AweSums

The Fun and Joy of Mathematics

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TASTER LECTURE

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Evening Course, UCD, Autumn 2020

Outline

Introduction

Beautiful Spirals

The Golden Ratio

Symmetry

Recreational Mathematics

Outline

WELCOME TO
AweSums
The Fun and Joy of Mathematics

Taster Lecture

The course AweSums will run over six (6) lectures from 5 October to 16 November.

The aim of the course is to show you

- ▶ **The tremendous beauty of mathematics;**
- ▶ **Its great utility in our daily lives;**
- ▶ **The fun we can have studying maths.**

Taster Lecture

For several years I have taught a course called
- ► **Sum-enchanted Evenings.**

It has worked well, and this year's course,
- ► **AweSums: The Majesty of Maths**

will be similar, but with *much new material.*

**In this Taster Lecture I will give a sample
of some of the topics covered in the course.**

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ **(máthéma), meaning "knowledge" or "study" or "learning".**

It is the study of topics such as

- ► *Quantity* **(numbers)**
- ► *Structure* **(patterns)**
- ► *Space* **(geometry)**
- ► *Change* **(analysis).**

Outline

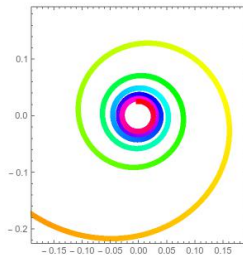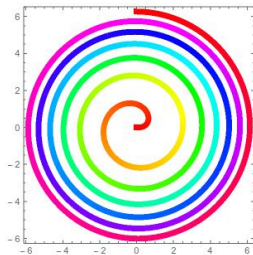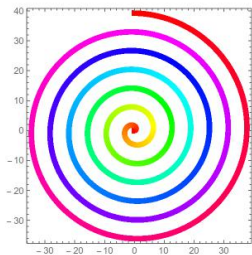# A Splendid Spiral in Booterstown

This sandbank, a beautiful spiral form, has slowly built up on the beach near Booterstown Station.

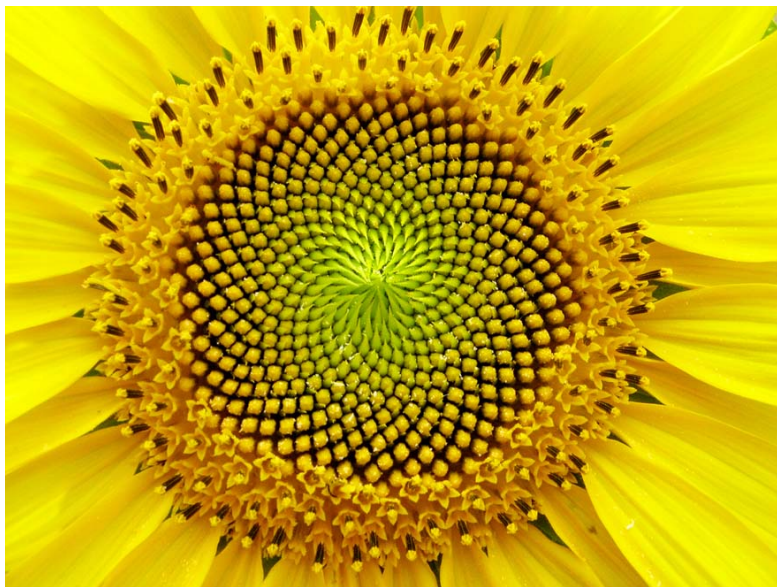**Spirals are found throughout the natural world.**

Some Mathematical Spirals

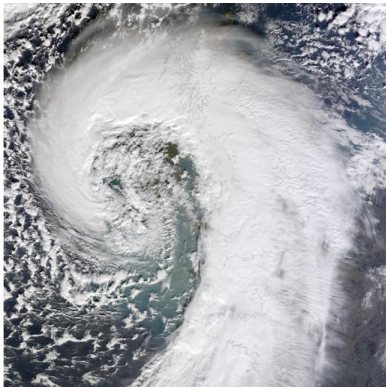**Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.**

The Nautilus Shell: *a logarithmic Spiral.*

The Sunflower: Groups of Spirals

Spirals in the Physical World

$\star$ $\quad$ $\star$ $\quad$ $\star$

https://thatsmaths.com/

Fibonacci Numbers

- **Count the petals on a flower.**
- **Count leaves on a stem or bumps on an asparagus.**
- **Look at patterns on pineapples/pine-cones.**
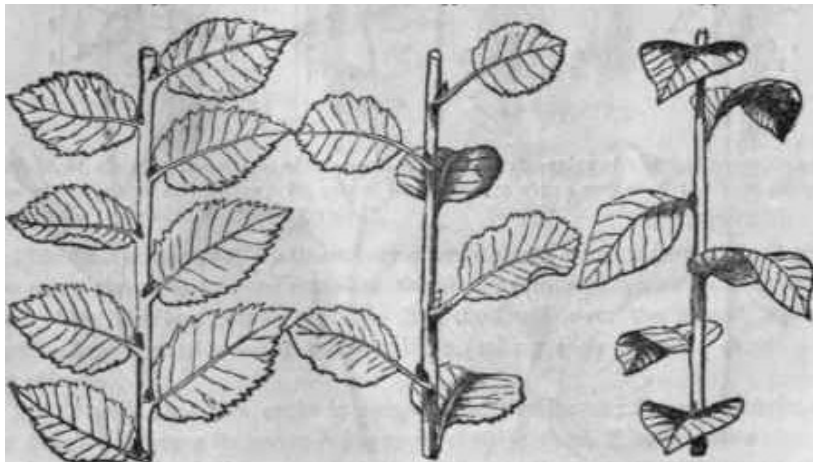- **Study the geometry of seeds on sunflowers.**

In all cases, we find numbers in the sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

This is the famous Fibonacci sequence.

Fibonacci and Phyllotaxis

Vi Hart's Videos

**There are several mathematical videos
on YouTube presented by Vi Hart.**

Some of the ones on Fibonacci Numbers are at:

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https://www.youtube.com/
    watch?v=ahXIMUkSXX0
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It is *much easier* to go to Youtube and search for

"Vi Hart   Fibonacci"

Let's take a peek!

Outline

Golden Ratio and Fibonacci Numbers

The Golden Ratio is a number defined as

$$\phi = \frac{1 + \sqrt{5}}{2} \approx 1.618.$$

**It is intimately connected with
the *Fibonacci Numbers.***

Golden Rectangle

Ratio of breath to height is $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$.

Golden Rectangle in Your Pocket

Aspect ratio is about $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$**.**

Fibonacci Numbers

The Fibonacci sequence is the sequence

$$\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \dots\}$$

where *each number is the sum of the previous two.*

The Fibonacci numbers obey a recurrence relation

$$F_{n+1} = F_n + F_{n-1}$$

with the *starting values* $F_0 = 0$ *and* $F_1 = 1$.

The explicit expression for the Fibonacci numbers is

$$F_n = \frac{1}{\sqrt{5}} \left[\frac{1 + \sqrt{5}}{2} \right]^n - \frac{1}{\sqrt{5}} \left[\frac{1 - \sqrt{5}}{2} \right]^n$$

Fibonacci Numbers

Let's consider the sequence of ratios of terms

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \cdots$$

The ratios get closer and closer to the golden number:

$$\frac{F_{n+1}}{F_n} \to \phi \quad \textbf{as} \quad n \to \infty$$

Exotic Expressions for ϕ

We can write ϕ as a *continued fraction*

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

We can also write it as a *continued root*

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}}$$

These extraordinary expressions are actually quite easy to demonstrate!

Fibonacci Numbers in Nature

Look at post

Sunflowers and Fibonacci: Models of Efficiency

on the *ThatsMaths* blog:

thatsmaths.com

Outline

Ubiquity and Beauty of Symmetry

Symmetry is all around us.

- **Many buildings are symmetric.**
- **Our bodies have bilateral symmetry.**
- **Crystals have great symmetry.**
- **Viruses can display stunning symmetries.**
- **At the sub-atomic scale, symmetry reigns.**
- **Galaxies have many symmetries.**

The Taj Mahal

A Face with Symmetry: Halle Berry

Halle Berry   Berry Halle

An Asymmetric Face: You know Who!

Symmetry and Group Theory

Symmetry is an essentially *geometric* concept.

The mathematical theory of symmetry is *algebraic.*

The key concept is that of a group.

A group is a *set of elements* such that any two elements can be combined to produce another.

Instead of giving the mathematical definition, I will give an example to make things clear.

The *Dihedral Group* D₁

The group of symmetries of the human face and of all biological forms with bilateral symmetry. We could call D_1 the *Janus Group.*

I : The Identity transformation

R : Reflection about central line

Table: First Dihedral Group D_1.

	I	R
I	I	R
R	R	I

This is how we combine, or *multiply* transformations.

From 2 to 3 Dimensional Symmetry
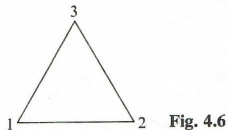
Fig. 4.6
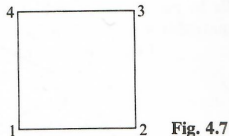
Fig. 4.7

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
(Animation)	(Animation)	(Animation)	(Animation)	(Animation)

Outline

Recreational Mathematics

Recreational mathematics puts the focus on insight, imagination and beauty.

Recreational Maths includes the study of

- **The culture of mathematics,**
- **Its relevance to art, music and literature,**
- **Its role in technology,**
- **Mathematical games and puzzles,**
- **The lives of the great mathematicians.**

Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine Plus:

`https://plus.maths.org/`.

All past content is available and is a valuable resource for school students and teachers.

Content of an Earlier Course

Lecture	Content
1	Outline of Course. Emergence of Numbers.
2	Georg Cantor. Set Theory.
3	Pythagoras. Irrational Numbers.
4	Hilbert. Gauss. The Real Number Line
5	Powers. Logarithms. Prime Numbers.
6	Functions. Archimedes. Natural Logs.
7	Exponential Growth. Euler. Sequences & Series.
8	Trigonometry. Taylor Series.
9	Basel Problem. Complex Numbers. Euler's Formula.
10	Prime Number Theorem. Riemann Hypothesis.

This year's course will be different: Better!

Thank you