Max Margules and his Tendency Equation

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The paper appearing below in translation was published in the Boltzmann Festschrift in 1904. In this short paper, originally entitled *Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung*, Margules examines the relationship between the continuity equation and changes in surface pressure. He concludes that any attempt to derive a reliable estimate of synoptic-scale changes in pressure, using the continuity equation alone, is doomed to failure.

1 Margules' Tendency Paper

Margules begins with the hydrostatic approximation that the pressure at a point is determined by the mass of air above that point (his Assumption (A)). Then the surface pressure is given by $p_0 = \int_0^\infty g\mu\,dz$, where μ is the density. He introduces vertically averaged velocities, which we may define as

$$\mathfrak{u} = \frac{1}{p_0}\int_0^{p_0} u\,dp\,; \qquad \mathfrak{v} = \frac{1}{p_0}\int_0^{p_0} v\,dp\,.$$

He then integrates the continuity equation from the earth's surface, assumed flat, to a height h. If h is assumed large, we arrive at his equation (2) which may be written

$$\frac{\partial p_0}{\partial t} + \nabla\cdot p_0\mathfrak{V} = 0\,, \qquad (1)$$

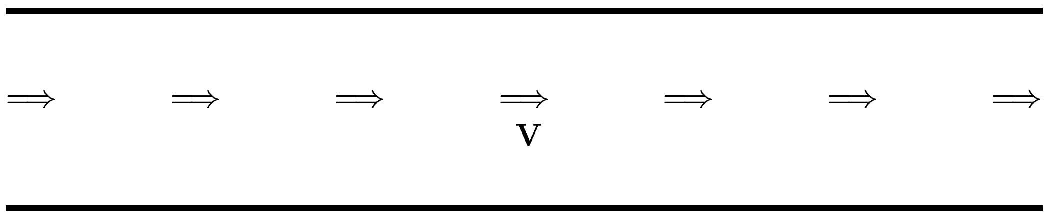
where $\mathfrak{V} = (\mathfrak{u},\mathfrak{v})$ is the vertically averaged horizontal velocity vector. We note that this is equivalent to the more familiar form

$$\frac{\partial p_0}{\partial t} + \int_0^{p_0} \nabla\cdot\mathbf{V}\,dp = 0\,, \qquad (2)$$

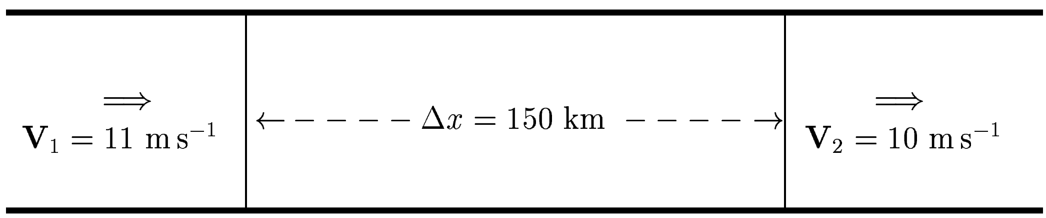
where $\mathbf{V}$ is the horizontal velocity. This is the familiar tendency equation, which we may justifiably call Margules' Equation (Wallace and Hobbs, 1977).

The tendency equation gives us a means of calculating changes in the surface pressure, given the vertical distribution of the wind velocity $\mathbf{V}$. The central result of Margules (1904) is that this procedure is fraught with difficulty. He cites, as motivation for his study, earlier publications of Exner and Trabert which addressed the problem of predicting pressure changes over a day with a view to forecasting the weather. He demonstrates that, if an accurate pressure tendency is to be obtained, the winds must be known to a precision quite beyond what is practically feasible. The ineluctable conclusion is that Margules' Equation alone does not provide a useful means of calculating synoptic pressure changes.

Margules considers various simple cases in which the tendency equation can be easily solved for changes in pressure. We give one example here, based on his special case (2_1). Let us consider uniform zonal flow in a channel



We assume that the velocity $\mathbf{V} = 10\,\mathrm{m\,s^{-1}}$ is constant with height and that the surface pressure is uniform, $p_0 = 10^5\,\mathrm{Pa}$. Clearly, the divergence vanishes identically, so the tendency equation implies that the pressure remains constant. Now let us assume that the velocity is measured at two points, one 150 km downstream from the other. If the observation at the first point overestimates the true value by 10%, we obtain $\mathbf{V}_1 = 11\,\mathrm{m\,s^{-1}}$ and $\mathbf{V}_2 = 10\,\mathrm{m\,s^{-1}}$:



Such a magnitude of the error is typical of realistic observations. We next estimate the divergence in the box between the two points by a finite difference ratio:

$$\nabla \cdot \mathbf{V} \approx \frac{\Delta \mathbf{V}}{\Delta x} = \frac{-1\,\mathrm{m\,s^{-1}}}{1.5 \times 10^5\,\mathrm{m}} = -6.67 \times 10^{-6}\ \mathrm{s^{-1}}$$

But now Margules' Equation (2) gives, for the tendency,

$$\frac{\partial p_0}{\partial t} = -p_0 \nabla \cdot \mathbf{V} = 0.667\,\mathrm{Pa\ s^{-1}}$$

so the pressure in the box should rise by two-thirds of a Pascal every second. Perhaps this seems a small value, but recall the song from *The Pajama Game*:

> Seven and a half cents doesn't buy a heck-of-a-lot,
> Seven and a half cents doesn't mean a thing.
> But give it to me every hour, forty hours of every week,
> That's enough for me to be livin' like a king.

If this tendency is sustained over a long period, the resulting pressure rise is dramatic:

$$\boxed{0.667\,\mathrm{Pa\ s^{-1}} = 144\ \mathrm{hPa\ in\ 6\ hours!}} \qquad (3)$$

The implication is clear: if we apply the tendency equation over a synoptic period, the resulting pressure change may be utterly unrealistic.

Astute readers will be reminded of Richardson's calculated pressure change, of 145 hPa in six hours, almost identical to the value in (3). Richardson (1922) used Margules' Tendency Equation, in the form

$$\frac{\partial p_0}{\partial t} = -g \int_0^\infty \nabla \cdot \mu \mathbf{V}\, dz = 0\,,$$

to obtain this value, so it is hardly surprising that his forecast was unrealistic.

There is no reason to believe that Richardson was aware of Margules' paper; certainly, he makes no reference to it in his book. Its contents were summarized by Exner in his textbook *Dynamische Meteorologie*, which Richardson does cite, but without explicit reference to the relevant section. Since this book was published in 1917, Richardson could not have seen it until his return to Britain after the First World War, and after his trial forecast had been completed. Richardson ascribed the difficulties with his predicted tendency to spurious values of divergence arising from errors in the wind observations. This explanation, while incomplete, is consistent with the analysis of Margules. Had Richardson been aware at an earlier stage of Margules' results, he might well have decided not to proceed with his trial forecast, or sought a radically different approach (Platzman, 1967).

On the final page of his short paper, Margules investigated pressure changes due solely to vertical motion. He considered a column of air extending from the surface to a fixed height h and assumed that all horizontal fluxes vanished. He found that a persistent downward velocity of $1\,\mathrm{cm\,s^{-1}}$ at 10 km — roughly, tropopause height — would cause a pressure drop of 1 mm Hg at that height, or about 8 hPa in six hours. Thus, small persistent vertical velocities can result in large pressure changes. In general, there are both horizontal and vertical fluxes and it is impossible to determine the vertical velocity from the continuity equation alone.

2 Max Margules (1856–1920)

Many outstanding scientists were active in meteorological studies in Austria in the period 1890–1925, and great progress was made in dynamic and synoptic meteorology and in climatology during this time. Amongst the most important members of this 'Vienna School' were Julius Hann, Josef Pernter, Wilhelm Trabert, Felix Exner, Wilhelm Schmidt, Heinrich Ficker, Albert Defant and, of course, Max Margules. The Austrian Central Institute for Meteorology and Geodynamics (ZAMG) recently celebrated its 150th anniversary, in conjunction with which a beautiful book has been produced (Hammerl, *et al.*, 2001) containing contributions on the work of the Vienna School and on the many scientists who worked there (see Davies, 2001; Fortak, 2001; Pichler, 2001).

Margules, one of the founders of dynamical meteorology, was unquestionably a brilliant theoretician, the true value of whose work was adequately appreciated only after his death. The present biographical sketch is based on Khrgian (1959), Kutzbach (1979) and Gold (1920), and on several articles in Hammerl (2001). Margules was born in the town of Brody, in western Ukraine, in 1856. He studied mathematics and physics at Vienna University, and among his teachers was Ludwig Boltzmann.[1] After a two-year spell as a Volunteer at the Meteorological Institute in Vienna, Margules went to Berlin University in 1879. He returned to Vienna University the following year as a lecturer in physics. In 1882 he rejoined the Meteorological Institute as an Assistant, and continued to work there for 24 years.

[1] Indeed, the paper appearing below was first published in the Festschrift on the occasion of Boltzmann's 60th birthday.

Margules studied the diurnal and semi-diurnal variations in atmospheric pressure due to solar radiative forcing, analyzing the Laplace tidal equations and deriving two species of solutions, which he called 'Wellen erster Art' and 'Wellen zweiter Art' (Margules, 1893). This was the first identification of the distinct types of waves now known as inertia-gravity waves and rotational waves. This work, soon followed by Hough's closely related but independent study (Hough, 1898), foreshadowed the studies of atmospheric planetary waves some thirty years later and its full significance was appreciated only after the insights of Rossby (1939) and Haurwitz (1940).

Margules turned next to the study of the source of energy of storms. He demonstrated that the available potential energy associated with horizontal temperature contrasts within a cyclone was, if converted to kinetic energy, sufficient to explain the observed winds. In the course of this work, he derived an expression for the slope of inclination of the boundary between two air masses, a formula which bears his name and is occasionally found in modern textbooks. On the basis of observations carried out at Vienna and Bratislava on 3 December, 1899, Margules showed that surfaces of separation between distinct air masses actually exist in the atmosphere (Kutzbach, 1979). This work overturned the convective theory of cyclones and adumbrated the frontal theory which emerged about a decade later.

Margules was an introverted and lonely man, who never married and worked in isolation, not collaborating with other scientists. His published work was often abstruse and inaccessible and as a result was undervalued. It is fair to say that he was far ahead of his time. Felix Exner, a friend and colleague was one of the few who understood and appreciated Margules research. Margules was disappointed and disillusioned at the lack of recognition of his work and retired from the Meteorological Institute in 1906, aged only fifty, on a modest pension. After retirement he turned his back on meteorology and spent his energy exclusively on chemical studies. His last meteorological publication (Margules, 1906) opened with the surprisingly personal remark "Circumstances, on which I cannot elaborate here, compel me to write down this paper and supplement in haste, and to bid farewell to meteorology" (Fortak, 2001).

In a moving appreciation of Margules shortly after his death, Exner (1920) wrote that Austria had lost an outstanding scientist and that, taking a broad perspective, Margules could be accurately described as one of the first ever theoretical meteorologists. His altogether tragic fate greatly saddened Exner, who described him as one of the loneliest men he had ever known. Gold wrote in an obituary (1920) that "meteorology lost him some fifteen years ago, and is forever the poorer for a loss which one feels might and ought to have been prevented". However, Exner and other colleagues had tried their best to prevent this loss, making repeated offers of help, which Margules resolutely resisted. The value of his pension was severely eroded during the First World War so that his 400 crowns per month was worth about one Euro, insufficient for more than the most meagre survival. He was awarded the Hann Medal by the Austrian Meteorological Society in 1919 but declined the honorarium which accompanied it, with the stoical comment "I would accept the offer if it could be of help but, as things stand, the Institute could put the money to better use" (Hammerl, *et al.*, 2001, page 134). Margules died of starvation just one year later.

In 1987 the computer building of ZAMG was completely renovated and named Max-Margules-Haus. This is most appropriate: Margules' 1904 paper played an important rôle in helping us to understand the problems of numerically integrating the primitive equations used in numerical weather prediction today. In his day, Margules considered that any attempt to predict the evolution of atmospheric flow, that is, to forecast the weather, was premature and indeed futile. He was quoted by Exner as saying that forecasting was "immoral and damaging to the character of a meteorologist". The paper appearing in translation here gives some insight into why Margules held this view.

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