AweSums

Marvels and Mysteries of Mathematics

•

LECTURE 1

Peter Lynch
School of Mathematics & Statistics
University College Dublin

Evening Course, UCD, Autumn 2021

Outline

Outline

Introduction

Overview

Beautiful Spirals

Recreational Mathematics

Distraction 1: A Piem

The Golden Ratio

Symmetry

Visual Maths 1

Aim of the Course

AweSums

The course will run over eight (8) lectures,
from 27 September to 22 November.

No lecture on 25th October.
So, the course splits into 4 + 4.

Aims of the course: to show you

- The great beauty of mathematics;
- Its tremendous utility in our daily lives;
- The fun we can have studying maths.

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "lesson" or "learning".

It is the study of topics such as

- Quantity
- Structure
- Space
- Change

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It is the study of topics such as

- Quantity: [Numbers. Arithmetic]
- Structure: [Patterns. Algebra]
- Space: [Geometry. Topology]
- Change: [Analysis. Calculus]

Thatsmaths.com: A valuable website

Every Thursday, I post a piece to my blog

`https://thatsmaths.com/`

Some of the articles are simple; some are advanced.

There is a search-bar, where you can
find posts on particular topics.

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Let's have a peak, and seek Tom Lerher.

Tom Lehrer: Mathematician, Musician and Comic Genius

Tom Lerher, best known as a brilliant comical songwriter, was also a mathematician.

Several of his songs have a distinct mathematical flavour.

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Several of his songs have a distinct mathematical flavour.

- `/Users/peter/Dropbox/Music/Videos.html`
- Run Video (vsn 2).

Keywords in Tom Lehrer's Song

Counting. Fair Sharing/Division. Folding.

Bouncing balls (dynamics). Recipes/Algorithms.

Money. Noon on the Moon. Getting to Infinity.

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We will certainly be looking at infinity.

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Notes and Slides

- All the slides will be available online:
  http://mathsci.ucd.ie/∼plynch/AweSums
  [just Google for "Peter Lynch UCD"]
- *No notes* are to be provided.
  Why Not? See next slide.
- Additional Reading Recommendations.
- Optional Puzzles and Problems.

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- Additional Reading Recommendations.
- Optional Puzzles and Problems.
- No Assignments!
- No Assessments!
- No Examinations!

Why No Notes?

- Maths is NOT a Spectator Sport
- Active engagement is essential to understanding.
- You should take your own notes:
  - This involves repetition of what you hear.
  - This involves repetition of what you see.
  - What you write passes through your mind!
  - This process is a great help to memory.

Lectures

- Classes run from 7pm to 9pm.
- 120 minutes intensive lecturing too long (both for you and for me).
- Educational Theory:
  - Attention & retention both decrease with time.
- Class will be broken into short sections.

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If you cannot attend a class:

- Please do not bother to email me.
- There is no need to give any reasons.
- The presentation slides will be available.

Communications

In the unlikely event that a class has to be cancelled
I will notify “UCD Adult & Lifelong Learning”.

You may wish to form a WhatsApp group.

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I will also tell you about other mathematical events in Ireland if I hear about them.

FOR EXAMPLE ....>

Hamilton Lecture, 2021

Glimpses into Hyperbolic Geometry
Caroline Series, Warwick University
Friday, October 15, 19:00
Free: booking at `www.ria.ie`

Ben Green Lecture

Louise and Richard K. Guy Lecture
Unsolved Problems in Number Theory
University of Calgary.

Ben Green, Oxford University

Wednesday, September 29, 2021

Time: Noon – 1:00 p.m. (MT) $=$ 19:00 UTC?

“Maths in Human Society”

LMS/IMA Joint Meeting 2021
Maths in Human Society
Location: Online
30 Sept. – 1 Oct. 2021

Google for
`` ``LMS/IMAJointMeeting2021" ``

“Typical” Structure of a Class

1. Problem: Background and Theory
2. Distraction (10 min)
3. Some History of the problem
4. Another Distraction
5. Exercises, Puzzles, History
6. Questions & Discussion

Total duration: about 120 minutes.

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I will (normally) be available after classes
to answer questions or offer clarifications.

Some Distractions

- Visual Awareness: Maths Everywhere
- Puzzles: E.g. Watermelon Puzzle
- Sieve of Eratosthenes
- The Greek Alphabet
- Lateral Thinking in Maths
- *Lecture sans paroles*
- How Cubic and Quartic Equations were cracked
- Four-colour Theorem
- Online Encyclopedia of Integer Sequences

Please ask me if you have a favorite topic!

It's Your Course!!!

I expect a group with a wide range of knowledge and "mathematical maturity".

Everybody should benefit from the course.

If anything is unclear, **SHOUT OUT!** or whisper!

If something is missing, let me know.

Feedback on the course is very welcome.

It’s Your Course!!!

Classes begin at 7 pm. and run till 9 pm.

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There seem to be two options:

- Break at 7:50 for 10, 15 or 20 minutes.
- Don’t break at all !!!

I have no strong views but I suspect that there might be a riot if we do not have a break.

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I have no strong views but I suspect that there might be a riot if we do not have a break.

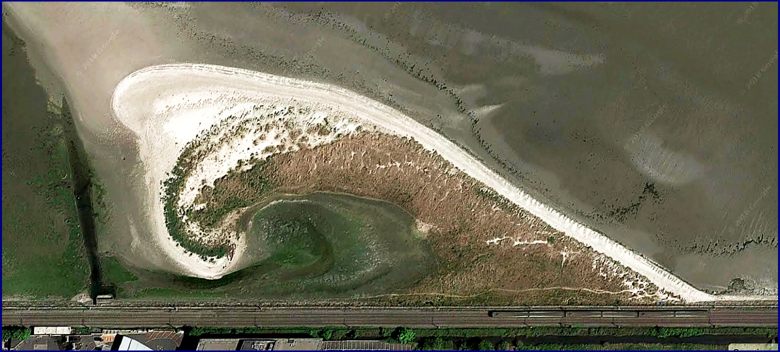
Let's have a poll: Who wants a break?

Popular Mathematics Books

1. John H Conway and Richard K Guy, 1996:
   The Book of Numbers. Copernicus, New York.
2. ♡ ⇒ John Darbyshire, 2004:
   Prime Obsession. Plume Publishing.
3. ♡ ⇒ William Dunham, 1991:
   Journey through Genius. Penguin Books.
4. Marcus Du Sautoy, 2004:
   The Music of the Primes. Harper Perennial.
5. ♡ ⇒ Richard Elwes, 2010:
   Mathematics 1001. Firefly Books.
6. Peter Lynch, 2016: *That's Maths.*
   Gill Books. Published in October 2016.

Outline

A Splendid Spiral in Booterstown



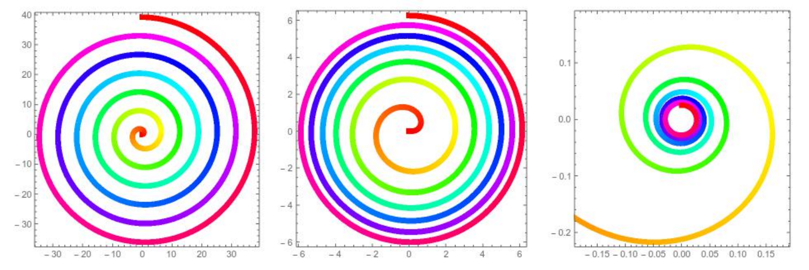
This sandbank, a beautiful spiral form, has slowly built up on the beach near Booterstown Station.

Spirals are found throughout the natural world.

A Splendid Spiral in Booterstown

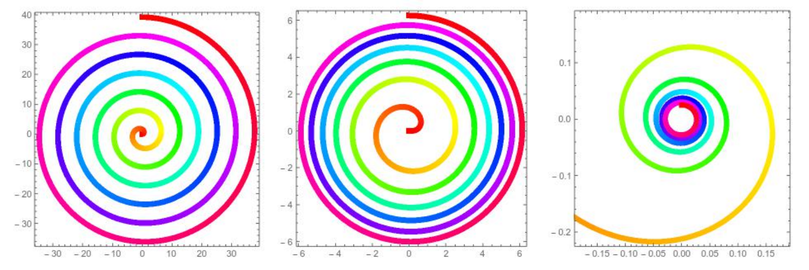
A recent update (Sept. 2021).

Some Mathematical Spirals



Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.

Some Mathematical Spirals

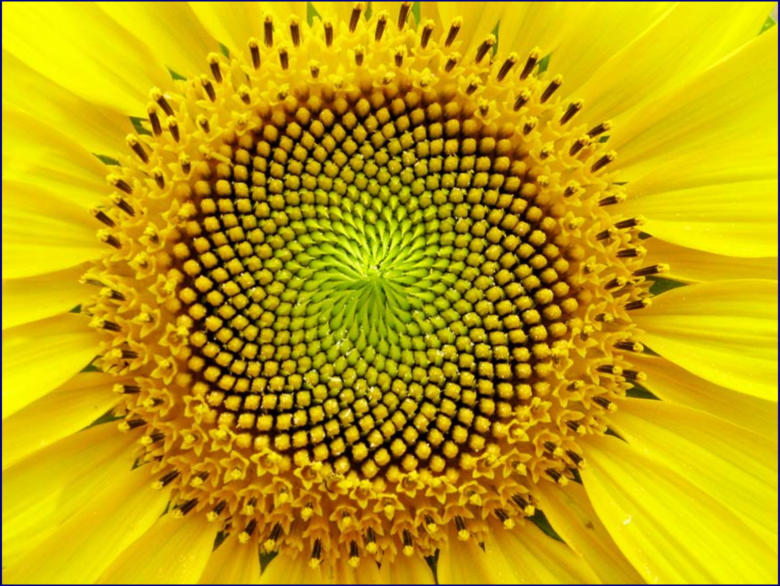


Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.

Challenge: Find mathematical equations for these.
Hint: Use polar coordinates (r, θ).

The Nautilus Shell: *a logarithmic Spiral.*

The Sunflower: Groups of Spirals



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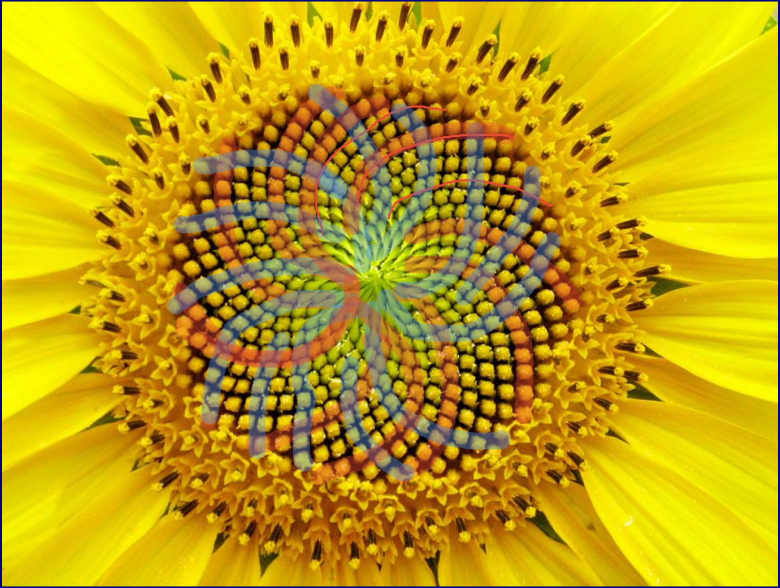
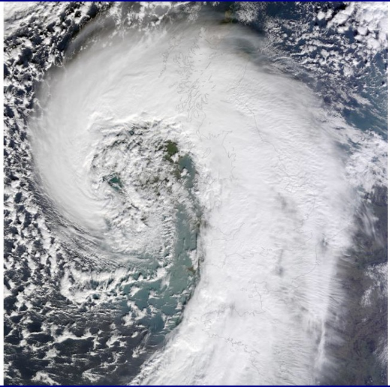


Spirals in the Physical World



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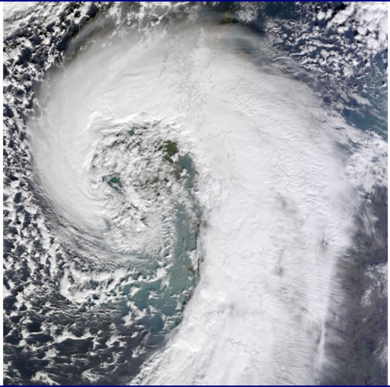
Spirals in the Physical World



⋆ ⋆ ⋆

https://thatsmaths.com/
[Search for "Spirals"]

Fibonacci Numbers

- Count the petals on a flower.
- Count leaves on a stem or
  bumps on an asparagus.
- Look at patterns on pineapples/pine-cones.
- Study the geometry of seeds on sunflowers.

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  bumps on an asparagus.
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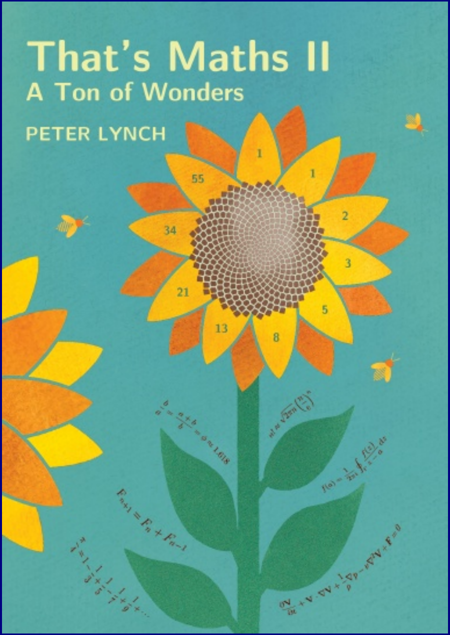
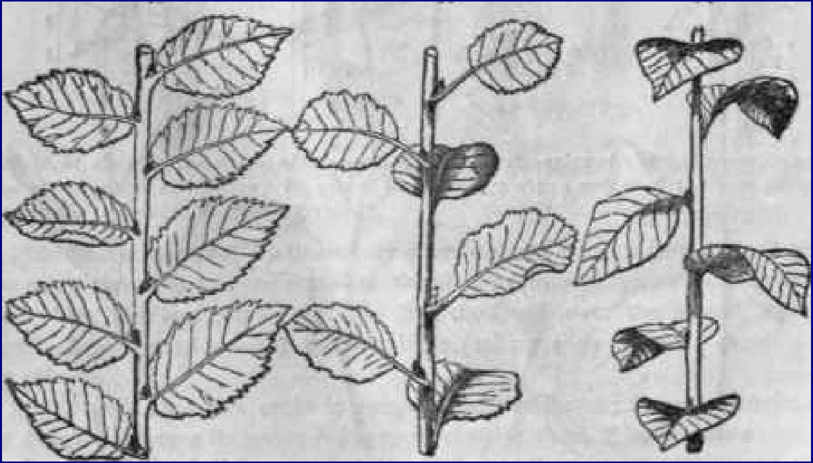
In all cases, we find numbers in the sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

This is the famous Fibonacci sequence.


That's Maths II
A Ton of Wonders
PETER LYNCH
1
1
2
3
5
8
13
21
34
55
$F_{n+1} = F_n + F_{n-1}$

Fibonacci and Phyllotaxis



Vi Hart's Videos

There are several mathematical videos on YouTube presented by Vi Hart.

Some of the ones on Fibonacci Numbers are at:

https://www.youtube.com/watch?v=ahXIMUkSXX0

It is *much easier* to go to YouTube and search for

"Vi Hart   Fibonacci"

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Let's take a peek!

Outline

ï»¿

Making Mathematics Accessible

We all enjoy Sports — watching or participating.

Music gives us enormous pleasure
even if we cannot compose a symphony.

It is the same with mathematics:
Great satisfaction is available to anyone
who invests a small effort in the subject.

Mathematics is widely considered formidable and intimidating. However, many people derive great joy and fulfilment through recreational maths.

Recreational Mathematics

Recreational mathematics puts the focus on
insight, imagination and beauty.

Recreational Maths includes the study of

- The culture of mathematics,
- Its relevance to art, music and literature,
- Its role in technology,
- The lives of the great mathematicians.
- Fractals, paradoxes, games, puzzles, etc.

Contributions of Amateurs

Some fields of mathematics have advanced through the activities of amateurs.

- Probability,
- Number theory,
- Graph Theory,
- Combinatorics.

Martin Gardner

Martin Gardner introduced millions of people to the wonder, variety and sheer fun of mathematics.

He turned thousands of children into mathematicians and thousands of mathematicians into children.

For decades, Gardner tried to convince educators that recreational mathematics should be included in the standard curriculum.

Sadly, “movement in that direction has been glacial.”

Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine Plus:
`https://plus.maths.org/`

Articles in Plus will open your eyes to the many uses of maths in the real world.

All past content is available and is a valuable resource for school students and teachers.

Many Resources Available

International Conferences on Recreational Maths.

Ludus (URL), biennial conference in Lisbon.
Electronic journal Recreational Mathematics Mag.

http://rmm.ludus-opuscula.org/

twice per year (free)

MOVES (Maths of Various Entertaining Subjects)

http://momath.org/moves-conference/

National Museum of Mathematics (Manhattan)

http://momath.org/

New Maths Gallery, Science Museum, Kensington.

http://www.sciencemuseum.org.uk

Outline

Distraction 1: Remember π

The ratio of circumference of circle to diameter is π.

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How I want a drink,
Alcoholic of course,
After the heavy lectures
involving quantum mechanics.

Distraction 1: Remember π

How I want a drink,
Lemonsoda of course,
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Distraction 1: Remember π

How I want a drink,
Lemonsoda of course,
After the heavy lectures
involving quantum mechanics.

How I want a drink,
Sugarfree of course,
After the heavy lectures
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Repeat: To Remember π

To 15-figure accuracy, π is equal to

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How can we remember this without much effort?

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Can I remember the reciprocal?

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Distraction 1: Remember $1/\pi$

The reciprocal of π is approximately 0.318310
Can I remember the reciprocal?

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3 1 8 3 10

Now you know π and $1/\pi$ to an accuracy greater than you are ever likely to need!

Outline

Golden Ratio and Fibonacci Numbers

The Golden Ratio is a number defined as

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618.$$

It is intimately connected with
the Fibonacci Numbers.

Golden Rectangle

Ratio of breath to height is $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$**.**

Golden Rectangle in Your Pocket

Aspect ratio is about $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$.

Fibonacci Numbers

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The explicit expression for the Fibonacci numbers is

$$F_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n - \frac{1}{\sqrt{5}}\left[\frac{1-\sqrt{5}}{2}\right]^n$$

Fibonacci Numbers

Let's consider the sequence of ratios of terms

$$\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \ldots$$

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The ratios get ever-closer to the golden number:

$$\frac{F_{n+1}}{F_n} \to \phi \quad \textbf{as} \quad n \to \infty$$

Exotic Expressions for ϕ

We can write ϕ as a **continued fraction**

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

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These extraordinary expressions are actually quite easy to demonstrate!

Fibonacci Numbers in Nature

Look at post

Sunflowers and Fibonacci: Models of Efficiency

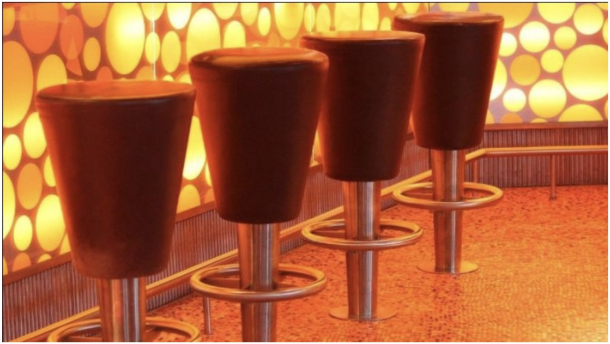
on the *ThatsMaths* blog:

`thatsmaths.com`

Round Bar Stool in a Square Room Puzzle

You have 14 round Bar Stools and a square room.
How can you place the Bar Stools along the walls so that you have the same number along each wall?

Please draw a diagram (hand drawn is fine) and submit as a JPEG file.



Send your answers to mathscareers@ima.org.uk

Outline

Ubiquity and Beauty of Symmetry

Symmetry is all around us.

- Many buildings are symmetric.
- Our bodies have bilateral symmetry.
- Crystals have great symmetry.
- Viruses can display stunning symmetries.
- At the sub-atomic scale, symmetry reigns.
- Galaxies have many symmetries.

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Like spirals, symmetry is found at all scales.

The Taj Mahal

A Face with Symmetry: Halle Berry

Halle Berry **Berry Halle**

An Asymmetric Face: You know Who!

Symmetry and Group Theory

Symmetry is an essentially geometric concept.

The mathematical theory of symmetry is algebraic.

The key concept is that of a group.

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The mathematical theory of symmetry is algebraic.

The key concept is that of a group.

A group is a set of elements such that any two elements can be combined to produce another.

Instead of giving the mathematical definition, I will give an example to make things clear.

The *Dihedral Group* D_1

The group of symmetries of the human face and of all biological forms with bilateral symmetry. We could call D_1 the *Janus Group.*

I : The Identity transformation

R : Reflection about central line

Table: First Dihedral Group D_1.

	I	R
I	I	R
R	R	I

This is how we combine, or multiply transformations.

From 2 to 3 Dimensional Symmetry

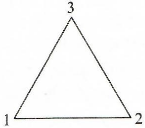


Fig. 4.6

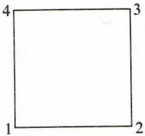
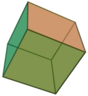
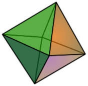
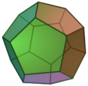
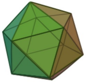


Fig. 4.7

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
 (Animation)	 (Animation)	 (Animation)	 (Animation)	 (Animation)

Outline

To Begin: An Optical Illusion

A cautionary tale:

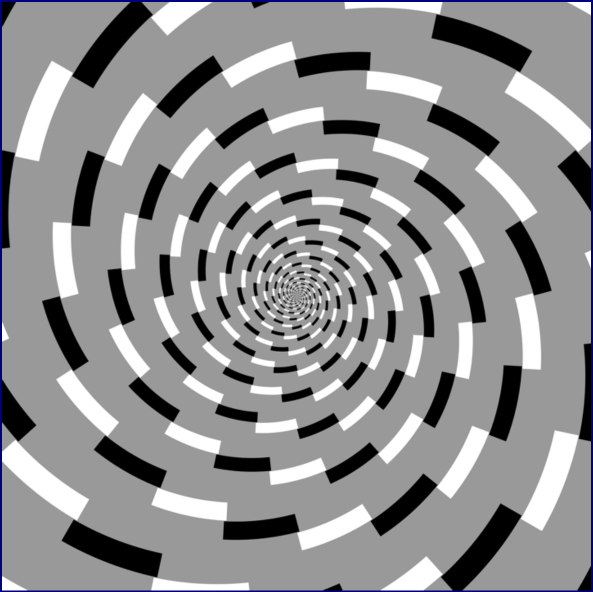
In maths we often use pictures to prove things.

This is usually very helpful.

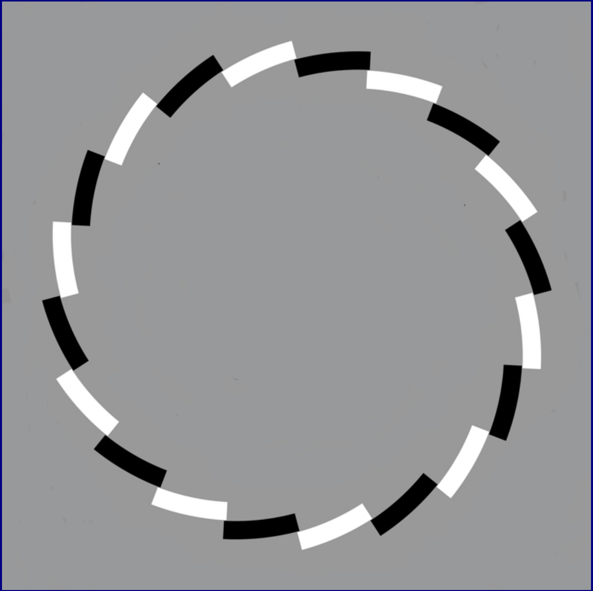
However, it can sometimes mislead us.

Let us look at the Fraser Spiral.

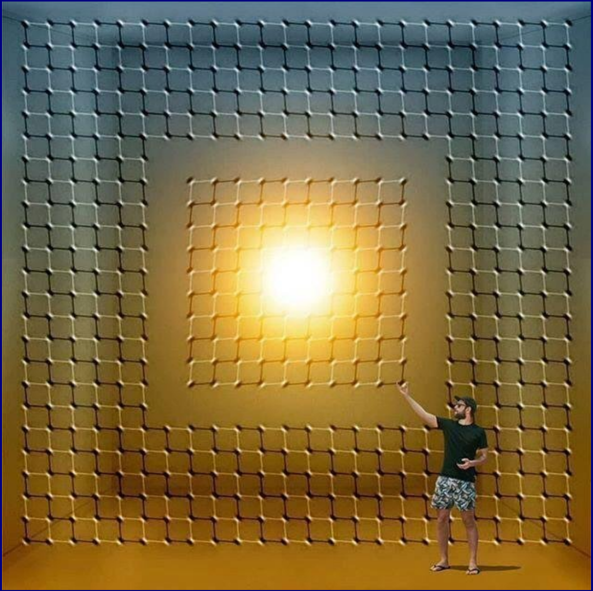
Fraser Spiral



Fraser Spiral



https://pbs.twimg.com/media/DI5ImeIU8AEpXB7.jpg



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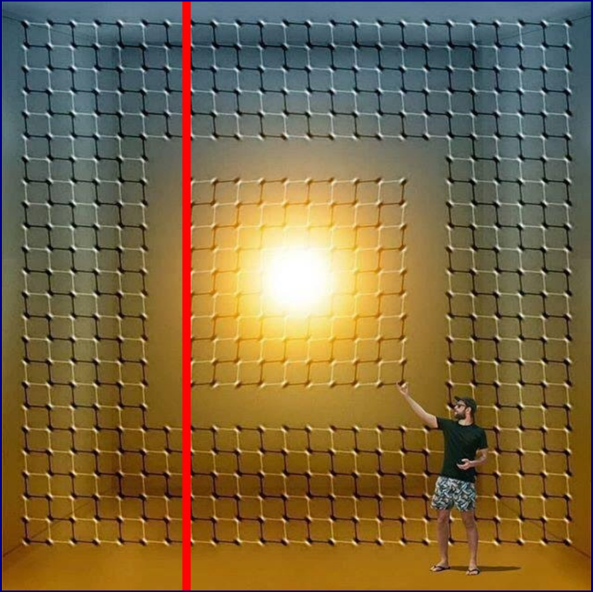
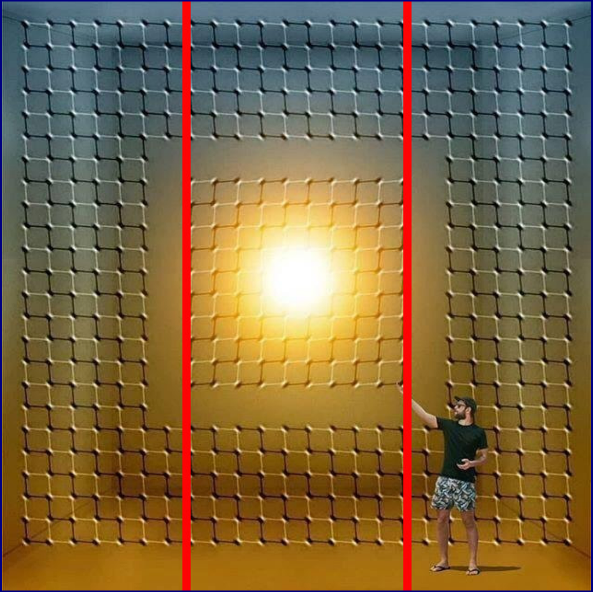


https://pbs.twimg.com/media/Dl5lmelU8AEpXB7.jpg



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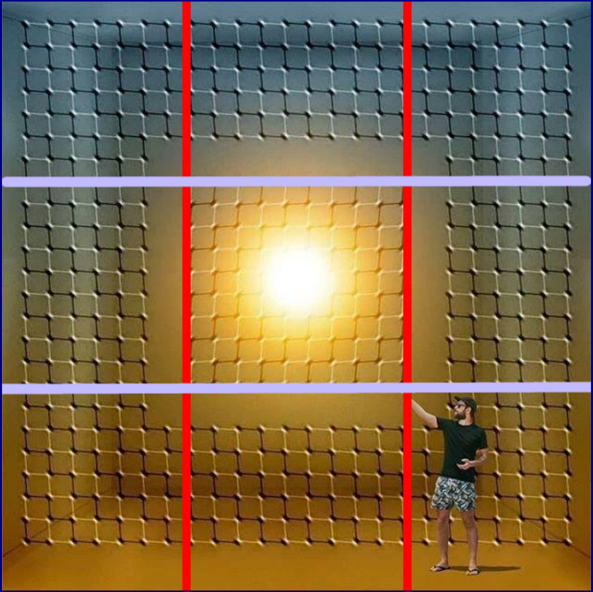

Visual Maths Proofs

Can the sum of an infinite number of quantities have a finite value?

Let's look at the infinite series

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

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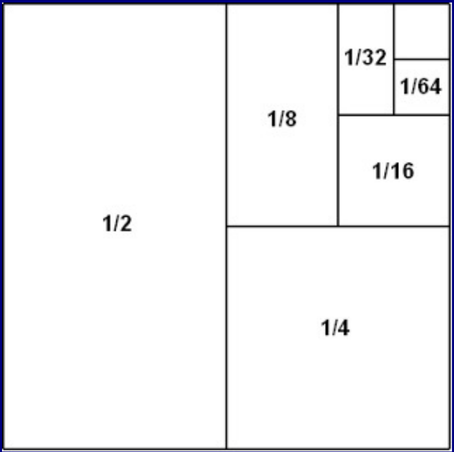
Each term is half the size of the preceding one.

The terms are getting smaller but it is not obvious that the series converges.

A picture makes everything clear:



Unit Square: At each stage, we add **half the remainder** of the square.

Conclusion

The infinite series

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

has a finite sum:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1$$

The terms are getting smaller quickly enough for the series to be convergent.

Another Simple Proof

What is the sum of the first *n* odd numbers?

$$1 = 1^2 \qquad (1+3) = 4 = 2^2 \qquad (1+3+5) = 9 = 3^2$$

Is this pattern continued? Can we prove it?

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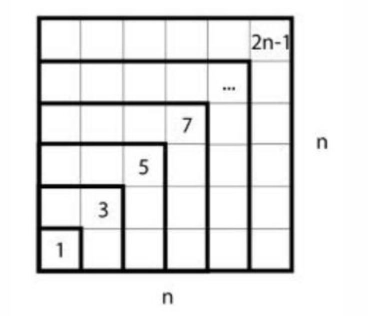

$$[1+3+5+7+\cdots+(2n-1)] = n^2$$

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$$S = 2 + 4 + 6 + 8 + \cdots 2n$$

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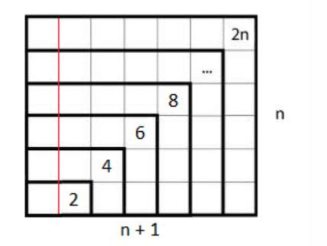

We just add a column on the left. This increases each term of the sequence of odd numbers by 1.

$$[2 + 4 + 6 + \cdots + 2n] = n(n + 1)$$

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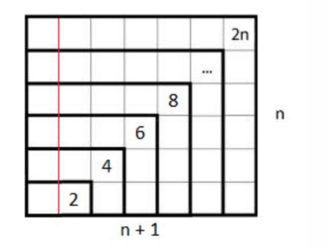

We just add a column on the left. This increases each term of the sequence of odd numbers by 1.

$$[2 + 4 + 6 + \cdots + 2n] = n(n+1)$$

Now divide both sides by 2 to get:

$$[1 + 2 + 3 + \cdots + n] = \frac{1}{2}n(n+1)$$

Thank you