AweSums

Marvels and Mysteries of Mathematics

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LECTURE 1

Peter Lynch
School of Mathematics & Statistics
University College Dublin

Evening Course, UCD, Autumn 2020

Outline

Introduction

Overview

Beautiful Spirals

The Golden Ratio

Symmetry

Recreational Mathematics

Visual Maths 1

Distraction 1: A Piem

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Aim of the Course

AweSums

The course will run over six (6) lectures, from 5 October to 16 November.

No lecture on 26th October.
So, the course splits into 3 + 3.

The aim of the course is to show you

- **The great *beauty* of mathematics;**
- **Its tremendous *utility* in our daily lives;**
- **The *fun* we can have studying maths.**

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "study" or "learning".

It is the study of topics such as

- **Quantity**
- **Structure**
- **Space**
- **Change**

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "study" or "learning".

It is the study of topics such as

- **Quantity:** ***[Numbers. Arithmetic]***
- **Structure:** ***[Patterns. Algebra]***
- **Space:** ***[Geometry. Topology]***
- **Change:** ***[Analysis. Calculus]***

Tom Lehrer: Mathematician, Musician and Comic Genius

ThatsMaths **article in** ***The Irish Times*** **in September 2018 about mathematician and comic genius Tom Lehrer.**

(You can find articles using the Search Box.)

`https://thatsmaths.com/`

- `/Users/peter/Dropbox/Music/Videos.html`
- Run Video (vsn 1)
- List Keywords
- Run Video (vsn 2) later.

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Notes and Slides

- All the slides will be available online:
  http://mathsci.ucd.ie/∼plynch/AweSums
  [just Google for "Peter Lynch UCD"]
- *No notes* are to be provided.
  Why Not? See next slide.
- Additional Reading Recommendations.
- Optional Exercises and Problems.
- *No Assignments!*
- *No Assessments!*
- *No Examinations!*

Why No Notes?

- **Maths is NOT a Spectator Sport**
- **Active engagement is essential to understanding.**
- ***You should take your own notes:***
  - **This involves repetition of what you hear.**
  - **This involves repetition of what you see.**
  - **What you write passes through your mind!**
  - **This process is a great help to memory.**

Lectures

- Classes run from 7pm to 9pm.
- 120 minutes intensive lecturing too long
  (both for you and for me).
- Educational Theory:
  - Attention & retention both decrease with time.
- Class will be broken into short sections.

If you cannot attend a class:

- Please do not bother to email me.
- There is no need to give any reasons.
- The presentation slides will be available.

Communications

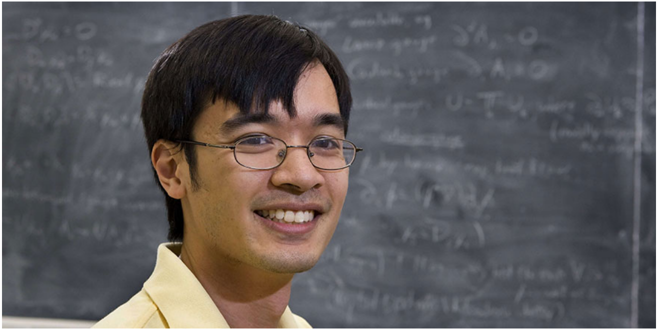
In the unlikely event that a class has to be cancelled,

- I will notify “Adult & Lifelong Learning”.
- You may wish to form a WhatsApp group.

I will also tell you about other mathematical events in Dublin if I hear about them.

FOR EXAMPLE ....>

Hamilton Lecture, 2020



The Cosmic Distance Ladder
Friday, October 16, 16:00
Free online event: booking essential
www.ria.ie

“Typical” Structure of a Class

1. Problem: Background and Theory
2. *Distraction* (10 min)
3. Some History of the problem
4. Another *Distraction*
5. Exercises, Puzzles, History
6. Questions & Discussion

Total duration: about 120 minutes.

I will (normally) be available after classes to answer questions or offer clarifications.

Some Distractions

- Visual Awareness: Maths Everywhere
- Puzzles: E.g. Watermelon Puzzle
- Sieve of Eratosthenes
- The Greek Alphabet
- Lateral Thinking in Maths
- *Lecture sans paroles*
- How Cubic and Quartic Equations were cracked
- Four-colour Theorem
- Online Encyclopedia of Integer Sequences

Please ask me if you have a favorite topic!

It's Your Course

I expect a group with a wide range of knowledge and "mathematical maturity".

Everybody should benefit from the course.

If anything is unclear, **SHOUT OUT!** or whisper!

If something is missing, let me know.

Feedback on the course is very welcome.

It’s Your Course

Classes begin at 7 pm. and run till 9 pm.

There seem to be two options:

- **Break at 7:50 for 10, 15 or 20 minutes.**
- **Don’t break at all !!!**

I have no strong views but I suspect that there might be a riot if we do not have a break.

Let’s have a poll: **Who wants a break?**

Popular Mathematics Books

1. John H Conway and Richard K Guy, 1996:
   The Book of Numbers. Copernicus, New York.
2. ♡ $\Rightarrow$ John Darbyshire, 2004:
   Prime Obsession. Plume Publishing.
3. ♡ $\Rightarrow$ William Dunham, 1991:
   Journey through Genius. Penguin Books.
4. Marcus Du Sautoy, 2004:
   The Music of the Primes. Harper Perennial.
5. ♡ $\Rightarrow$ Richard Elwes, 2010:
   Mathematics 1001. Firefly Books.
6. Peter Lynch, 2016: *That's Maths.*
   Gill Books. Published in October 2016.

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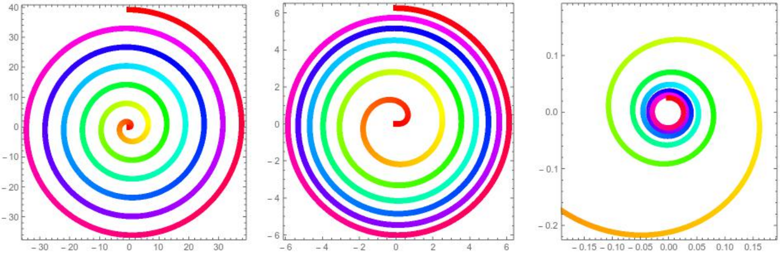
A Splendid Spiral in Booterstown

This sandbank, a beautiful spiral form, has slowly built up on the beach near Booterstown Station.

Spirals are found throughout the natural world.

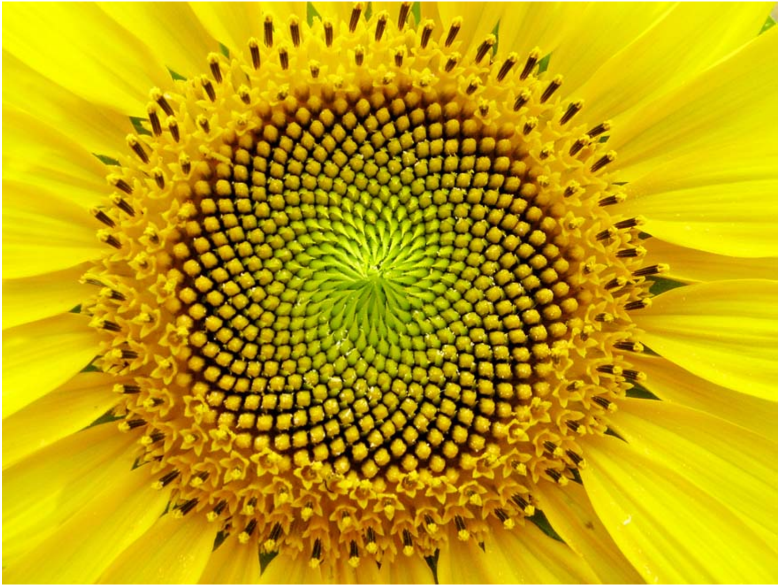
Some Mathematical Spirals



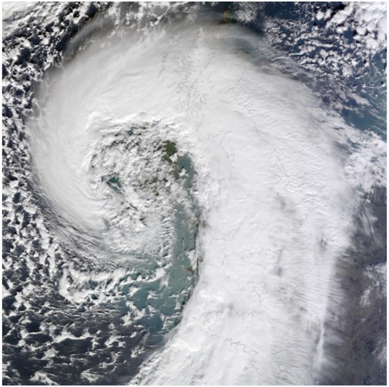
Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.

The Nautilus Shell: *a logarithmic Spiral.*

The Sunflower: Groups of Spirals



Spirals in the Physical World



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https://thatsmaths.com/

Fibonacci Numbers

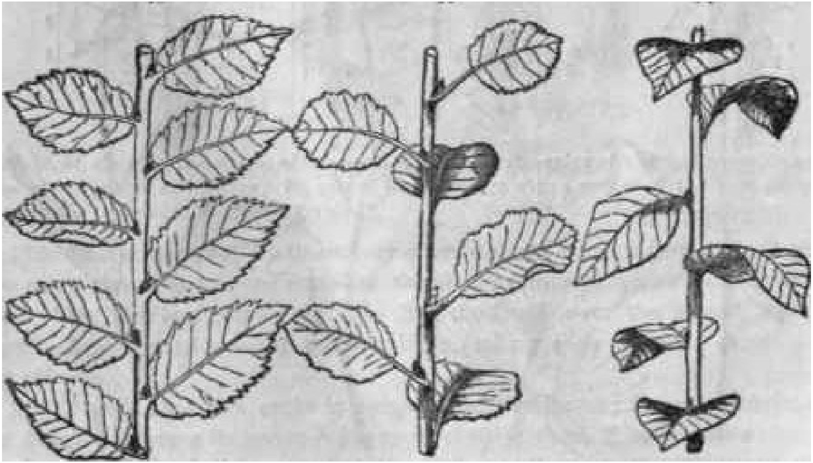
- Count the petals on a flower.
- Count leaves on a stem or
  bumps on an asparagus.
- Look at patterns on pineapples/pine-cones.
- Study the geometry of seeds on sunflowers.

In all cases, we find numbers in the sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

This is the famous Fibonacci sequence.

Fibonacci and Phyllotaxis



Vi Hart's Videos

There are several mathematical videos
on YouTube presented by Vi Hart.

Some of the ones on Fibonacci Numbers are at:

`https://www.youtube.com/watch?v=ahXIMUkSXX0`

It is *much easier* to go to Youtube and search for

"Vi Hart Fibonacci"

Let's take a peek!

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Golden Ratio and Fibonacci Numbers

The Golden Ratio is a number defined as

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618.$$

It is intimately connected with the *Fibonacci Numbers.*

Golden Rectangle

Ratio of breath to height is $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$**.**

Golden Rectangle in Your Pocket

Aspect ratio is about $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$.

Fibonacci Numbers

The Fibonacci sequence is the sequence

$$\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \dots\}$$

where *each number is the sum of the previous two.*

The Fibonacci numbers obey a recurrence relation

$$F_{n+1} = F_n + F_{n-1}$$

with the *starting values* $F_0 = 0$ *and* $F_1 = 1$*.*

The explicit expression for the Fibonacci numbers is

$$F_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n - \frac{1}{\sqrt{5}}\left[\frac{1-\sqrt{5}}{2}\right]^n$$

Fibonacci Numbers

Let's consider the sequence of ratios of terms

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \dots$$

The ratios get closer and closer to the golden number:

$$\frac{F_{n+1}}{F_n} \to \phi \quad \textbf{as} \quad n \to \infty$$

Exotic Expressions for ϕ

We can write ϕ as a *continued fraction*

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

We can also write it as a *continued root*

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}}$$

These extraordinary expressions are actually quite easy to demonstrate!

Fibonacci Numbers in Nature

Look at post

Sunflowers and Fibonacci: Models of Efficiency

on the *ThatsMaths* blog:

`thatsmaths.com`

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Ubiquity and Beauty of Symmetry

Symmetry is all around us.

- **Many buildings are symmetric.**
- **Our bodies have bilateral symmetry.**
- **Crystals have great symmetry.**
- **Viruses can display stunning symmetries.**
- **At the sub-atomic scale, symmetry reigns.**
- **Galaxies have many symmetries.**

The Taj Mahal

A Face with Symmetry: Halle Berry

Halle Berry **Berry Halle**

An Asymmetric Face: You know Who!

Symmetry and Group Theory

Symmetry is an essentially *geometric* concept.

The mathematical theory of symmetry is *algebraic.*

The key concept is that of a group.

A group is a *set of elements* such that any two elements can be combined to produce another.

Instead of giving the mathematical definition, I will give an example to make things clear.

The *Dihedral Group* D_1

The group of symmetries of the human face and of all biological forms with bilateral symmetry. We could call D_1 the *Janus Group.*

I : The Identity transformation

R : Reflection about central line

Table : First Dihedral Group D_1.

	I	R
I	I	R
R	R	I

This is how we combine, or *multiply* transformations.

From 2 to 3 Dimensional Symmetry

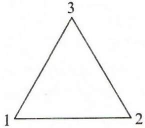


Fig. 4.6

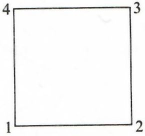
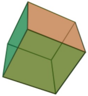
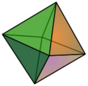
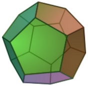
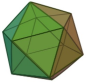


Fig. 4.7

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
 (Animation)	 (Animation)	 (Animation)	 (Animation)	 (Animation)

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Recreational Mathematics

Recreational mathematics puts the focus on insight, imagination and beauty.

Recreational Maths includes the study of

- **The culture of mathematics,**
- **Its relevance to art, music and literature,**
- **Its role in technology,**
- **Mathematical games and puzzles,**
- **The lives of the great mathematicians.**

Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine Plus:

`https://plus.maths.org/.`

All past content is available and is a valuable resource for school students and teachers.

Content of an Earlier Course

Lecture	*Content*
1	Outline of Course. Emergence of Numbers.
2	Georg Cantor. Set Theory.
3	Pythagoras. Irrational Numbers.
4	Hilbert. Gauss. The Real Number Line
5	Powers. Logarithms. Prime Numbers.
6	Functions. Archimedes. Natural Logs.
7	Exponential Growth. Euler. Sequences & Series.
8	Trigonometry. Taylor Series.
9	Basel Problem. Complex Numbers. Euler's Formula.
10	Prime Number Theorem. Riemann Hypothesis.

This year's course will be different: Better!

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To Begin: An Optical Illusion

A cautionary tale:

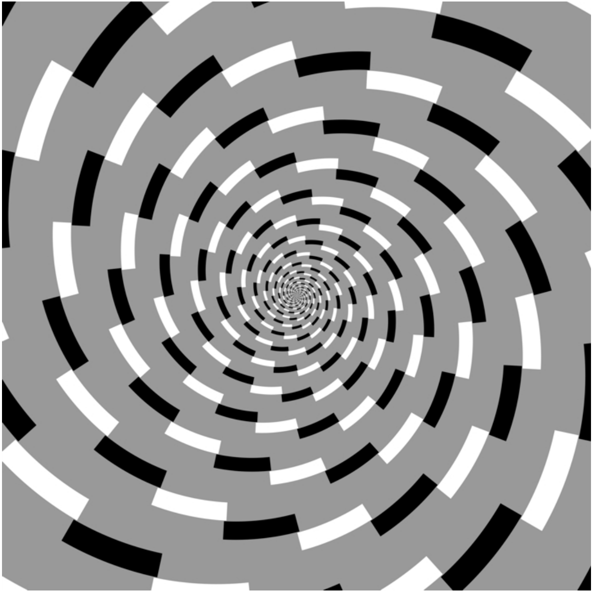
In maths we often use pictures to prove things.

This is usually very helpful.

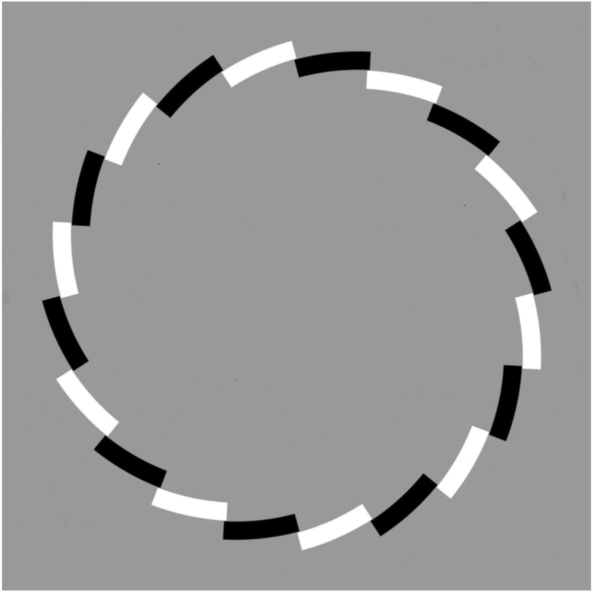
However, it can sometimes mislead us.

Let us look at the Fraser Spiral.

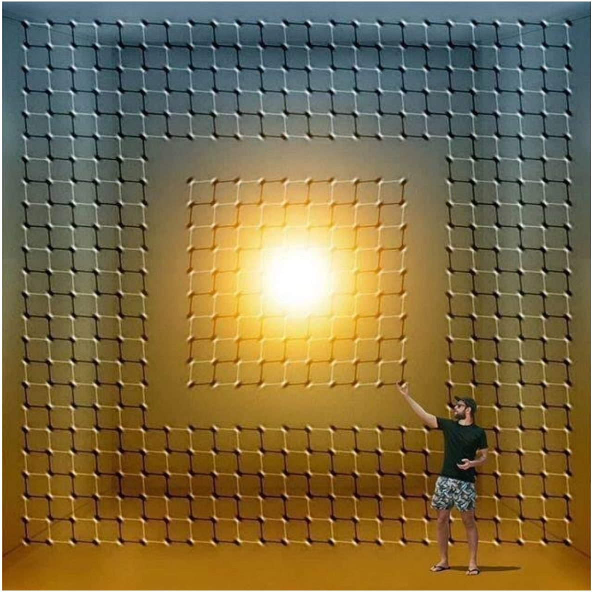
Fraser Spiral



Fraser Spiral



https://pbs.twimg.com/media/DI5ImeIU8AEpXB7.jpg



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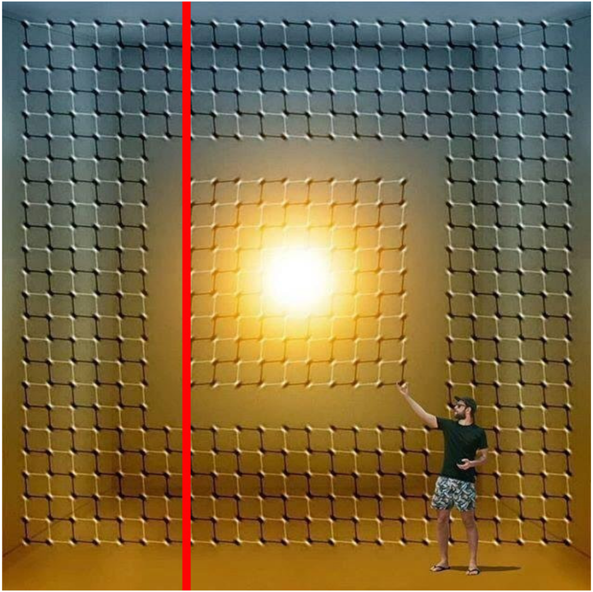

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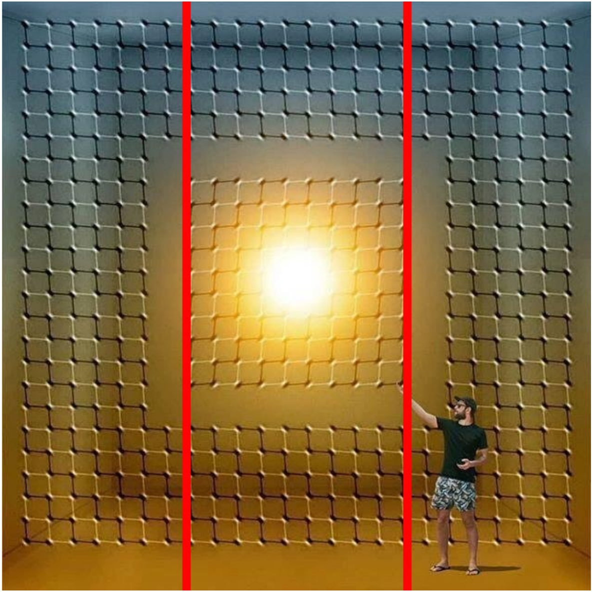

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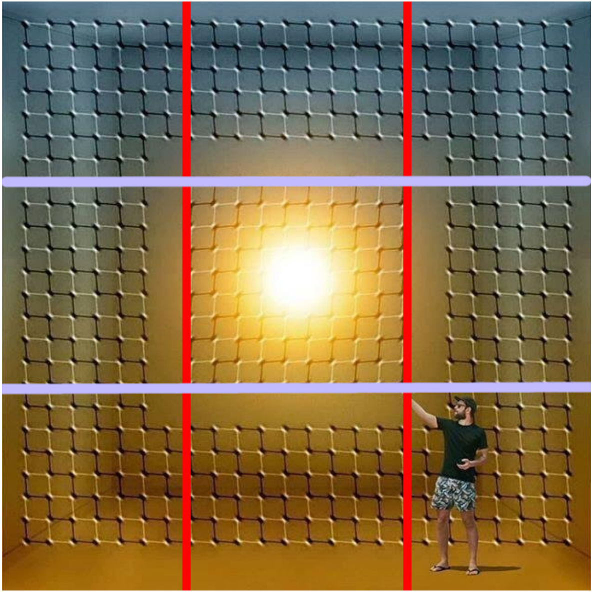


Visual Maths Proofs

Can the sum of an infinite number of quantities have a finite value?

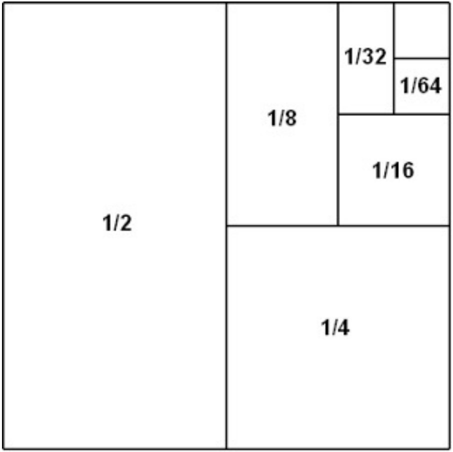
Let's look at the infinite series

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

Each term is half the size of the preceding one.

The terms are getting smaller but it is *not obvious* that the series converges.

A picture makes everything clear:



Unit Square: At each stage, we add *half the remainder* of the square.

Conclusion

The infinite series

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

has a *finite* sum:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1$$

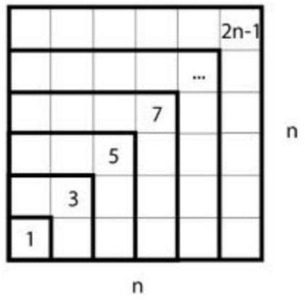
The terms are getting smaller quickly enough for the series to be convergent.

Another Simple Proof

What is the sum of the first n odd numbers?

$$1 = 1^2 \qquad (1+3) = 4 = 2^2 \qquad (1+3+5) = 9 = 3^2$$

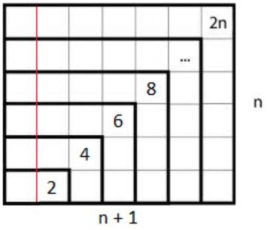
Is this pattern continued? *Can we prove it?*



$$[1+3+5+7+\cdots+(2n-1)] = n^2$$

What is the sum of the first n even numbers?

$$S = 2 + 4 + 6 + 8 + \cdots 2n$$



We just add a column on the left. This increases each term of the sequence of odd numbers by 1.

$$[2 + 4 + 6 + \cdots + 2n] = n(n+1)$$

Now divide both sides by 2 to get:

$$[1 + 2 + 3 + \cdots + n] = \frac{1}{2}n(n+1)$$

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Distraction 1: Remember π

The ratio of circumference of circle to diameter is π.

To 15-figure accuracy, π is equal to

$$3.14159265358979$$

How can we remember this without much effort?

Just remember this:

How I want a drink,
Alcoholic of course,
After the heavy lectures
involving quantum mechanics.

Distraction 1: Remember π

How I want a drink,
Lemonsoda of course,
After the heavy lectures
involving quantum mechanics.

How I want a drink,
Sugarfree of course,
After the heavy lectures
involving quantum mechanics.

Repeat: To Remember π

To 15-figure accuracy, π is equal to

$$3.14159265358979$$

How can we remember this without much effort?

Just remember this:

How I want a drink,
Alcoholic of course,
After the heavy lectures
involving quantum mechanics.

Distraction 1: Remember $1/\pi$

The reciprocal of π is approximately 0.318310
Can I remember the reciprocal?

How I remember the reciprocal!
3 1 8 3 10

Now you know π and $1/\pi$ to an accuracy greater than you are ever likely to need!

Thank you