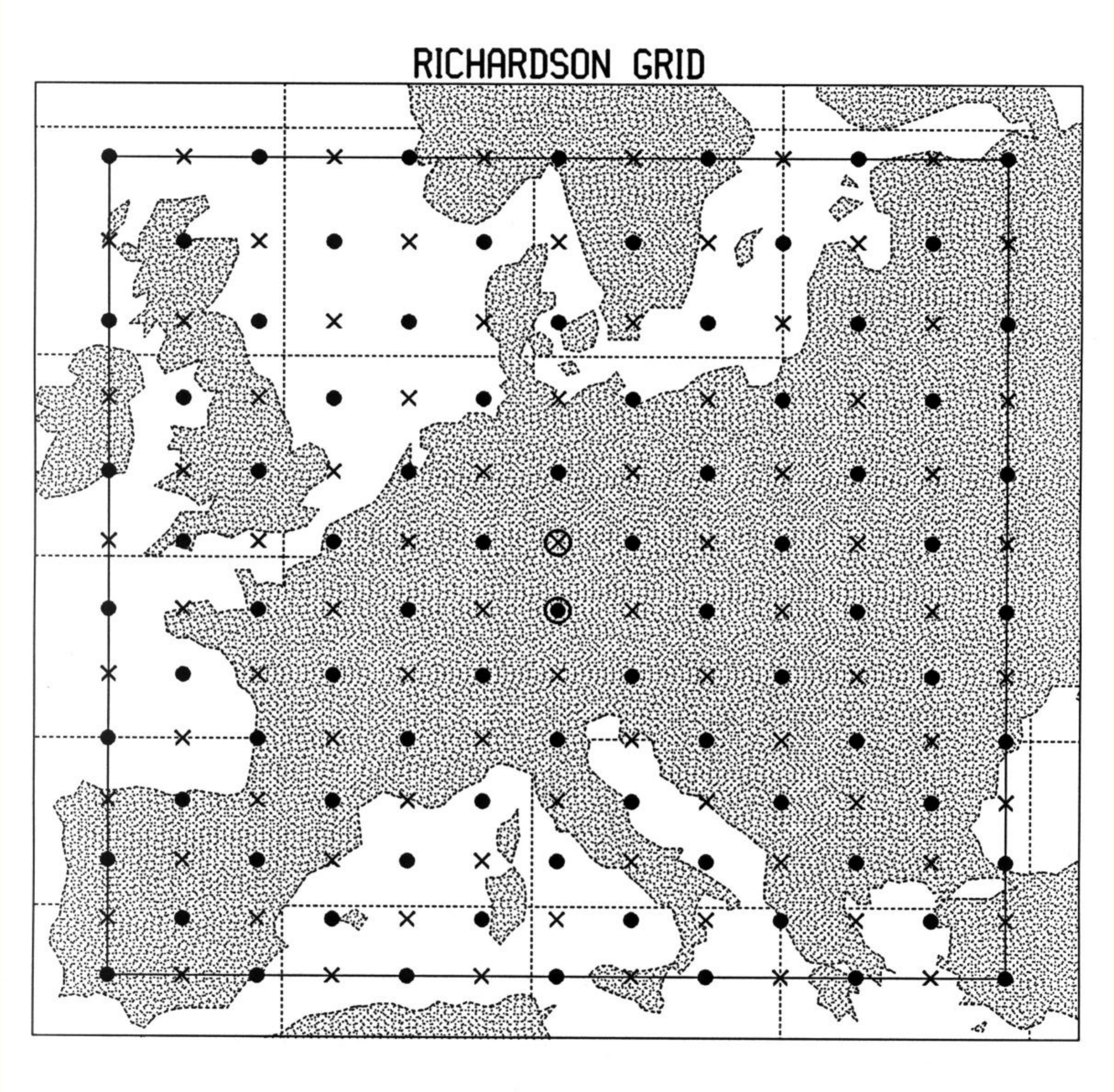
Richardson's Smoothing Methods

Richardson devoted a short chapter of his book to smoothing.

He outlined five smoothing methods:

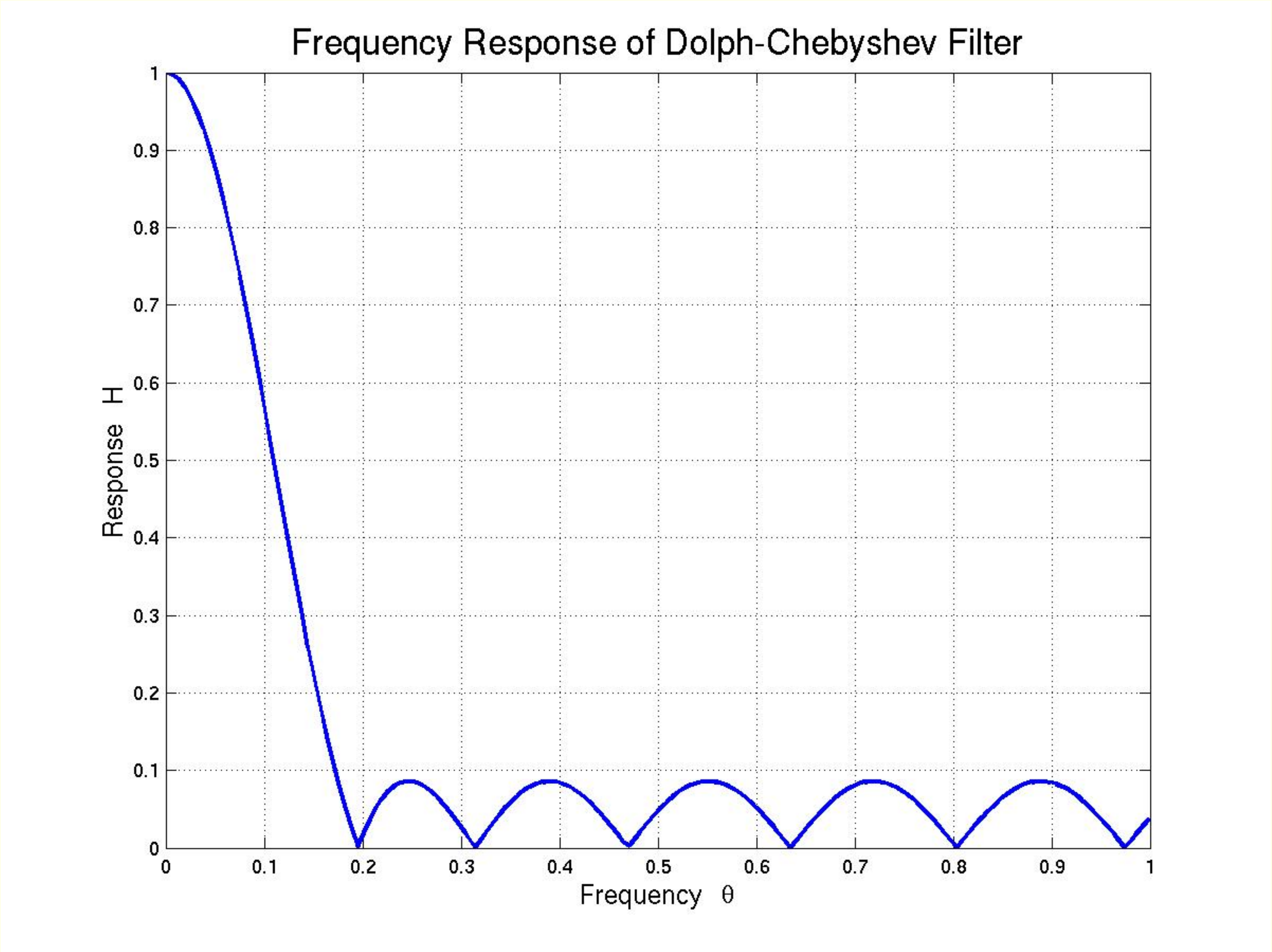
- **A. Space Means.**
- **B. Time Means.**
- **C. Potential Function.**
- **D. Stream Function.**
- **E. Smoothing during the Forecast.**

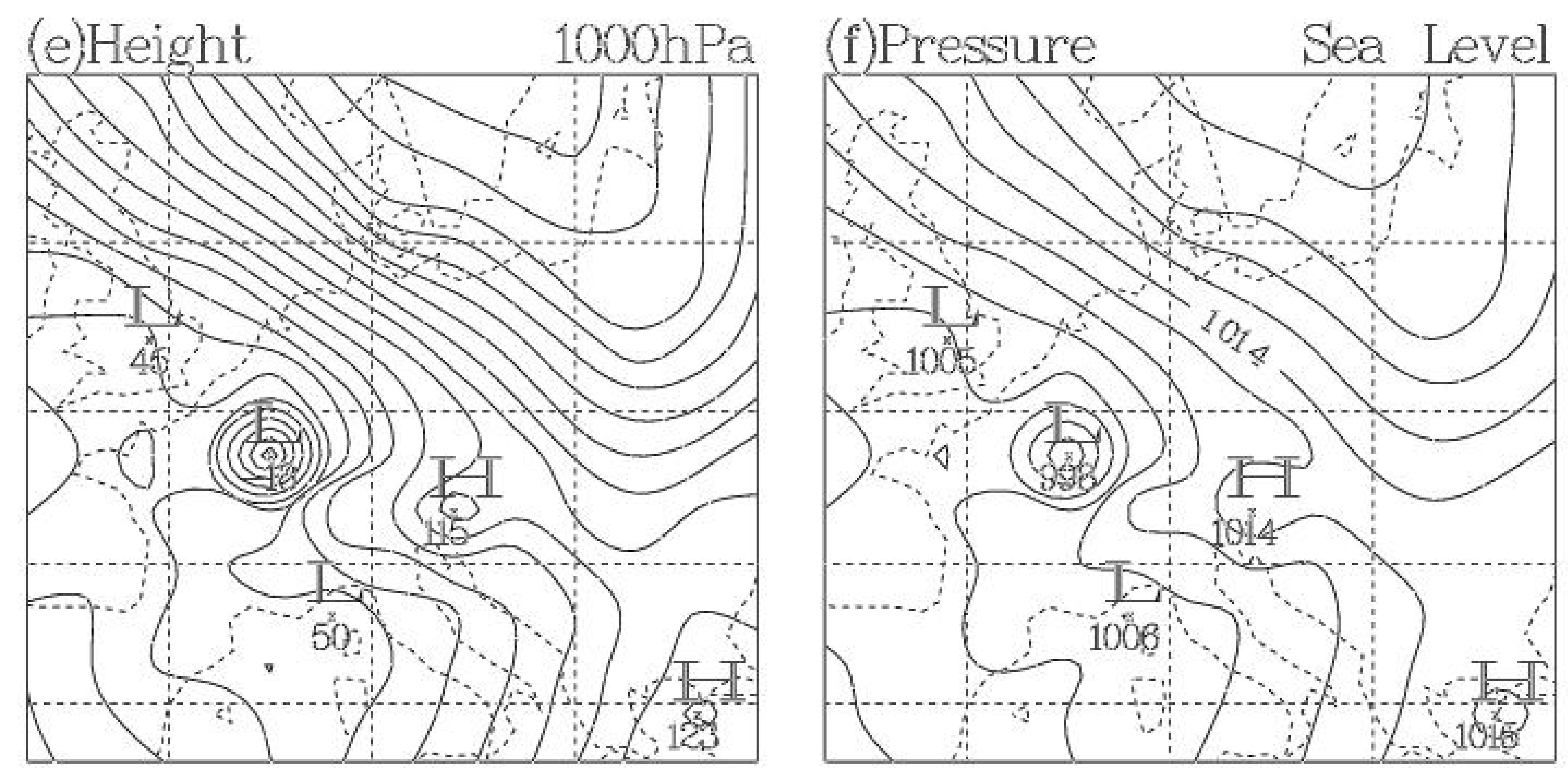
Richardson's *Method B* is a close cousin of Digital Filtering Initialization, which has some current popularity.



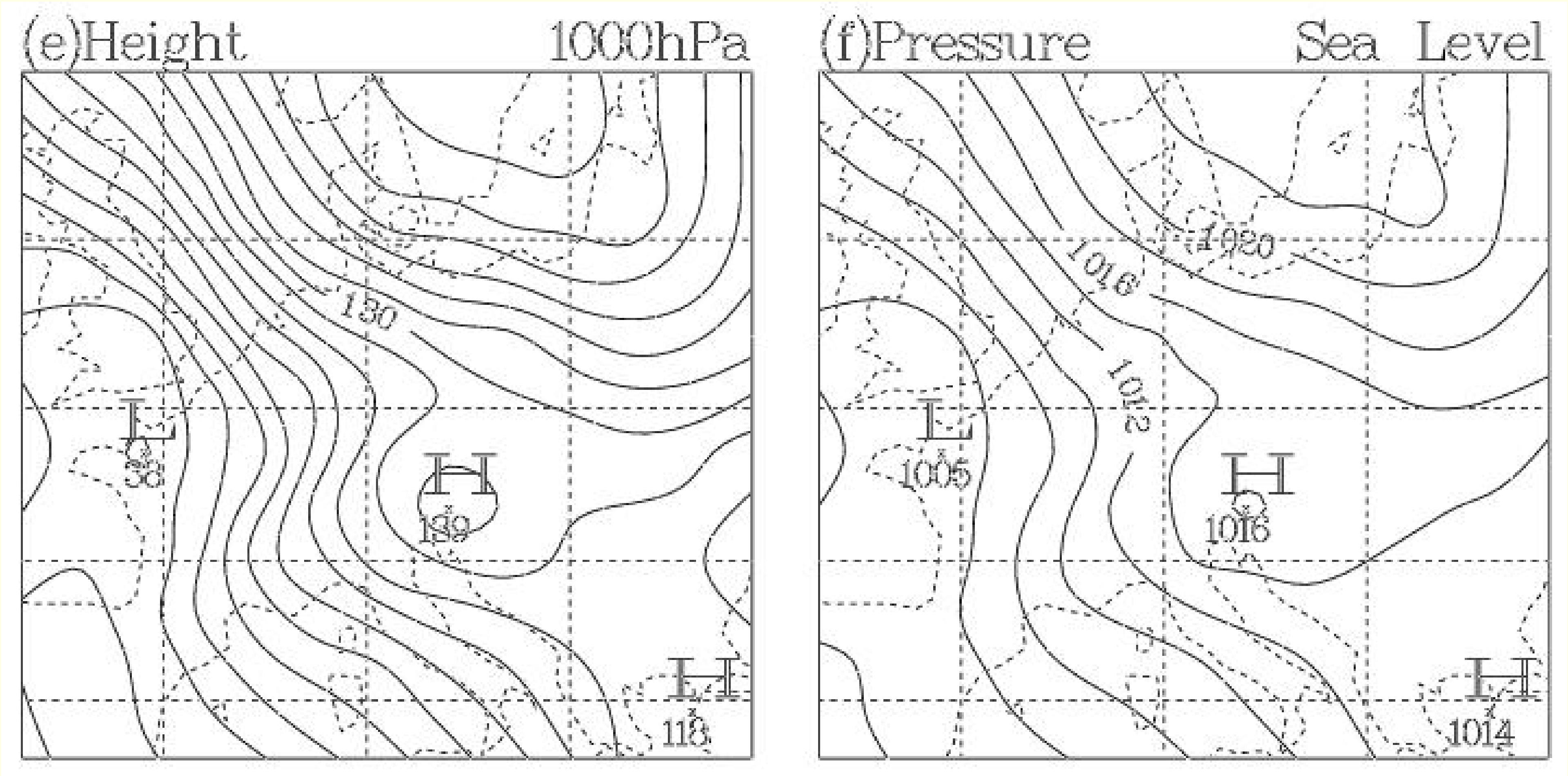
Grid for extending Richardson's forecast

Digital Filter Response



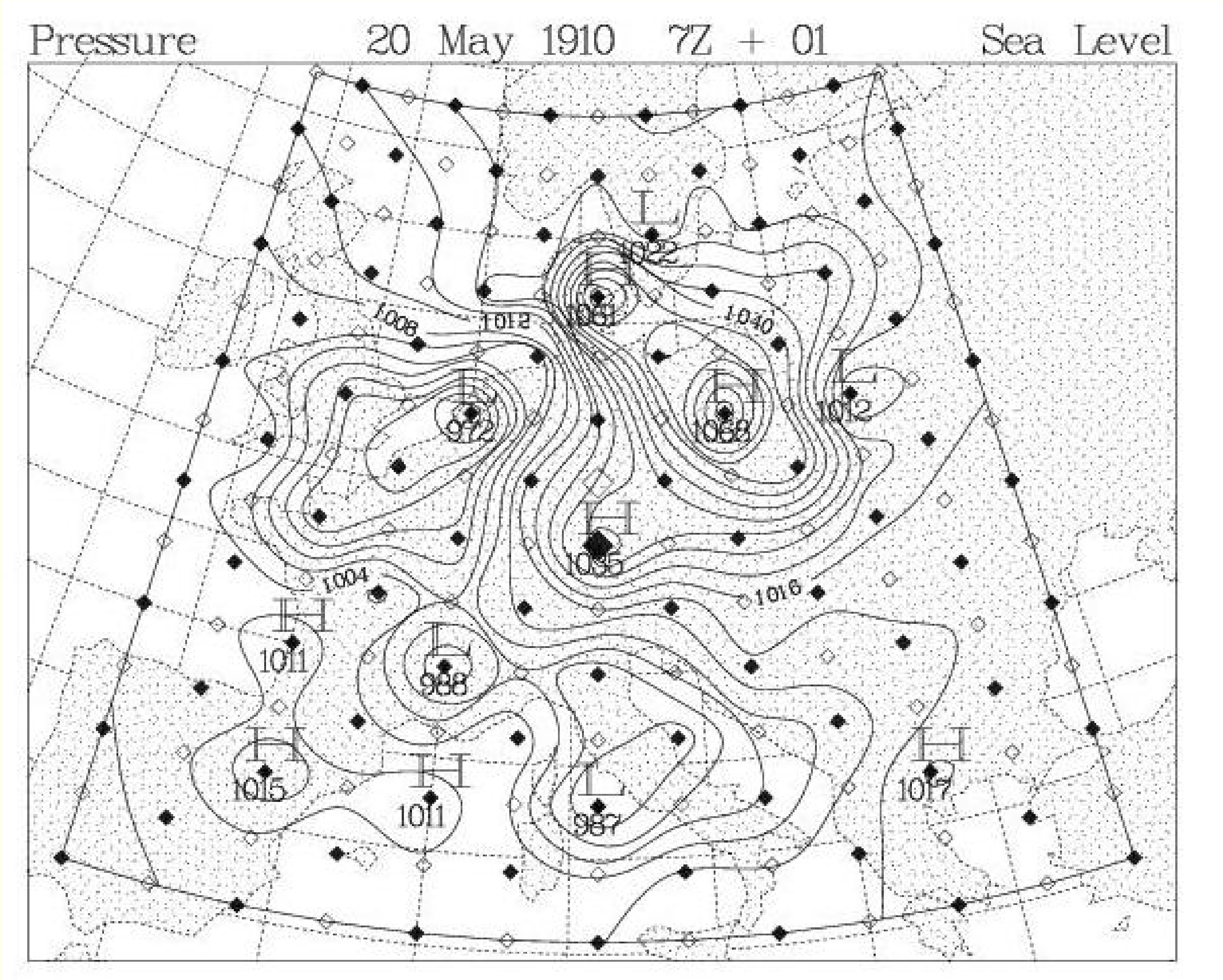


1000 hPa height and surface pressure before initialization



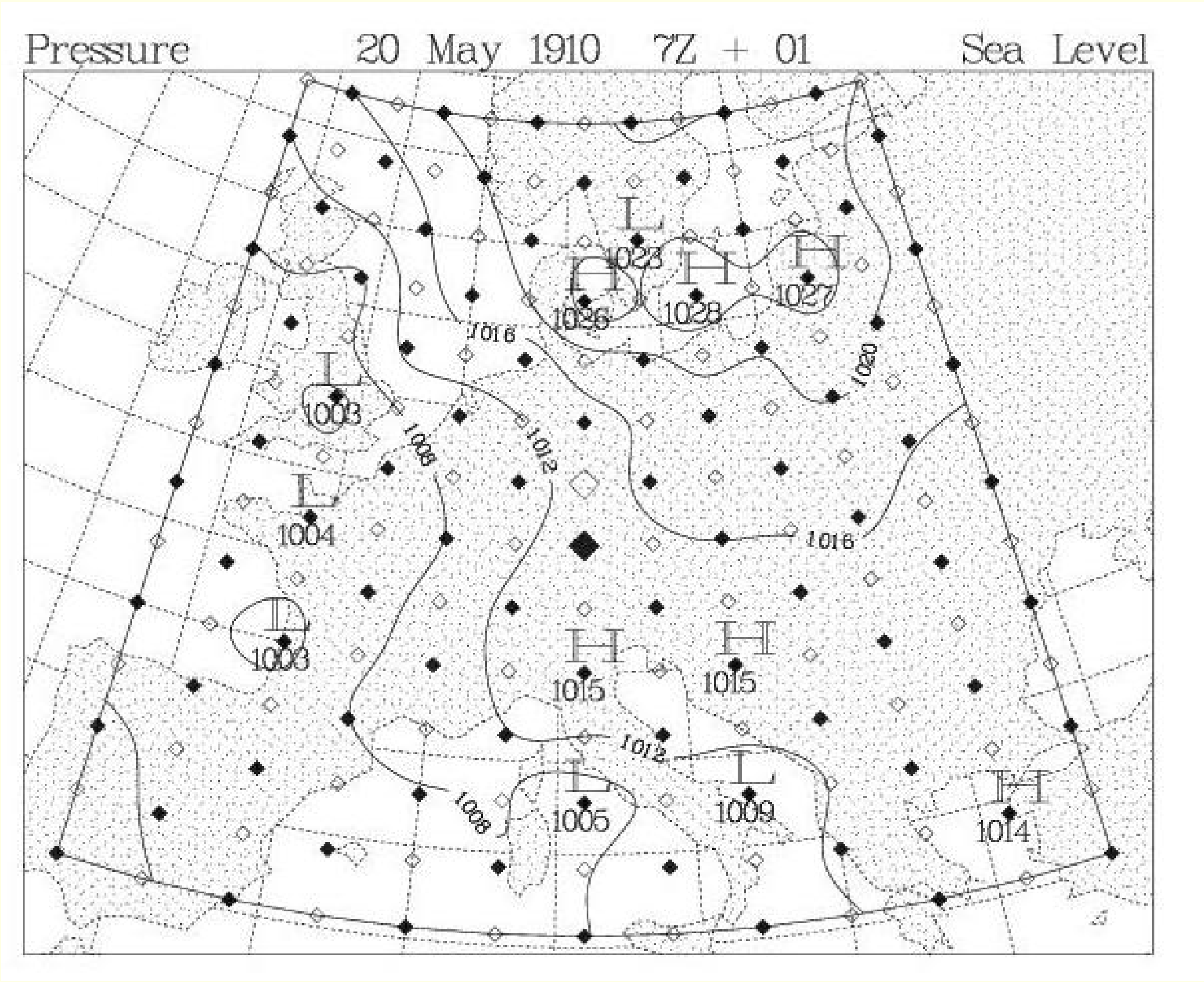
1000 hPa height and surface pressure after initialization

Forecast without Filtering



Short-range forecast of sea-level pressure, from *uninitialized data.* The contour interval is 4 hPa. Single forward time step of size $\Delta t = 3600\,\mathrm{s}$.

Forecast with Filtering



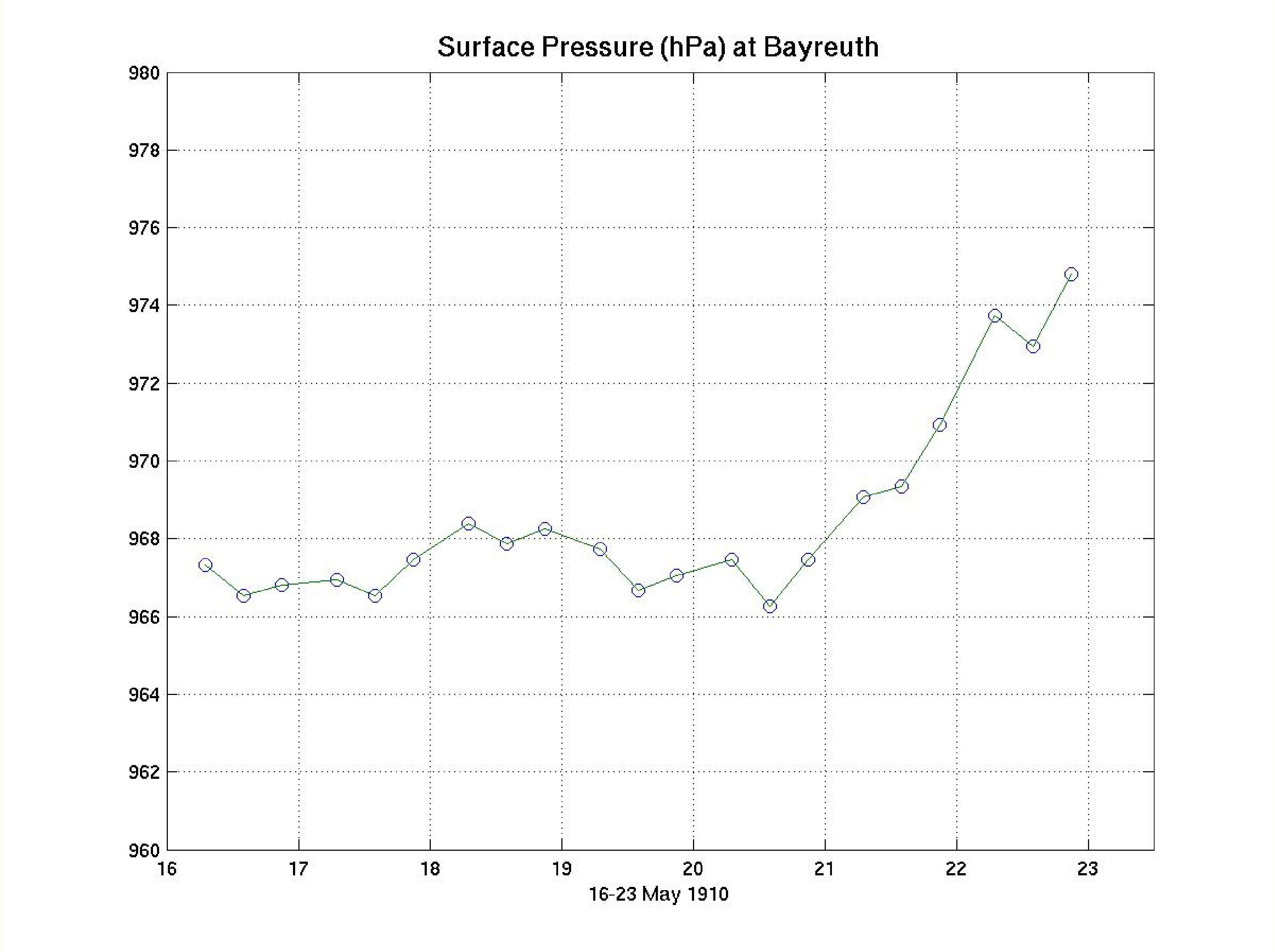
Short-range forecast of sea-level pressure, from *filtered data.* The contour interval is 4 hPa. Single forward time step of size $\Delta t = 3600\,$s.

Table 1: Analysis of the pressure changes (hPa) across each layer, and the pressure change at the base of each layer.

		Total Divergence	Change in Pressure Thickness	Change in Base Pressure
Layer	Level	$\nabla \cdot \mathbf{U} + [\rho w]$	$\frac{\partial [p]}{\partial t}\Delta t$	$\frac{\partial p}{\partial t}\Delta t$
I		-0.0229	$+48.3$	
	1			$+48.3$
II		-0.0136	$+28.7$	
	2			$+77.1$
III		-0.0124	$+26.2$	
	3			$+103.2$
IV		-0.0110	$+23.3$	
	4			$+126.5$
V		-0.0088	$+18.6$	
	S			**+145.1**

Table 2: Analysis of the pressure changes (hPa) for the forecast from data after Digital Filtering.

		Total Divergence	Change in Pressure Thickness	Change in Base Pressure
Layer	Level	$\nabla \cdot \mathbf{U} + [\rho w]$	$\frac{\partial [p]}{\partial t}\Delta t$	$\frac{\partial p}{\partial t}\Delta t$
I		$+0.0001$	-0.2	
	1			-0.2
II		$+0.0011$	$\mathbf{-2.4}$	
	2			-2.6
III		$+0.0002$	-0.4	
	3			-3.0
IV		$+0.0000$	-0.1	
	4			-3.1
V		-0.0010	$\mathbf{+2.1}$	
	S			$\mathbf{-0.9}$



Surface pressure at Bayreuth for a week in May, 1910

Crucial Advances, 1920–1950

- ***Dynamic Meteorology***
  - Rossby Waves
  - Quasi-geostrophic Theory
  - Baroclinic Instability
- ***Numerical Analysis***
  - CFL Criterion
- ***Atmopsheric Observations***
  - Radiosonde
- ***Electronic Computing***
  - ENIAC

Electronic Computer Project

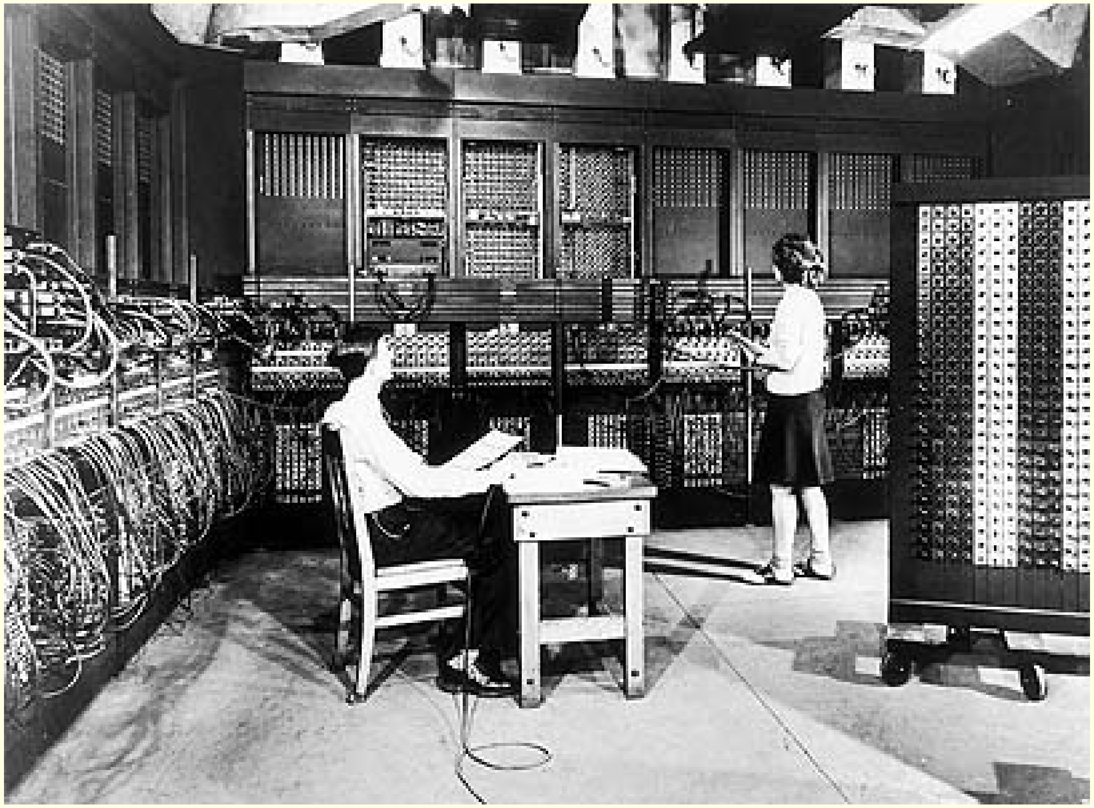
Von Neumann's idea (1946):

Weather forecasting was, *par excellence*, **a scientific problem suitable for solution using a large computer.**

Objective:

To predict the weather by simulating the dynamics of the atmosphere using a digital electronic computer.

The ENIAC



The ENIAC was the first multi-purpose programmable electronic digital computer.
It had:

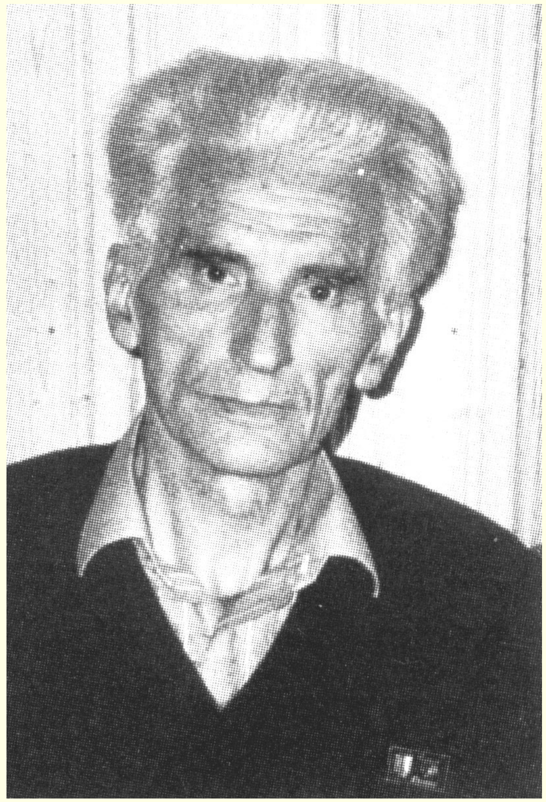
- **18,000 vacuum tubes**
- **70,000 resistors**
- **10,000 capacitors**
- **6,000 switches**
- **Power: 140 kWatts**

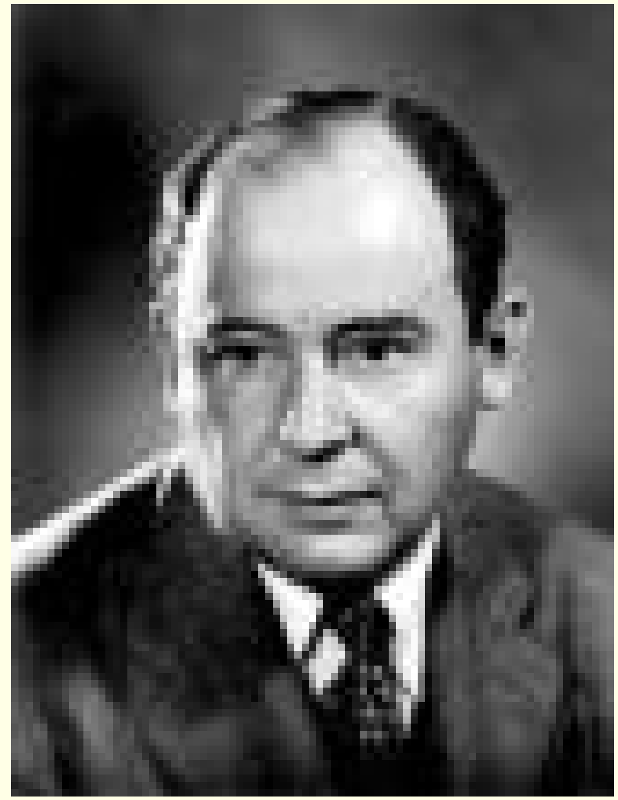
Evolution of the Meteorology Project:

- **Plan A: Integrate the Primitive Equations**
  Problems similar to Richardson's would arise
- **Plan B: Integrate baroclinic Q-G System**
  Too computationally demanding
- **Plan C: Solve barotropic vorticity equation**
  Very satisfactory initial results

$$\frac{d}{dt}(\zeta + f) = 0$$

Charney, Fjørtoft, von Neumann

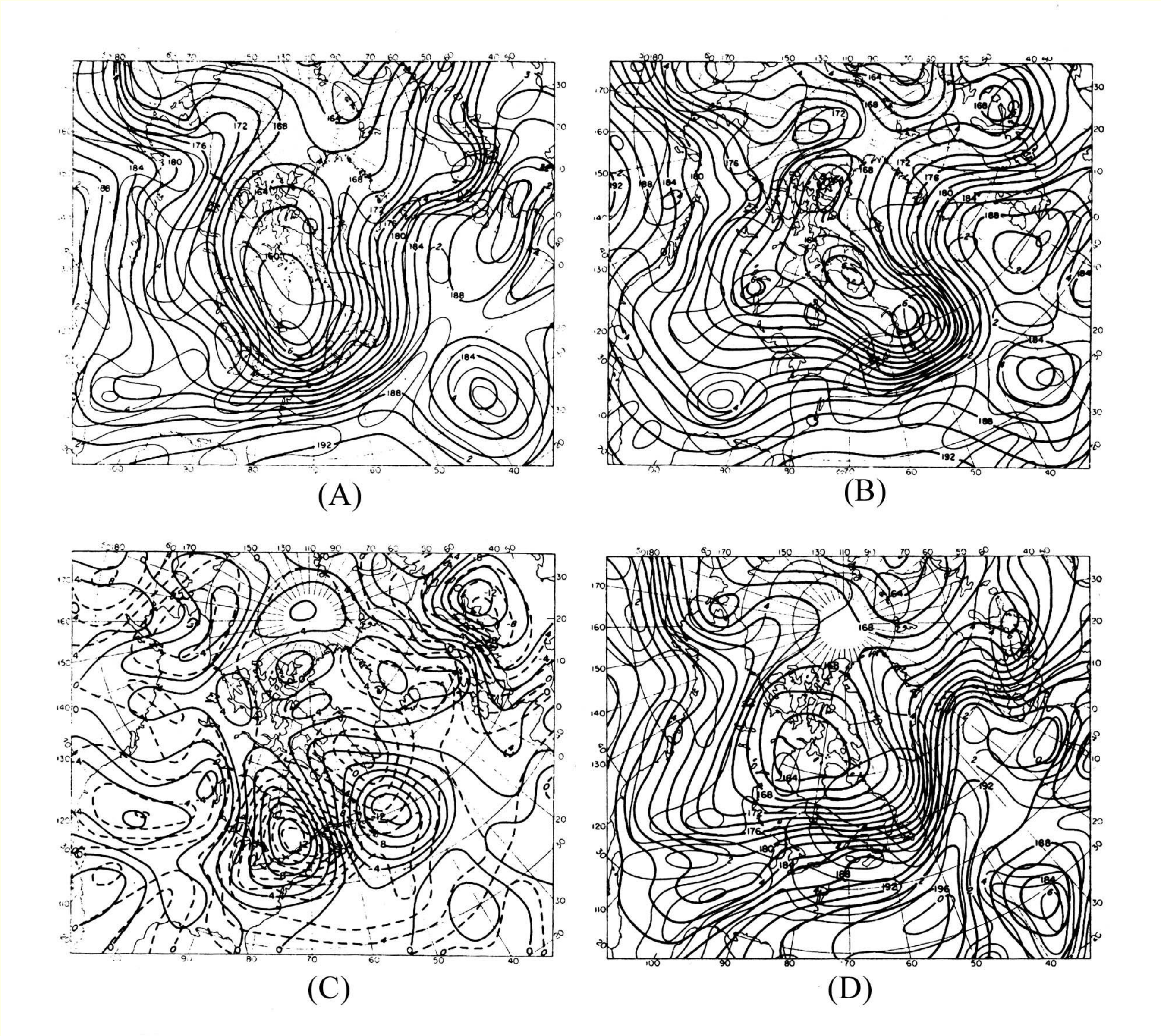




Charney, J.G., R. Fjørtoft and J. von Neumann, 1950:

Numerical integration of the barotropic vorticity equation. *Tellus*, 2, 237–254.

ENIAC: First Computer Forecast



Richardson's reaction

- ***"Allow me to congratulate you ... on the remarkable progress which has been made.***
- ***"This is ... an enormous scientific advance on the ... result in Richardson (1922)."***

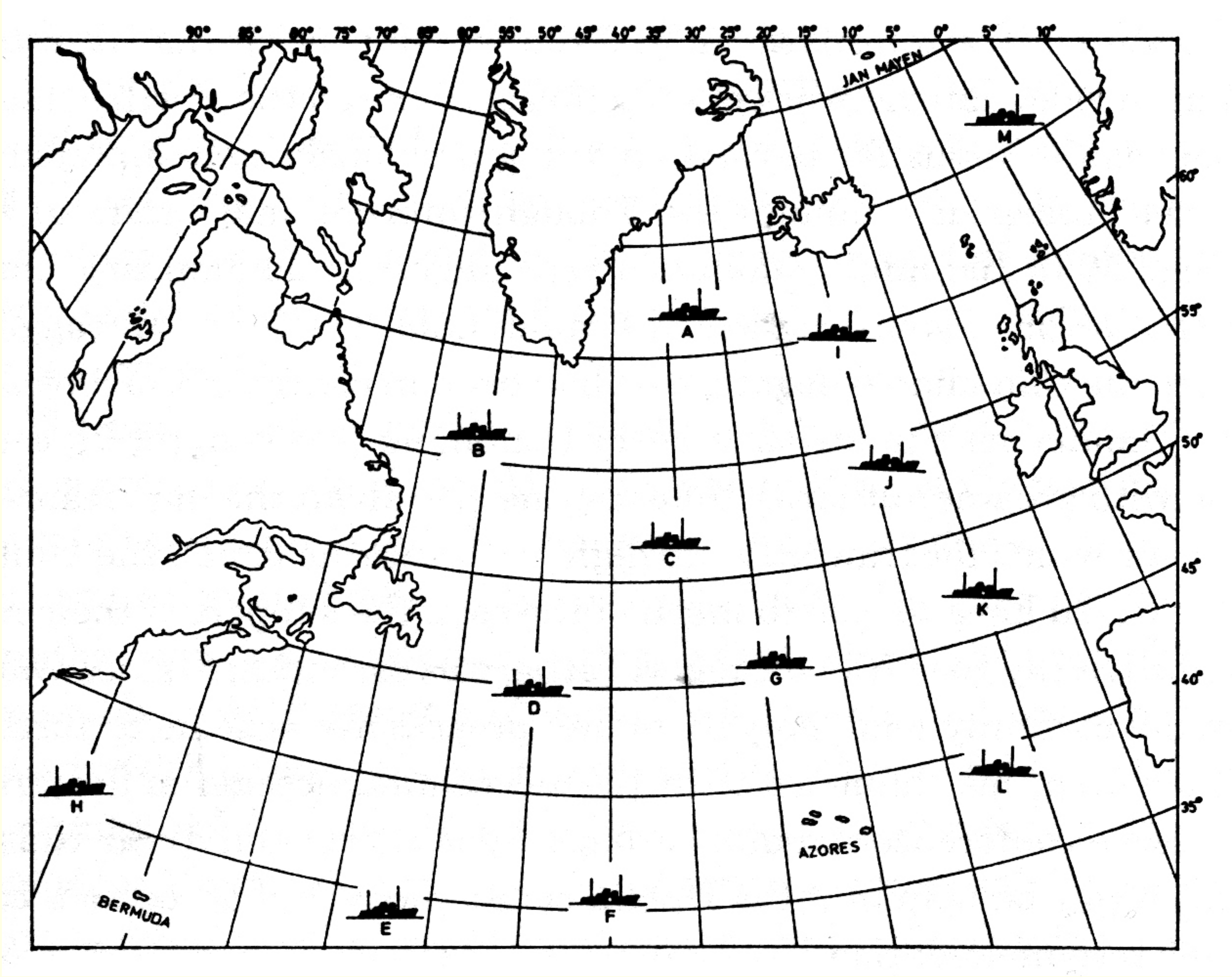
NWP Operations

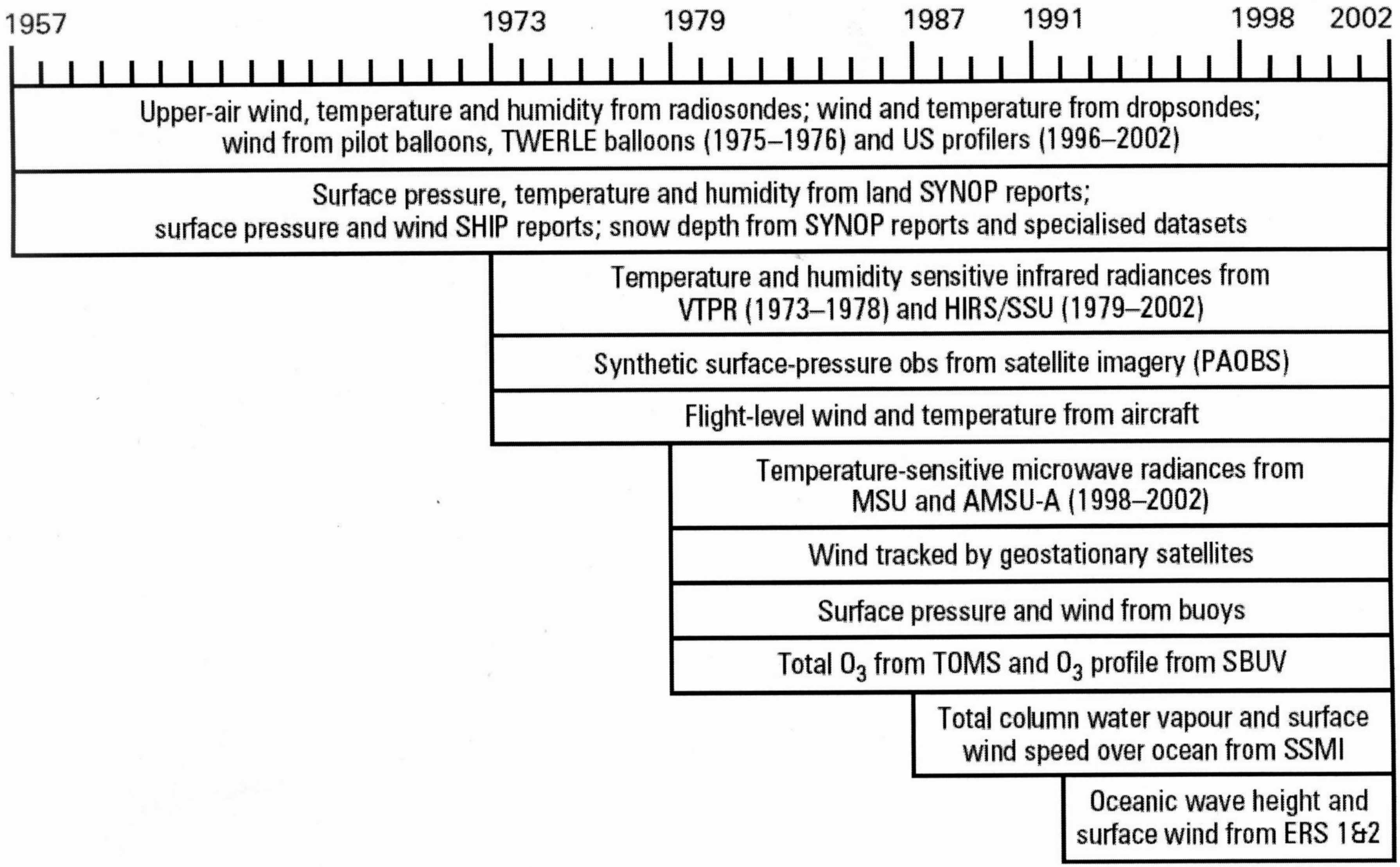
The Joint Numerical Weather Prediction Unit was established on July 1, 1954:

- ***Air Weather Service of US Air Force***
- ***The US Weather Bureau***
- ***The Naval Weather Service.***

Operational numerical forecasting began in May, 1955, with a three-level quasi-geostrophic model.

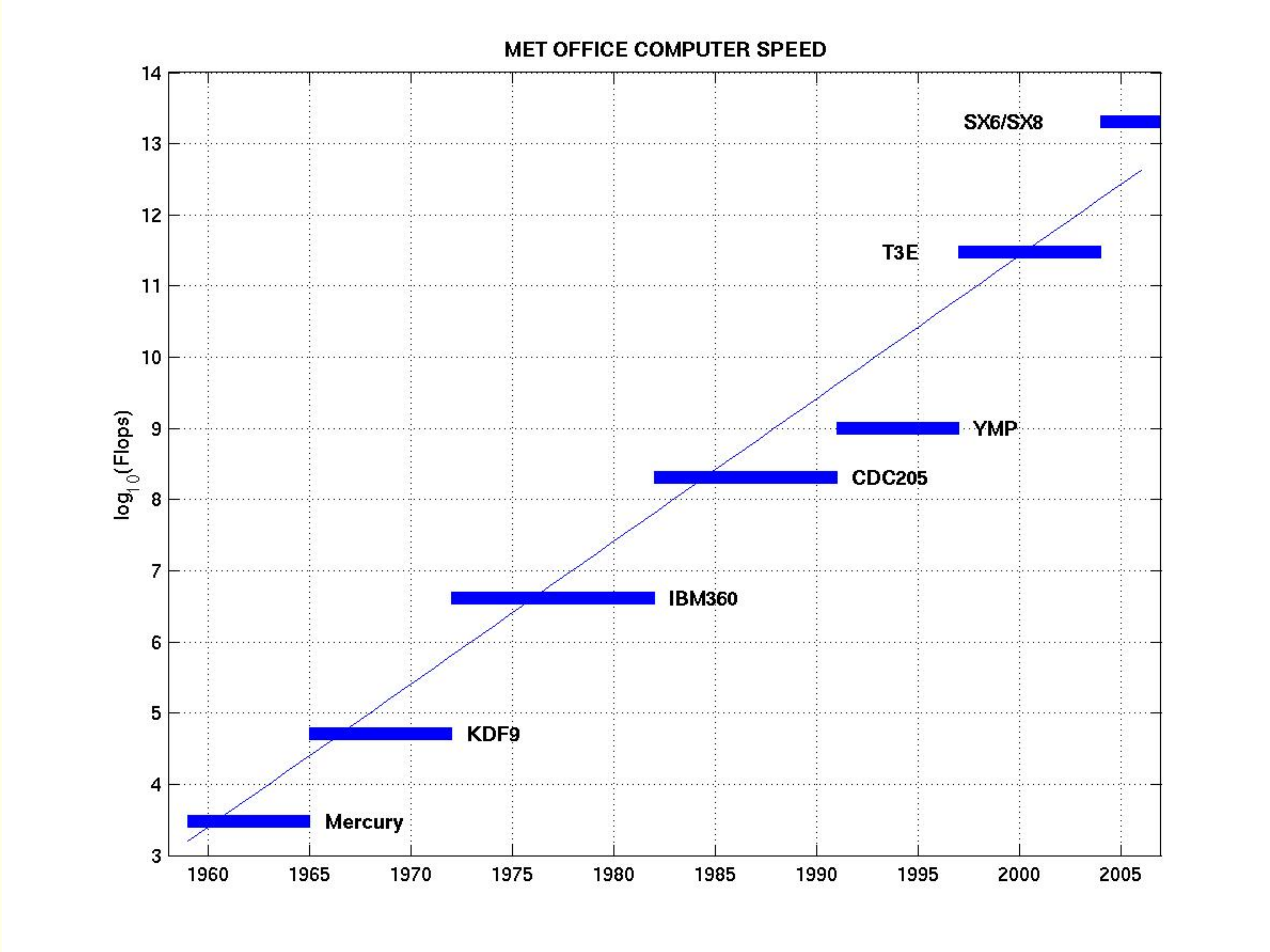
The PICAO Array





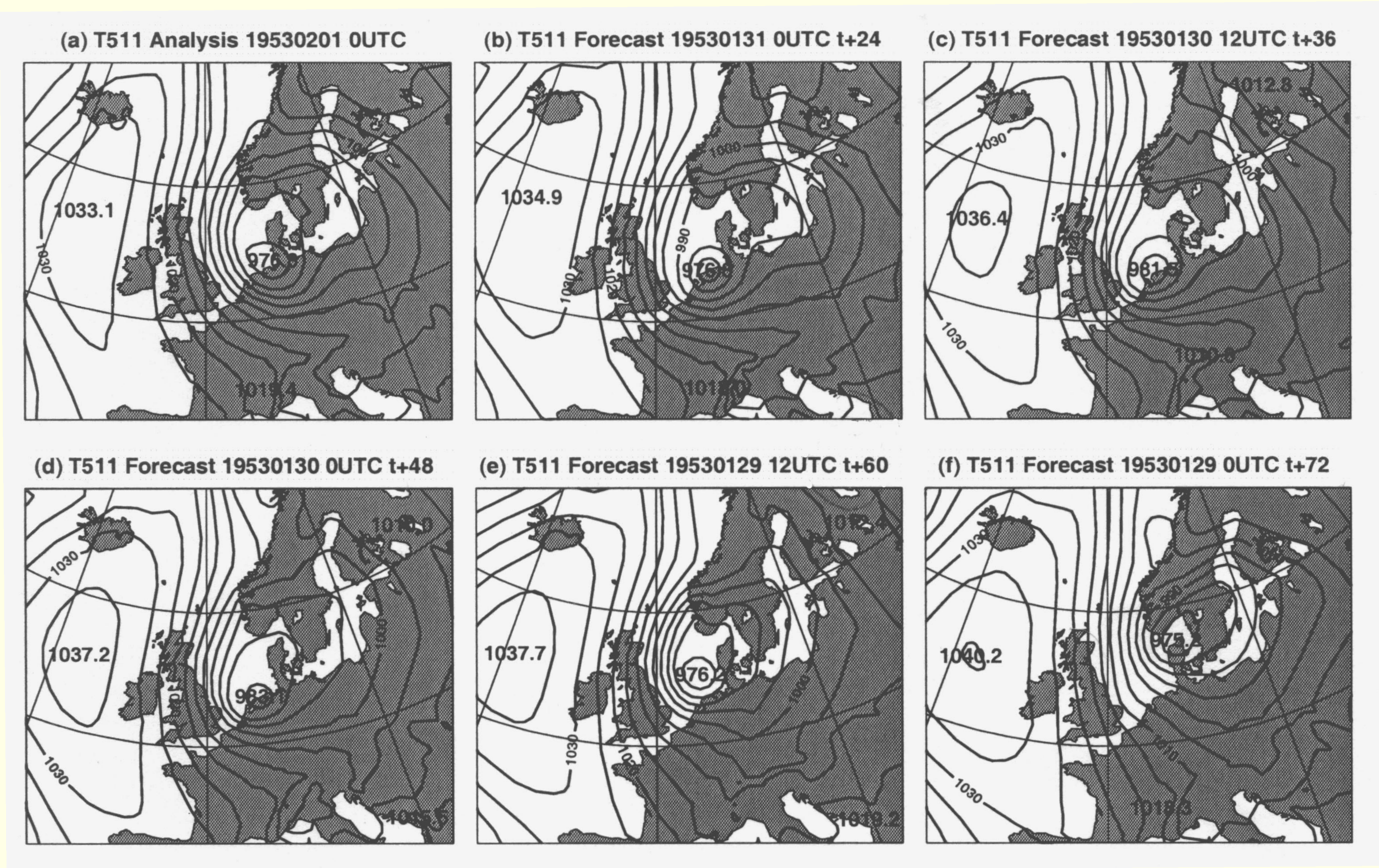
Development of global observing system over 50 years.

(Uppala, *et al.*, 2006)



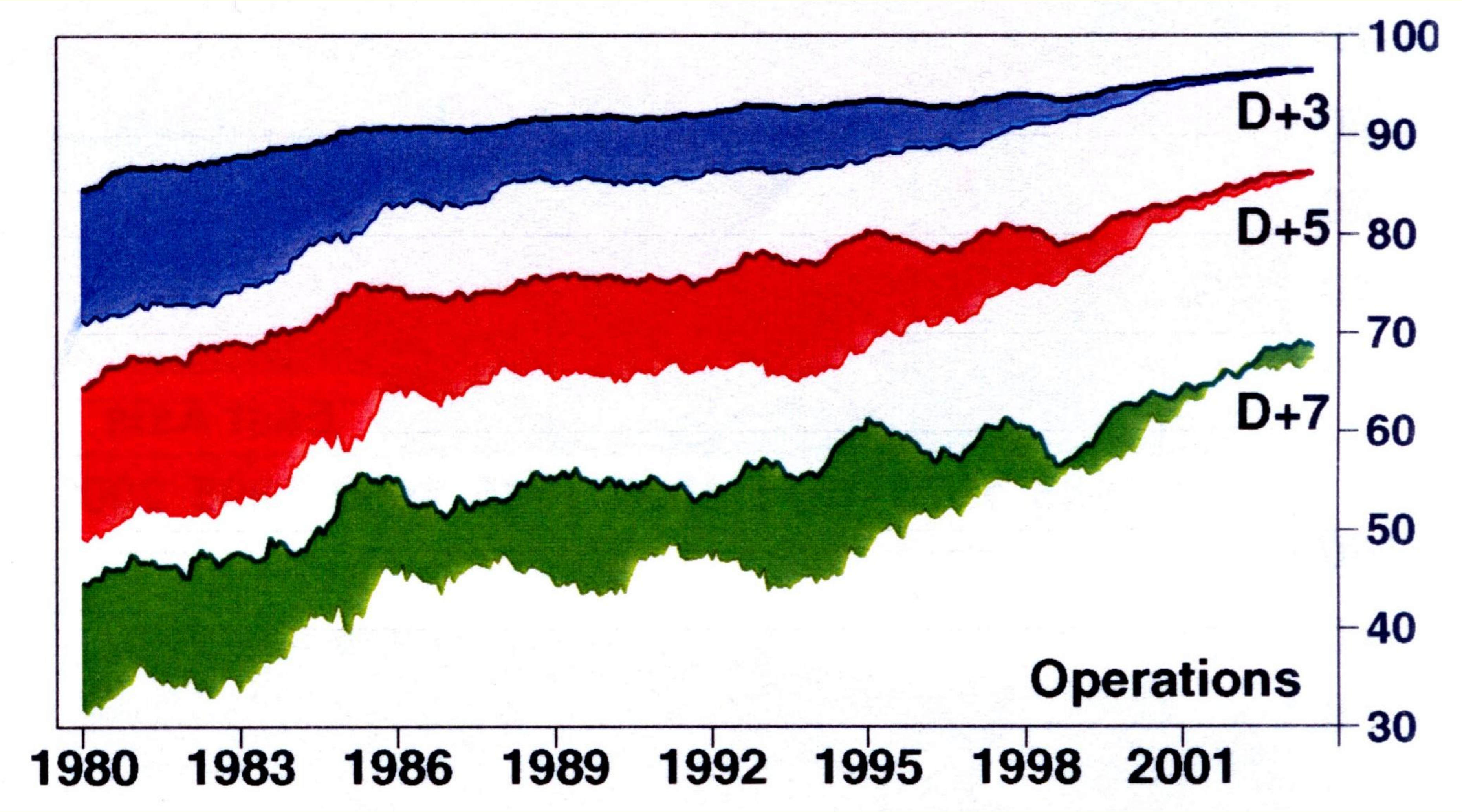
Processing speed of Met Office computers 1959–2005

(data from http://www.metoffice.gov.uk)



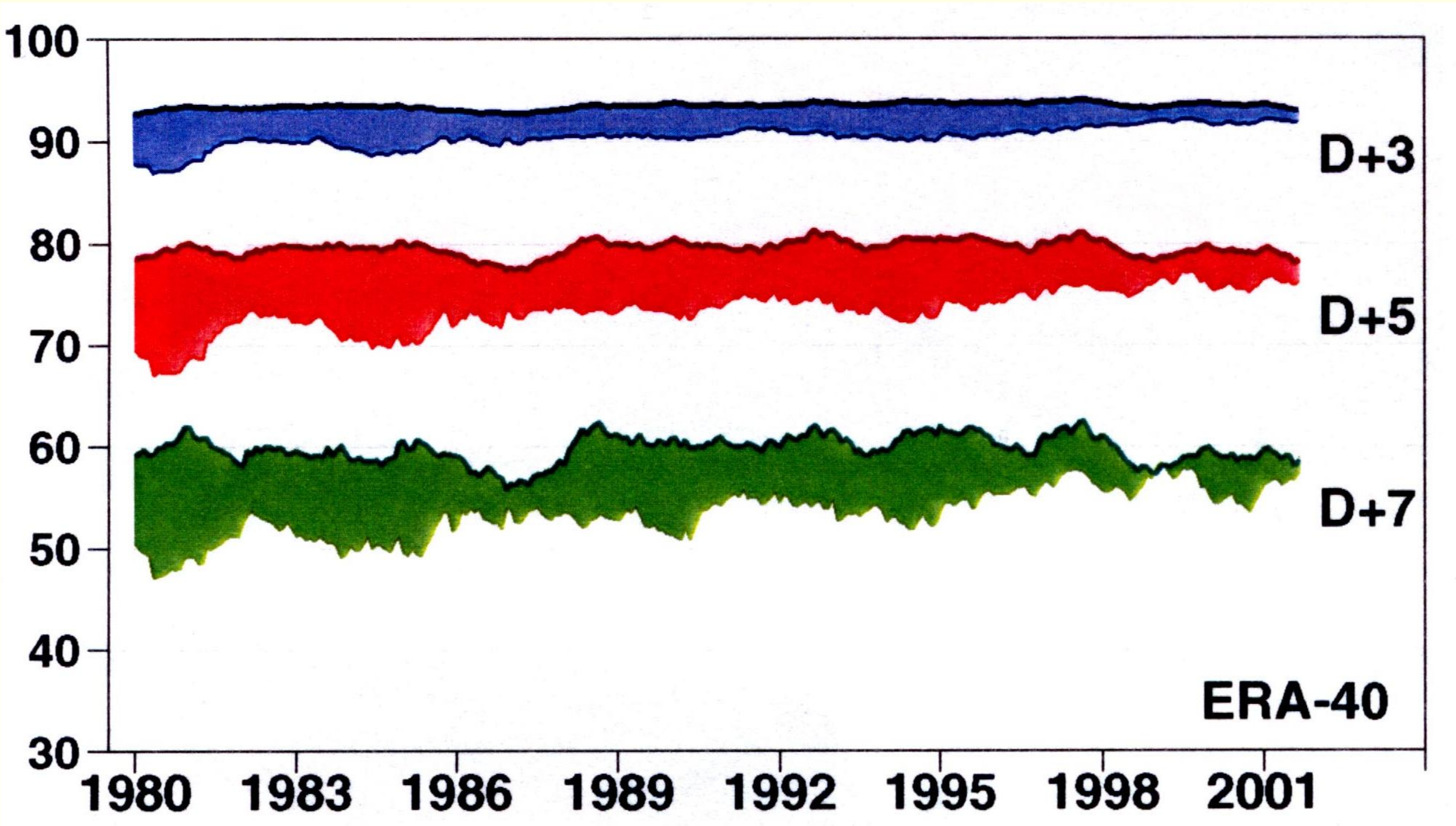
The Dutch Storm: 00 UTC on 1 February, 1953. Analysis and forecasts out to three days.

Anomaly Correlations: Operational



Anomaly Correlation of 3, 5 and 7 day 500 hPa forecasts for extra-tropical northern and southern hemispheres.

Anomaly Correlations: ERA40

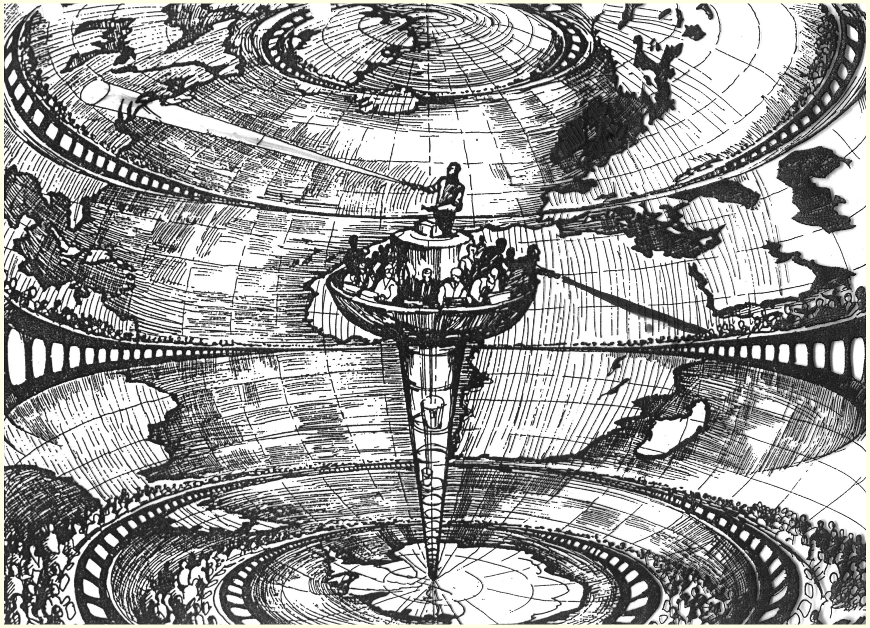


Anomaly Correlation of 3, 5 and 7 day 500 hPa forecasts for extra-tropical northern and southern hemispheres.

Richardson's Dream

"Perhaps some day in the dim future it will be possible to advance the computations faster than the weather advances . . . . But that is a dream."

Forecast Factory *versus* MPP



Richardson envisaged a ***Forecast Factory*** with 64,000 (human) computers.

The fastest computer in the TOP500 list as of June 2005 was the IBM BlueGene/L with 65,536 processors!

The IBM machine is rated at 136.8 TFlops (www.top500.org)

The BlueGene is about nine orders of magnitude faster than Richardson's Forecast Factory.

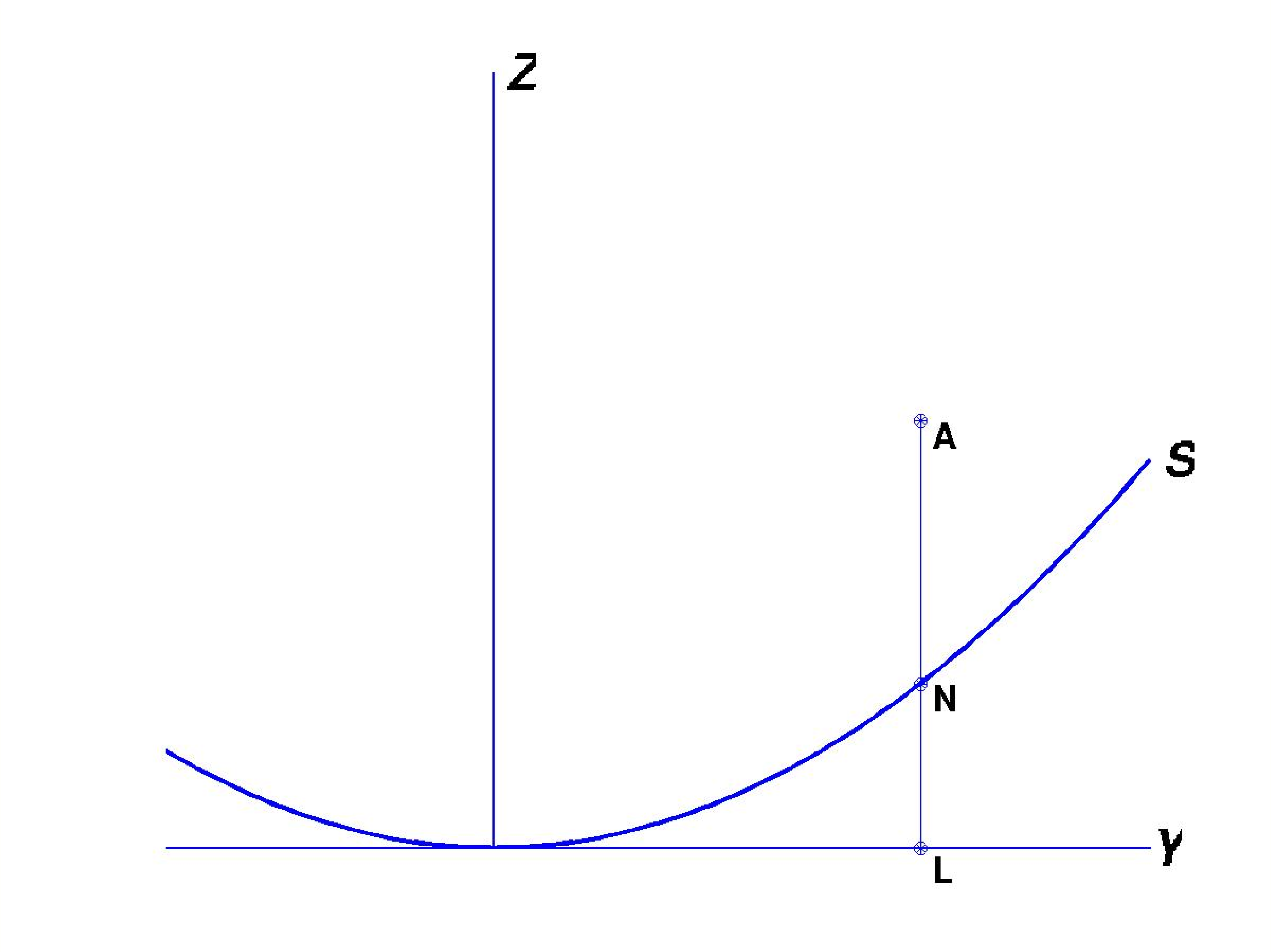
Yet, the logical structures of the Forecast Factory and the MPP have much in common: message-passing, domain decomposition, synchronization and control.

Concluding Remarks

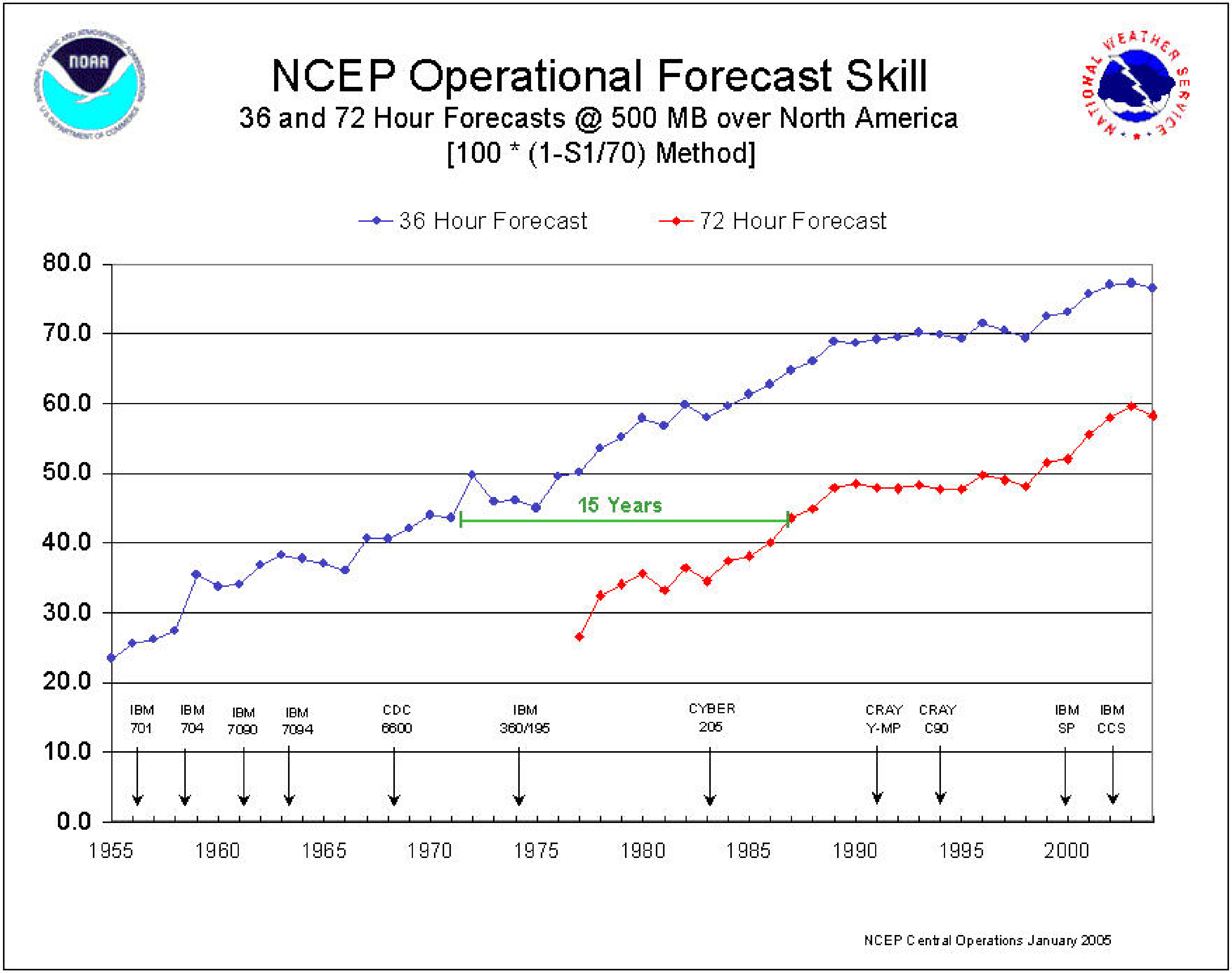
- ***Weather Prediction is now based on solid scientific foundations***
- ***Forecast skill is increasing by one day per decade***
- ***Richardson had the vision and audacity to put his ideas to a practical test***
- ***His method is essentially that in use throughout the globe today.***

The End

Typesetting Software: TEX, *Textures*, LATEX, `hyperref`, `texpower`, Adobe Acrobat 4.05
Graphics Software: Adobe Illustrator 9.0.2
LATEX Slide Macro Packages: Wendy McKay, Ross Moore



The slow manifold



NMC/NCEP Scores: The longest verification series in existence.