AweSums

The Fun and Joy of Mathematics

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TASTER LECTURE

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School of Mathematics & Statistics
University College Dublin

Evening Course, UCD, Autumn 2021

Outline

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WELCOME TO

AweSums

Marvels and Mysteries of Mathematics

Taster Lecture

The course AweSums will have eight lectures from 27 September to 22 November, 2021.

Splits into two groups of four lectures.
Sessions on Mondays at 7:00 pm.
No Lecture on 25 October.

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The aim of the course is to show you

- **The tremendous beauty of mathematics;**
- **Its great utility in our daily lives;**
- **The fun we can have studying maths.**

Taster Lecture

This is the sixth time I have taught a popular maths course. Sometimes it was called

- Sum-enchanted Evenings.

Other years the title was something like

- AweSums: The Majesty of Maths

The course is broarly similar from year to year, but I generally include some new material each time.

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IF THERE IS A TOPIC YOU'LD LIKE, PLEASE LET ME KNOW. MAYBE, I CAN INCLUDE IT!

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "lesson" or "learning".

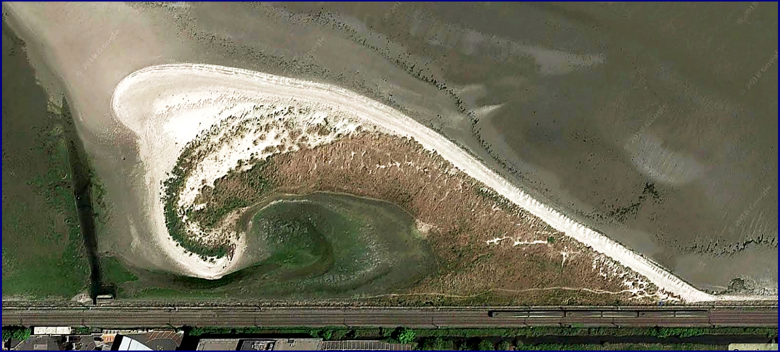
It is the study of topics such as

- Quantity (numbers)
- Structure (patterns)
- Space (geometry)
- Change (analysis).

Quote Ian Stewart in *From Here to Infinity*.

Outline

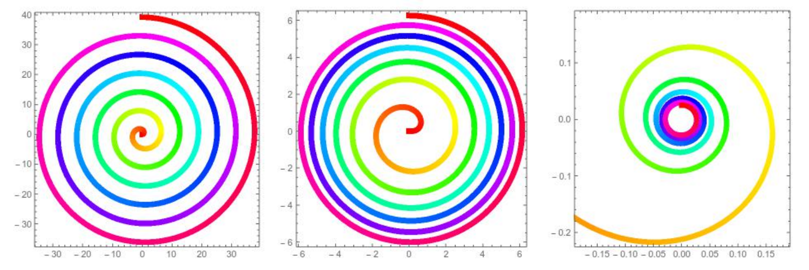
A Splendid Spiral in Booterstown



This sandbank, a beautiful spiral form, has slowly built up on the beach near Booterstown Station.

Spirals are found throughout the natural world.

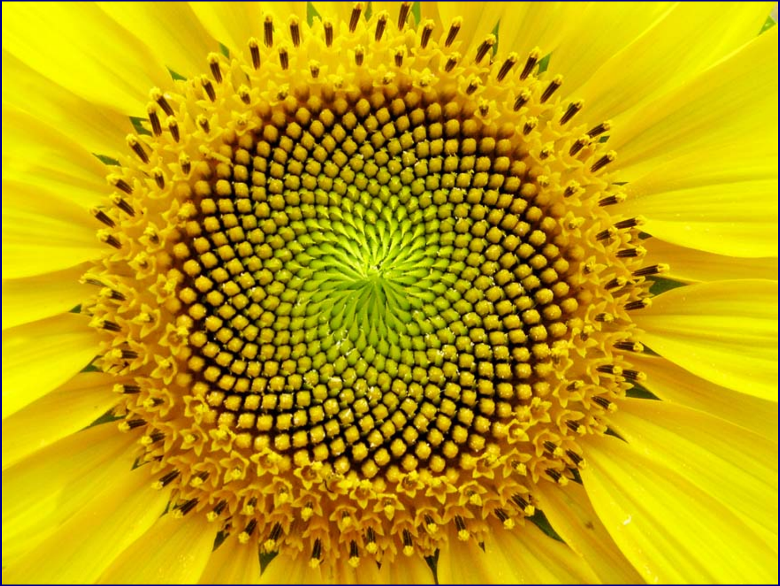
Some Mathematical Spirals



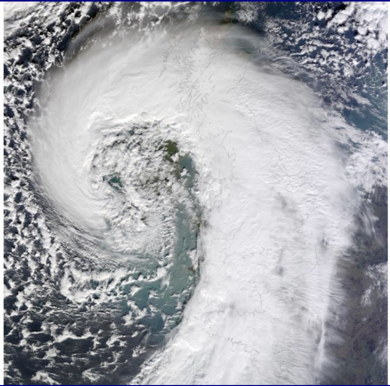
Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.

The Nautilus Shell: *a logarithmic Spiral.*

The Sunflower: Groups of Spirals



Spirals in the Physical World



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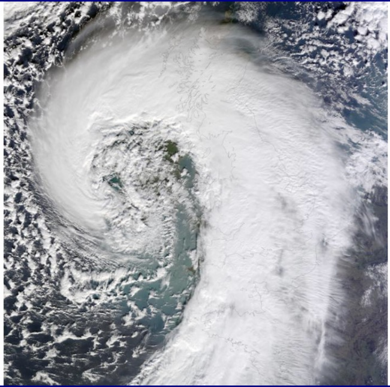

⋆ ⋆ ⋆

https://thatsmaths.com/
[Search for "Spirals"]

Fibonacci Numbers

- Count the petals on a flower.
- Count leaves on a stem or
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- Look at patterns on pineapples/pine-cones.
- Study the geometry of seeds on sunflowers.

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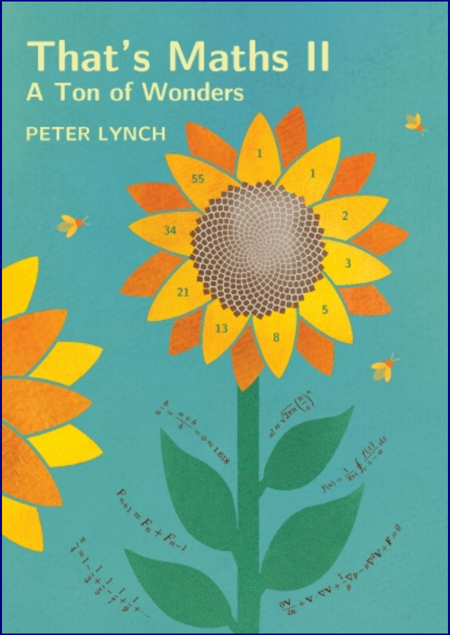
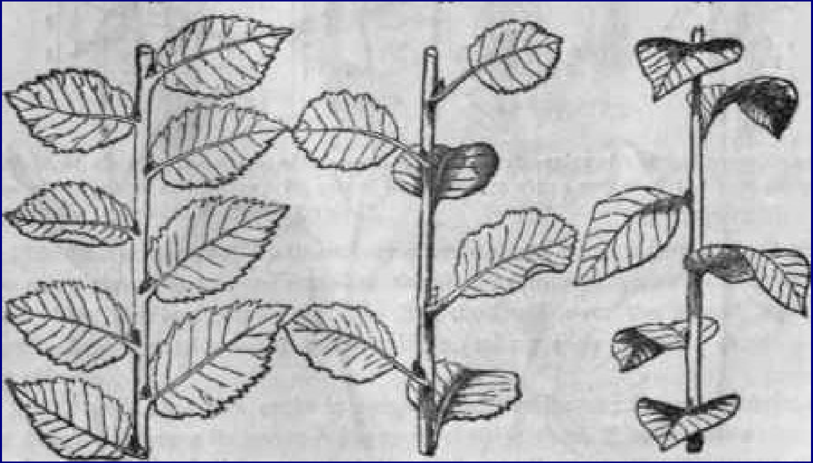
In all cases, we find numbers in the sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

This is the famous Fibonacci sequence.


That's Maths II
A Ton of Wonders
PETER LYNCH
1
1
2
3
5
8
13
21
34
55
F_{n+1} = F_n + F_{n-1}

Fibonacci and Phyllotaxis



Vi Hart's Videos

There are several mathematical videos on YouTube presented by Vi Hart.

Some of the ones on Fibonacci Numbers are at:

`https://www.youtube.com/watch?v=ahXIMUkSXX0`

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Let's take a peek!

Outline

Golden Ratio and Fibonacci Numbers

The Golden Ratio is a number defined as

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618.$$

It is intimately connected with the Fibonacci Numbers.

Golden Rectangle

Ratio of breath to height is $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$.

Golden Rectangle in Your Pocket

Aspect ratio is about $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$.

Fibonacci Numbers

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The explicit expression for the Fibonacci numbers is

$$F_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n - \frac{1}{\sqrt{5}}\left[\frac{1-\sqrt{5}}{2}\right]^n$$

Fibonacci Numbers

Let's consider the sequence of ratios of terms

$$\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \ldots$$

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The ratios get ever-closer to the golden number:

$$\frac{F_{n+1}}{F_n} \to \phi \quad \textbf{as} \quad n \to \infty$$

Exotic Expressions for ϕ

We can write ϕ as a continued fraction

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

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These extraordinary expressions are actually quite easy to demonstrate!

Fibonacci Numbers in Nature

Look at post

Sunflowers and Fibonacci: Models of Efficiency

on the *ThatsMaths* blog:

`thatsmaths.com`

Outline

Ubiquity and Beauty of Symmetry

Symmetry is all around us.

- Many buildings are symmetric.
- Our bodies have bilateral symmetry.
- Crystals have great symmetry.
- Viruses can display stunning symmetries.
- At the sub-atomic scale, symmetry reigns.
- Galaxies have many symmetries.

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Like spirals, symmetry is found at all scales.

The Taj Mahal

A Face with Symmetry: Halle Berry

Halle Berry | Berry Halle

An Asymmetric Face: You know Who!

Symmetry and Group Theory

Symmetry is an essentially geometric concept.

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The key concept is that of a group.

A group is a set of elements such that any two elements can be combined to produce another.

Instead of giving the mathematical definition, I will give an example to make things clear.

The *Dihedral Group* D_1

The group of symmetries of the human face and of all biological forms with bilateral symmetry. We could call D_1 the *Janus Group.*

I : The Identity transformation

R : Reflection about central line

Table: First Dihedral Group D_1.

	I	R
I	I	R
R	R	I

This is how we combine, or multiply transformations.

From 2 to 3 Dimensional Symmetry

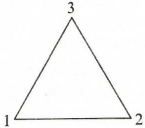


Fig. 4.6

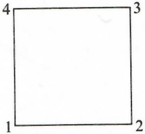
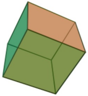
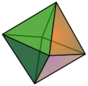
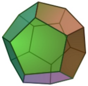
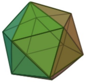


Fig. 4.7

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
				
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Outline

Recreational Mathematics

Recreational mathematics puts the focus on
insight, imagination and beauty.

Recreational Maths includes the study of

- The culture of mathematics,
- Its relevance to art, music and literature,
- Its role in technology,
- Mathematical games and puzzles,
- The lives of the great mathematicians.

Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine Plus:

`https://plus.maths.org/.`

All past content is available and is a valuable resource for school students and teachers.

Content of an Earlier Course

Lecture	*Content*
1	Outline of Course. Emergence of Numbers.
2	Georg Cantor. Set Theory.
3	Pythagoras. Irrational Numbers.
4	Hilbert. Gauss. The Real Number Line
5	Powers. Logarithms. Prime Numbers.
6	Functions. Archimedes. Natural Logs.
7	Exponential Growth. Euler. Sequences & Series.
8	Trigonometry. Taylor Series.
9	Basel Problem. Complex Numbers. Euler's Formula.
10	Prime Number Theorem. Riemann Hypothesis.

This year's course will be different: Better!

Thank you