

AweSums

Marvels and Mysteries of Mathematics



LECTURE 2

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**School of Mathematics & Statistics
University College Dublin**

Evening Course, UCD, Autumn 2020



Outline

Introduction

The Beginnings

Shackleton's Rescue Voyage

Babylonian Numeration Game

Distraction 2A: Simpsons

The Nippur Tablet

Georg Cantor

Distraction 2B: Books

Set Theory I



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Meaning and Content of Mathematics

The word **Mathematics** comes from Greek *μαθημα* (*máthéma*), meaning “knowledge” or “study” or “learning”.

It is the study of topics such as

- ▶ Quantity (numbers)
- ▶ Structure (patterns)
- ▶ Space (geometry)
- ▶ Change (analysis).



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The Ancient Origins of Mathematics

Basic social living was possible without numbers

... but ...

elementary comparisons and measures are needed to ensure fairness and avoid conflicts.

The need for mathematical thinking arose in problems like fair division of food.

Problem: How do you divide a woolly mammoth?



Division of Food

To divide a collection of apples, the idea of a *one-to-one correspondence* arose.

There was no direct need for *numbers* yet: the apples did not need to be counted, just broken into batches.

The problem of dividing up a slaughtered animal is more tricky: The forequarters and hindquarters of a woolly mammoth are not the same!



Fair Division: Main Idea

- ▶ **Divide a set of goods or resources between several people.**
- ▶ **Each person should receive his/her due share.**
- ▶ **Each person should be satisfied after the division (this is an *envy-free solution*).**

This problem arises in various real-world settings: auctions, divorce settlements, electronic spectrum and frequency allocation, airport traffic management.

It is an active research area in Mathematics, Economics, Conflict Resolution, and more.



I Cut and You Choose

For two people or two families, the familiar technique “I cut and you choose” should keep everyone happy.

This is the method used by children to divide a cake.

It works even for an inhomogeneous cake, say, half chocolate and half lemon sponge.



To divide fairly between all members of a family is *much more difficult* (as many of you know!).

Exercise: Try to devise a generalization of the “cut-and-choose” method that works for three people ... and one that works for four people.

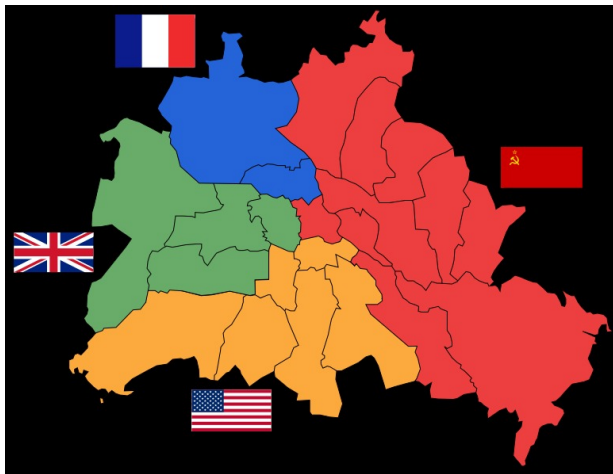
This is a difficult problem

It seems like an abstract problem, but it has *practical implications*:

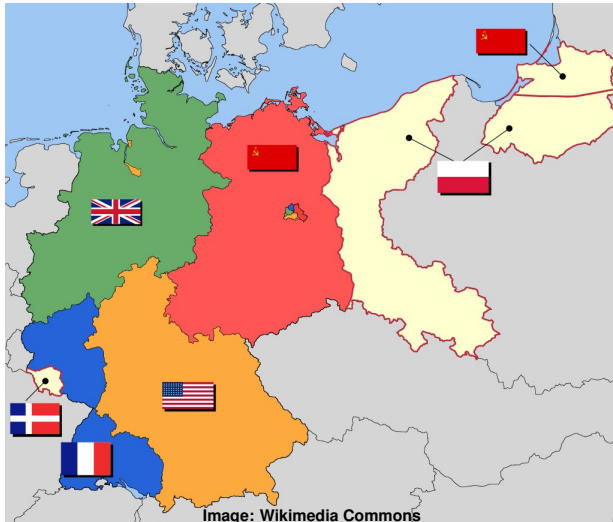
Consider the partition of Berlin



Partition of Berlin (Potsdam Agreement, 1945)

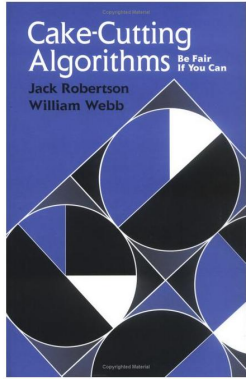
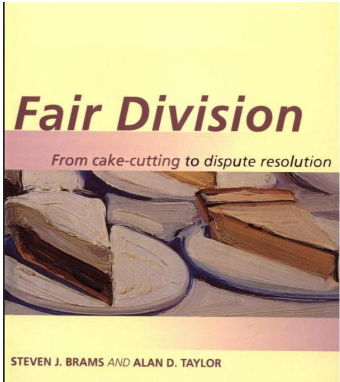


Partition of Germany (Potsdam Agreement, 1945)



Books on Fair Division

Two books devoted exclusively to this problem and its variations

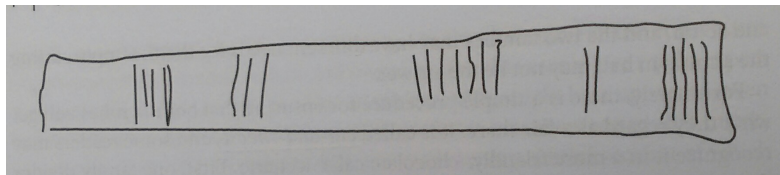


Tally Sticks

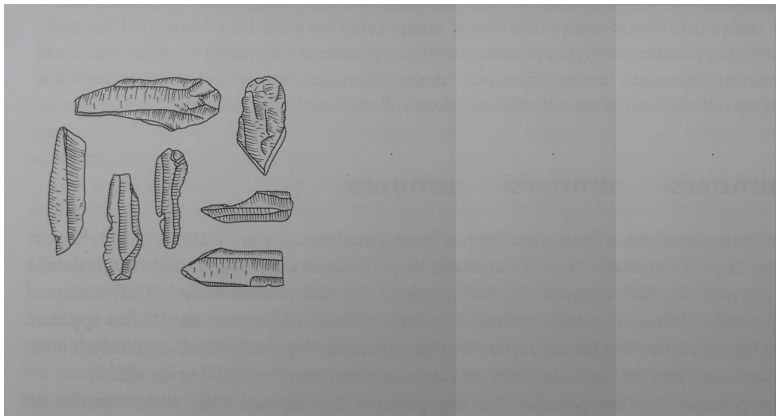
Keeping an account of sheep and such animals was done using a tally stick.

The number of notches corresponds to the number of sheep.

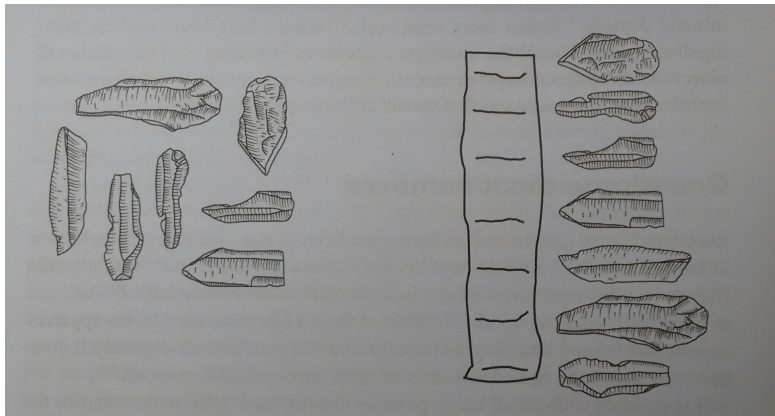
Again, for small flocks, no concept of *actual numbers* was essential.



Keeping Stock without Counting



Keeping Stock without Counting



The origin of the number line ???



Numbers

At some stage, the general notion of a number arose. Even in considering the fingers of a hand, numbers up to five would arise.

Gradually the idea of five as a concept would emerge. Placing two hands together immediately gives us the idea of a one-to-one correspondence:

Both hands have five fingers.

Through repetition and familiarity, the concept of five would become natural. Any set of objects that are in one-to-one correspondence with the fingers of the hand must have five elements.



Numerals

Gradually all the small natural numbers, at least up to about 10, came into use.

Sometimes, the connection between say two sheep and two bushels of corn was obscured.

Irish has distinct words for two apples and two people

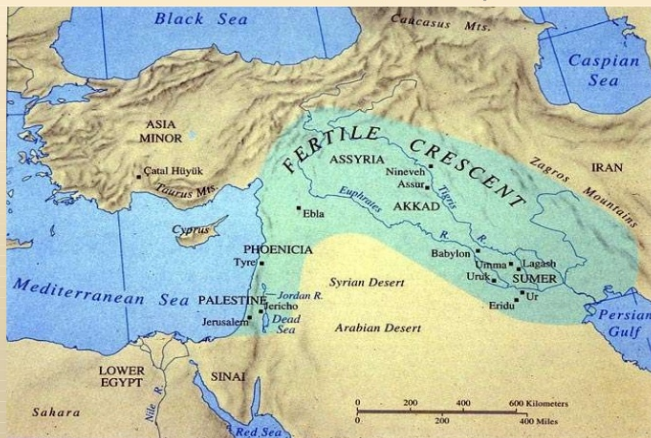
Eventually, numerals, or symbols for the numbers, emerged.

Much numerical material is found in writings from Mesopotamia and from Ancient Egypt.



The Fertile Crescent

The Fertile Crescent/Mesopotamia



Mesopotamia

Loosely called the Babylonian civilisation.

A vast number of cuneiform tablets survive.



**WE WILL RETURN TO BABYLON PRESENTLY
AND READ A CUNEIFORM TABLET!**



Bartering & Money

One group might have surplus *fish* while another group have excess *fruit*. Both gain by agreeing to an exchange.

A common measure was needed. This eventually led to the idea of money.

In several cultures, objects like *cowrie shells* were used as a medium of exchange.

In some cases, the currency had some inherent value or at least scarcity. In others, it had not.

Exercise: Discuss the opinion of Aristotle in his *Ethics*: “With money we can measure everything.”



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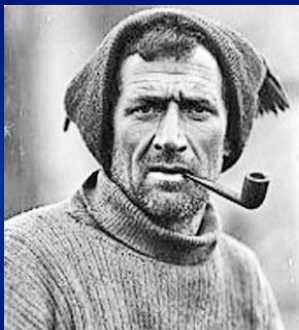
Who is this?



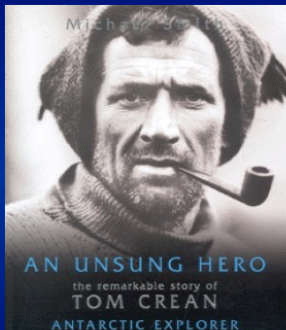
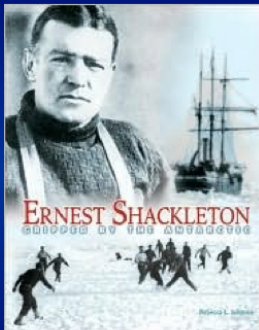
Who is this?



Who is this?



Ernest Shackleton Tom Crean



Shackleton's Imperial Trans-Antarctic Expedition (1914)



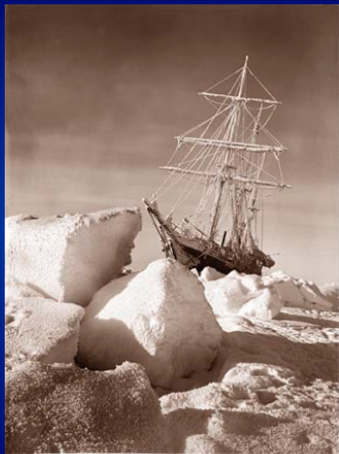
Shackleton's Imperial Trans-Antarctic Expedition (1914)



Shackleton's Imperial Trans-Antarctic Expedition (1914)



Endurance is Icebound



Shackleton's Imperial Trans-Antarctic Expedition (1914)



Shackleton's Imperial Trans-Antarctic Expedition (1914)





Six men sailed 800 miles across the Southern Ocean to South Georgia.



**Six men sailed 800 miles across the
Southern Ocean to South Georgia.**

How did they find their way?

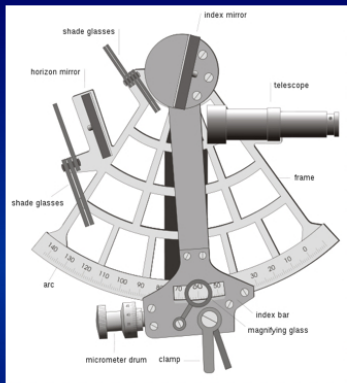


Six men sailed 800 miles across the Southern Ocean to South Georgia.

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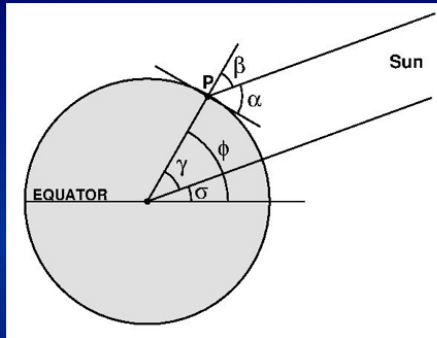
MATHEMATICS !!!





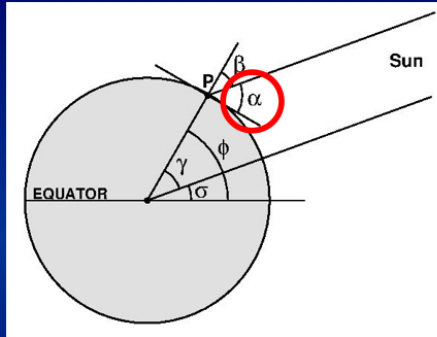
A sextant, used to determine latitude.





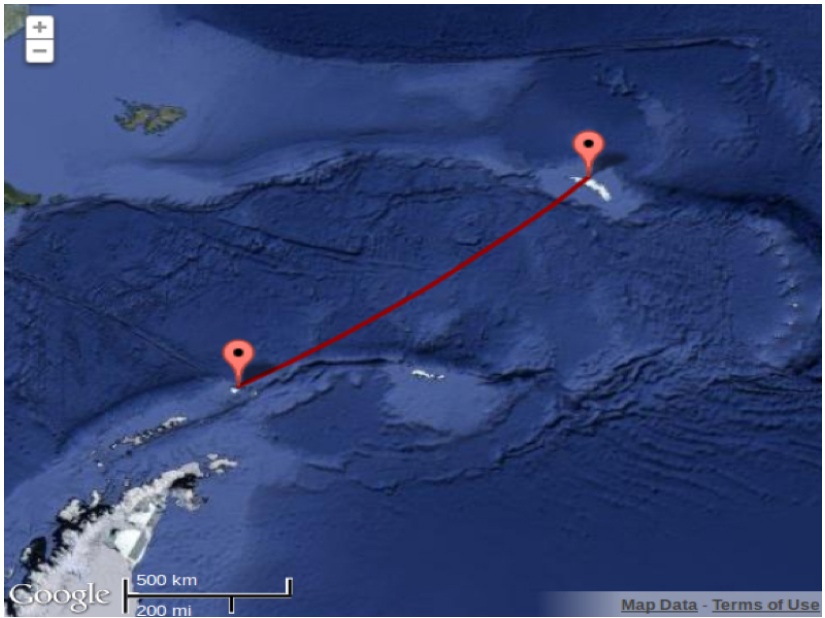
Angles used to calculate the latitude.





Angles used to calculate the latitude.





**The boat journey to South Georgia
was a spectacular feat of navigation.**

It resulted in the saving of 28 lives.

**This was possible thanks to
elementary geometry.**



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That's Maths!



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Reading a Tablet

On the next slide we will see a cuneiform tablet. It was discovered in the Sumerian city of Nippur (in modern-day Iraq), and dates to around 1500 BC.

We're not completely sure what this is, but most scholars suspect that it is a *homework exercise*.

It is not preserved perfectly, and dealing with this is part of the challenge (and part of the fun).

If you study the picture closely, you should be able to discover a lot about Babylonian numerals.



The Nippur Tablet



The Nippur Tablet Challenge



1. How do Babylonian numerals work?
2. Describe the maths on this tablet.
3. Write the number 72 in Babylonian numerals.

Does this seem impossible? Have faith in yourself!



Pause to Decode the Nippur Tablet



The Sexagesimal System

The Babylonian numerical system used 60 as its base. Why?

It is uncertain why, but reasonable to speculate that, since there are about 360 days in a year 60 was chosen to facilitate astronomical calculations.



The Babylonian Numerals

𐎶 1	𐎠𐎺 11	𐎠𐎶𐎶 21	𐎠𐎶𐎶𐎶 31	𐎠𐎶𐎶𐎶𐎶 41	𐎠𐎶𐎶𐎶𐎶𐎶 51
𐎶𐎶 2	𐎠𐎶𐎶𐎶 12	𐎠𐎶𐎶𐎶𐎶 22	𐎠𐎶𐎶𐎶𐎶𐎶 32	𐎠𐎶𐎶𐎶𐎶𐎶𐎶 42	𐎠𐎶𐎶𐎶𐎶𐎶𐎶𐎶 52
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The Sexagesimal System

The great advantage is that 60 has many divisors:
1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30.

This obviously facilitates all the division problems.

In Babylon, they wrote $70 = [1 \mid 10]$ and $254 = [4 \mid 14]$

We can add these: $324 = [5 \mid 24]$.

Thus, basic arithmetic is possible with this system.



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Distraction: The Simpsons



Several writers of the Simpsons scripts have advanced mathematical training.

They “sneak” jokes into the programmes.



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The Nippur Tablet



- **What is the last line?**
The last line states that

$$13 \times 13 = 2 \times 60 + 49 = 169$$

But it could be

$$13 \times 13 = 2 \times 60^2 + 40 \times 60 + 9$$

which comes to 9609.
Babylonian numeration is
ambiguous.

There is no zero!



The Nippur Tablet

What purpose could the Nippur Tablet have had?

What use could there be for a list of squares?

Perhaps it was used for multiplication!

**After a brief refresher on school maths,
we show how this can be done.**



Refresher: Some School Maths

How do we do multiplication of binomials

$$(a + b) \times (c + d) ?$$

This can be evaluated by expanding twice:

$$a \cdot (c + d) + b \cdot (c + d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d$$

A special case where the two factors are equal:

$$(a + b) \cdot (a + b) = a \cdot a + a \cdot b + b \cdot a + b \cdot b$$

so that

$$(a + b)^2 = a^2 + 2ab + b^2$$



Multiplication by Squaring

Let a and b be any two numbers:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Subtracting, we get

$$(a + b)^2 - (a - b)^2 = 4ab$$

Thus, we can find the product using squares:

$$ab = \frac{1}{4} \left[(a + b)^2 - (a - b)^2 \right].$$

Every product is the difference of two squares ($\div 4$).



Multiplication by Squaring

$$\frac{1}{4} \left[(a + b)^2 - (a - b)^2 \right] = ab$$

Let us take a particular example: $37 \times 13 = ?$

$$a = 37 \quad b = 13 \quad a + b = 50 \quad a - b = 24.$$

$$\begin{aligned} \frac{1}{4} [50^2 - 24^2] &= \frac{1}{4} [2500 - 576] \\ &= \frac{1}{4} [1924] \\ &= 481 \\ &= 37 \times 13. \end{aligned}$$

Perhaps this was the function of the Nippur tablet.



Practicalities in Babylon

$$ab = \frac{1}{4} \left[(a+b)^2 - (a-b)^2 \right].$$

Suppose it was important to be able to multiply numbers up to, say, 100.

A full multiplication table would have 10,000 entries. With 20 products on each tablet, this would mean 500 clay tablets!

A table of squares up to 200 would require only 10 clay tablets.



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Georg Cantor



Inventor of Set Theory

**Born in St. Petersburg,
Russia in 1845.**

**Moved to Germany in
1856 at the age of 11.**

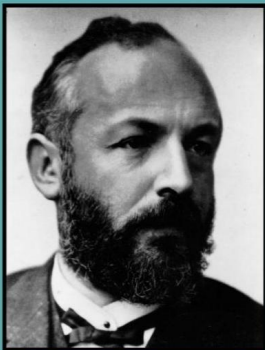
**His main career was at
the University of Halle.**



Dauben Biography of Cantor

GEORG CANTOR

*His Mathematics and
Philosophy of the Infinite*



Joseph Warren Dauben



Georg Cantor (1845–1918)

- ▶ **Invented Set Theory.**
- ▶ **One-to-one Correspondence.**
- ▶ **Infinite and Well-ordered Sets.**
- ▶ **Cardinal and Ordinal Numbers.**
- ▶ **Proved: $\#(\mathbb{Q}) = \#(\mathbb{N})$.**
- ▶ **Proved: $\#(\mathbb{R}) > \#(\mathbb{N})$.**
- ▶ **Infinite Hierarchy of Infinities.**

Outline Galileo's arguments on infinity.



Set Theory: Controversy

Cantor was strongly criticized by

- ▶ **Leopold Kronecker.**
- ▶ **Henri Poincaré.**
- ▶ **Ludwig Wittgenstein.**

Cantor is a “corrupter of youth” (LK).

Set Theory is a “grave disease” (HP).

Set Theory is “nonsense; laughable; wrong!” (LW).

Adverse criticism like this may well have contributed to Cantor’s mental breakdown.



Set Theory: A Difficult Birth

Set Theory brought into prominence several *paradoxical results*.

Many mathematicians had great difficulty accepting some of the stranger results.

Some of these are still the subject of discussion and disagreement today.



Set Theory: A Difficult Birth

Cantor's Set Theory was of profound philosophical interest.

It was *so innovative* that many mathematicians could not appreciate its fundamental value and importance.

Gösta Mittag-Leffler was reluctant to publish it in his *Acta Mathematica*. He said the work was “100 years ahead of its time”.

**David Hilbert said:
“We shall not be expelled from the paradise that Cantor has created for us.”**



A Passionate Mathematician

In 1874, Cantor married Vally Guttmann.

They had six children. The last one, a son named Rudolph, was born in 1886.

According to Wikipedia:

“During his honeymoon in the Harz mountains, Cantor spent much time in mathematical discussions with Richard Dedekind.”

[Cantor had met the renowned mathematician Dedekind two years earlier while he was on holiday in Switzerland.]



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Books on a Shelf



Ten books are arranged on a shelf.
They include an Almanac (A) and a Bible (B).

Suppose A must be to the left of B
(not necessarily beside it).

How many possible arrangements are there?



Books on a Shelf



Ten books are arranged on a shelf.
They include an Almanac (A) and a Bible (B).

BIG IDEA: SYMMETRY.

*Every SOLUTION corresponds to a NON-SOLUTION:
Just switch the positions of A and B!*

The total number of arrangements is 10!.
For half of these, A is to the left of B.

So, answer is $\frac{1}{2}(10 \times 9 \times \cdots \times 1) = \frac{1}{2} \times 10!$

Q.E.D.



Outline

Introduction

The Beginnings

Shackleton's Rescue Voyage

Babylonian Numeration Game

Distraction 2A: Simpsons

The Nippur Tablet

Georg Cantor

Distraction 2B: Books

Set Theory I



Set Theory I

The concept of *set* is very general.

Sets are the basic building-blocks of mathematics.

Definition: A *set* is a collection of objects.

The objects in a set are called the elements.

Examples:

- ▶ All the prime numbers, \mathbb{P}
- ▶ All even numbers: $\mathbb{E} = \{2, 4, 6, 8, \dots\}$
- ▶ All the people in Ireland: See Census returns.
- ▶ The colours of the rainbow: {Red, ..., Violet}.
- ▶ Light waves with wavelength $\lambda \in [390 - 700\text{nm}]$



Do You Remember Venn?

John Venn was a logician and philosopher, born in Hull, Yorkshire in 1834.

He studied at Cambridge University, graduating in 1857 as sixth Wrangler.

Venn introduced his diagrams in *Symbolic Logic*, a book published in 1881.





Venn Diagrams



Venn diagrams are very valuable for showing elementary properties of sets.

They comprise a number of overlapping circles.

The interior of a circle represents a collection of numbers or objects or perhaps a more abstract set.



The Universe of Discourse

We often draw a rectangle to represent the *universe*, the set of all objects under current consideration.

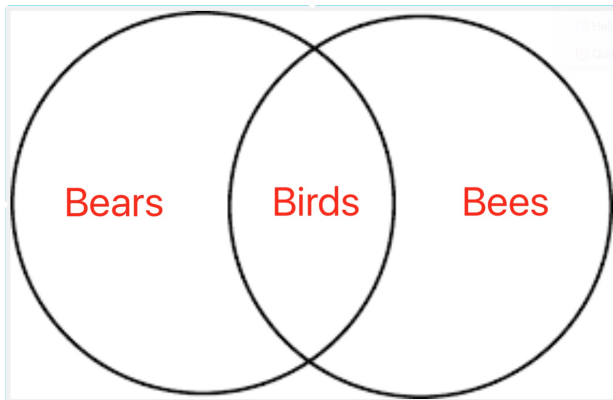
For example, suppose we consider all species of animals as the universe.

A rectangle represents this universe.

Two circles indicate subsets of animals of two different types.



The Birds and the Bees



Two-legged Animals

Flying Animals

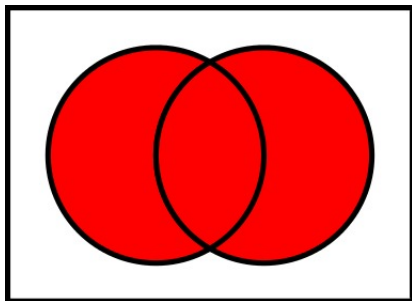
Where do we fit in this diagram?



The Union of Two Sets

The aggregate of two sets is called their union.

Let one set contain all two-legged animals and the other contain all flying animals.



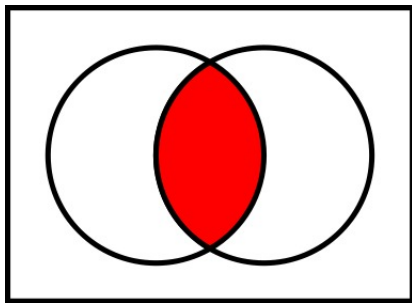
Bears, birds and bees (and we) are in the union.



The Intersection of Two Sets

The elements in both sets make up the intersection.

Let one set contain all two-legged animals
and the other contain all flying animals.



Birds are in the intersection. Bears and bees are not.

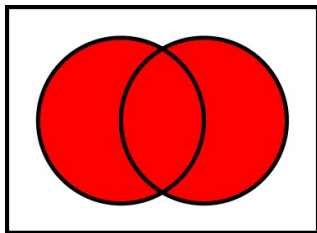


The Notation for Union and Intersection

Let A and B be two sets

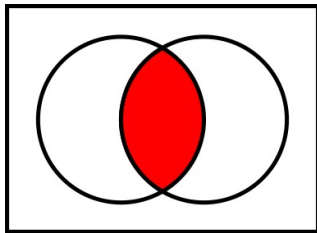
The union of the sets is

$$A \cup B$$



The intersection is

$$A \cap B$$



The Technical (Logical) Definitions

Let A and B be two sets.

The union of the sets $A \cup B$ is defined by

$$[x \in A \cup B] \iff [(x \in A) \vee (x \in B)]$$

The intersection of the sets $A \cap B$ is defined by

$$[x \in A \cap B] \iff [(x \in A) \wedge (x \in B)]$$

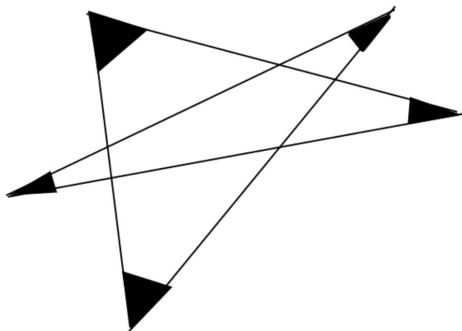
*There is an intimate connection between
Set Theory and Symbolic Logic.*



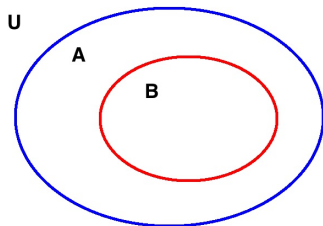
Digression: A Simple Puzzle

Puzzle - Seeing Stars

What is the sum of all the marked angles in the five-pointed star?



Subset of a Set



For two sets A and B we write

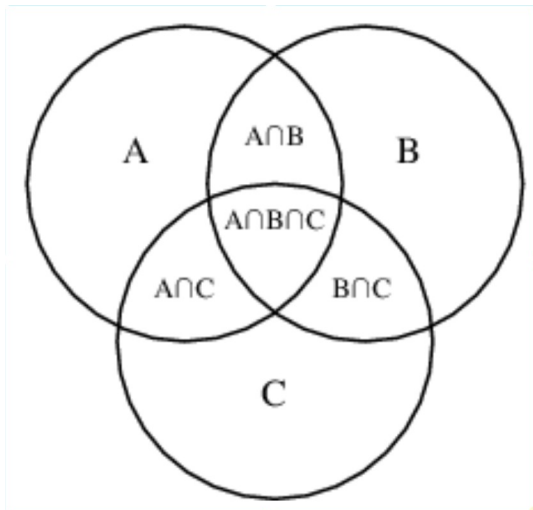
$$B \subset A \quad \text{or} \quad B \subseteq A$$

to denote that B is a subset of A .

For more on set theory, see website of Claire Wladis
<http://www.cwladis.com/math100/Lecture4Sets.htm>



Intersections between 3 Sets

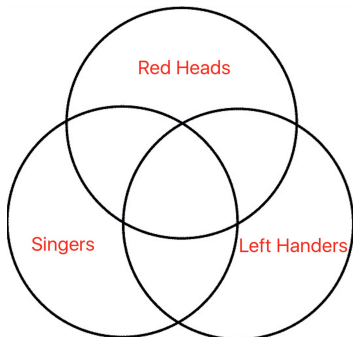


Example: Intersection of 3 Sets

In the diagram the elements of the universe are all the people from Connacht.

Three subsets are shown:

- ▶ Red-heads
- ▶ Singers
- ▶ Left-handers.

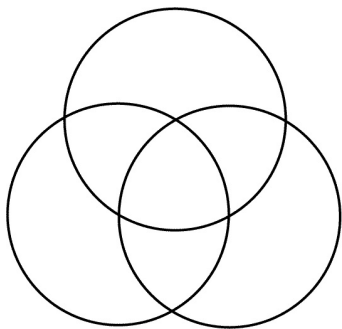


All are from Connacht.

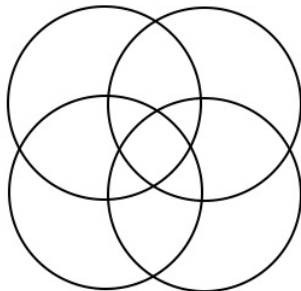
These sets overlap and, indeed, there are some copper-topped, crooning cithogues in Connacht.



Three and Four Sets



8 Domains



14 Domains



How Many Possibilities?

With just one set A , there are 2 possibilities:

$$x \in A \quad \text{or} \quad x \notin A$$

With two sets, A and B , there are 4 possibilities:

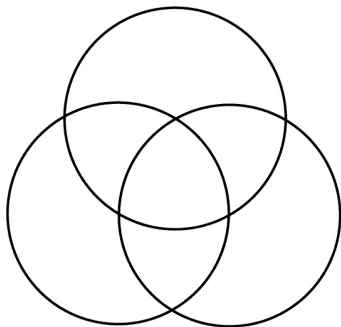
$$\begin{aligned} (x \in A) \wedge (x \in B) & \quad \text{or} \quad (x \in A) \wedge (x \notin B) \\ (x \notin A) \wedge (x \in B) & \quad \text{or} \quad (x \notin A) \wedge (x \notin B) \end{aligned}$$

With three sets there are 8 possible conditions.

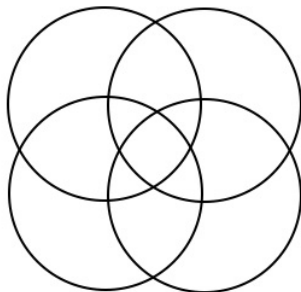
With four sets there are 16 possible conditions.



Three and Four Sets



8 Domains



14 Domains

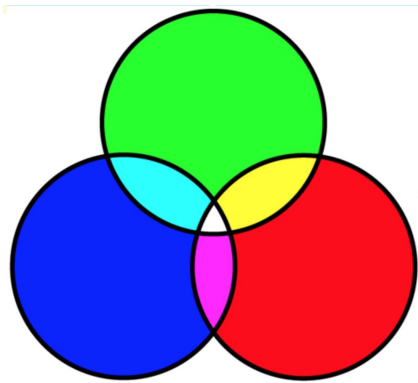
**With three sets there are 8 possible conditions.
With four sets there are 16 possible conditions.**



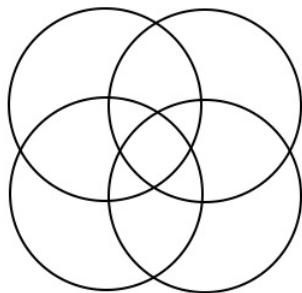
The Intersection of 3 Sets

The three overlapping circles have attained an iconic status, seen in a huge range of contexts.

It is possible to devise Venn diagrams with four sets, but the simplicity of the diagram is lost.



Exercise: Four Set Venn Diagram



***Can you modify the 4-set diagram to cover all cases.
(You will not be able to do it with circles only)***



Hint: Venn Diagrams for 5 and 7 Sets

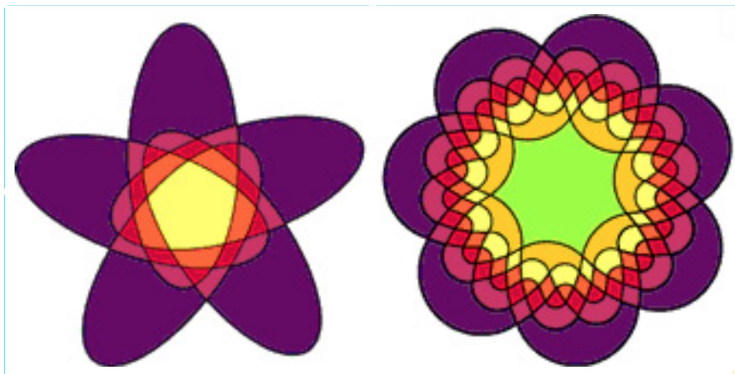
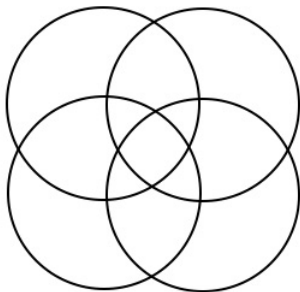


Image from Wolfram MathWorld: Venn Diagram



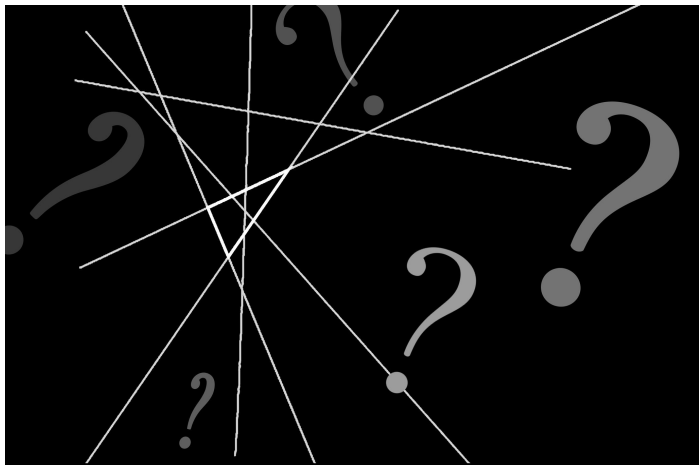
Solution: Next Week (if you are lucky)



We will find a surprising connection with a Cube



Digression: A Simple Puzzle



Thank you

