AweSums

Marvels and Mysteries of Mathematics

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LECTURE 4

Peter Lynch
School of Mathematics & Statistics
University College Dublin

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Outline

Outline

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "study" or "learning".

It is the study of topics such as

- **Quantity (numbers)**
- **Structure (patterns)**
- **Space (geometry)**
- **Change (analysis).**

Outline

Topology: a Major Branch of Mathematics

Topology is all about continuity and connectivity.

We will look at a few examples of connectivity.

- **A Circle**
- **A Square**
- **A Triangle**

What makes them different?
What makes them the same?

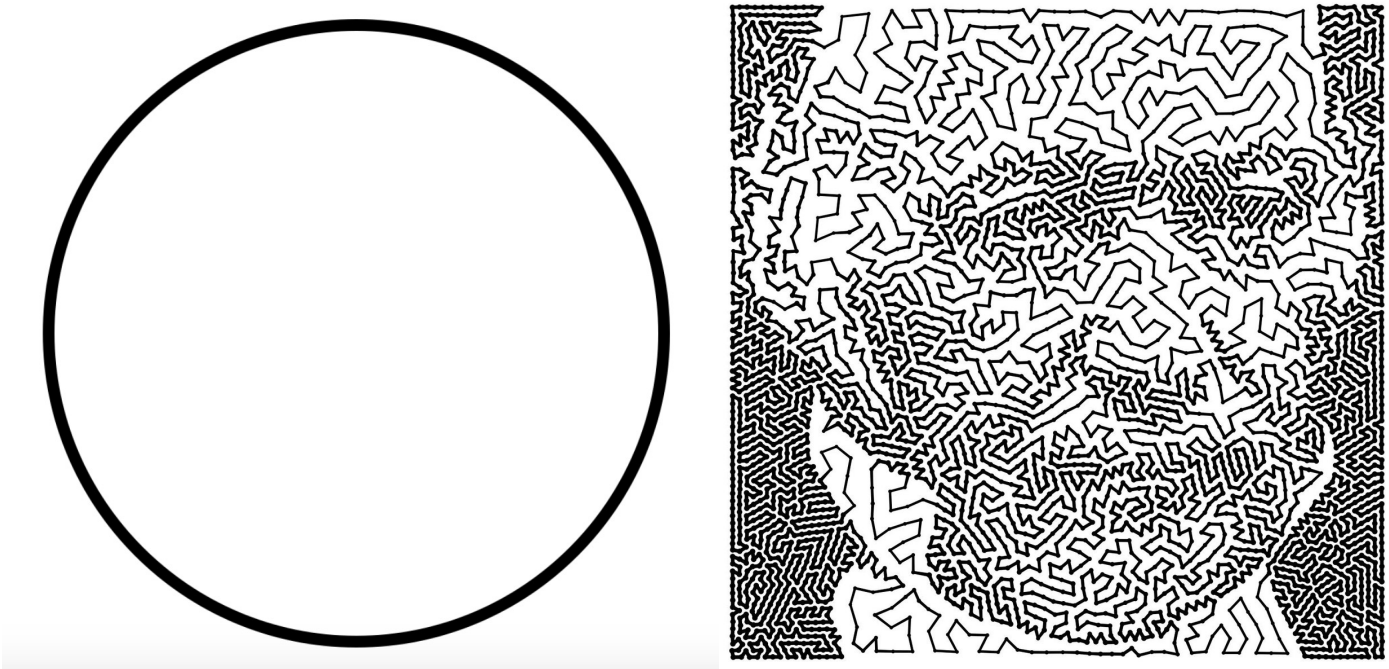


Figure: Topographically equivalent curves in the plane

Jordan Curves are topologically equivalent to a circle.

They are also called simple closed curves and are important for the *Travelling Salesman Problem.*

Topology: a Major Branch of Mathematics

Topology is all about continuity and connectivity.

We will look at a few aspects of Topology.

- **The Bridges of Königsberg**
- **Doughnuts and Coffee-cups**
- **Knots and Links**
- **Nodes and Edges: Graphs**
- **The Möbius Band**

Let us start with the *London Underground Map.*

The London Underground Map

Figure: Topographical map of the Underground

The London Underground Map

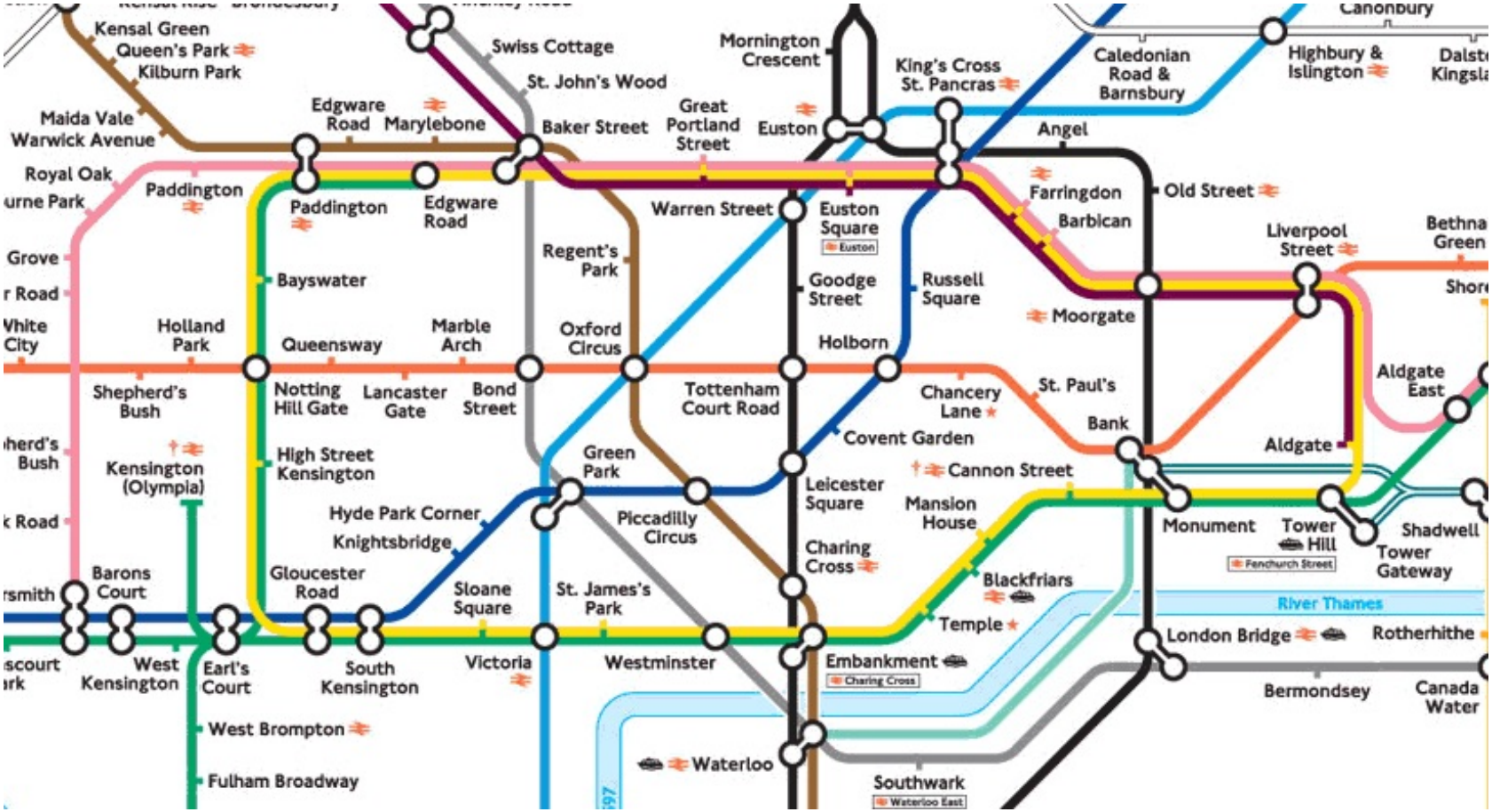


Figure: Topological map of the Underground

The London Underground Map

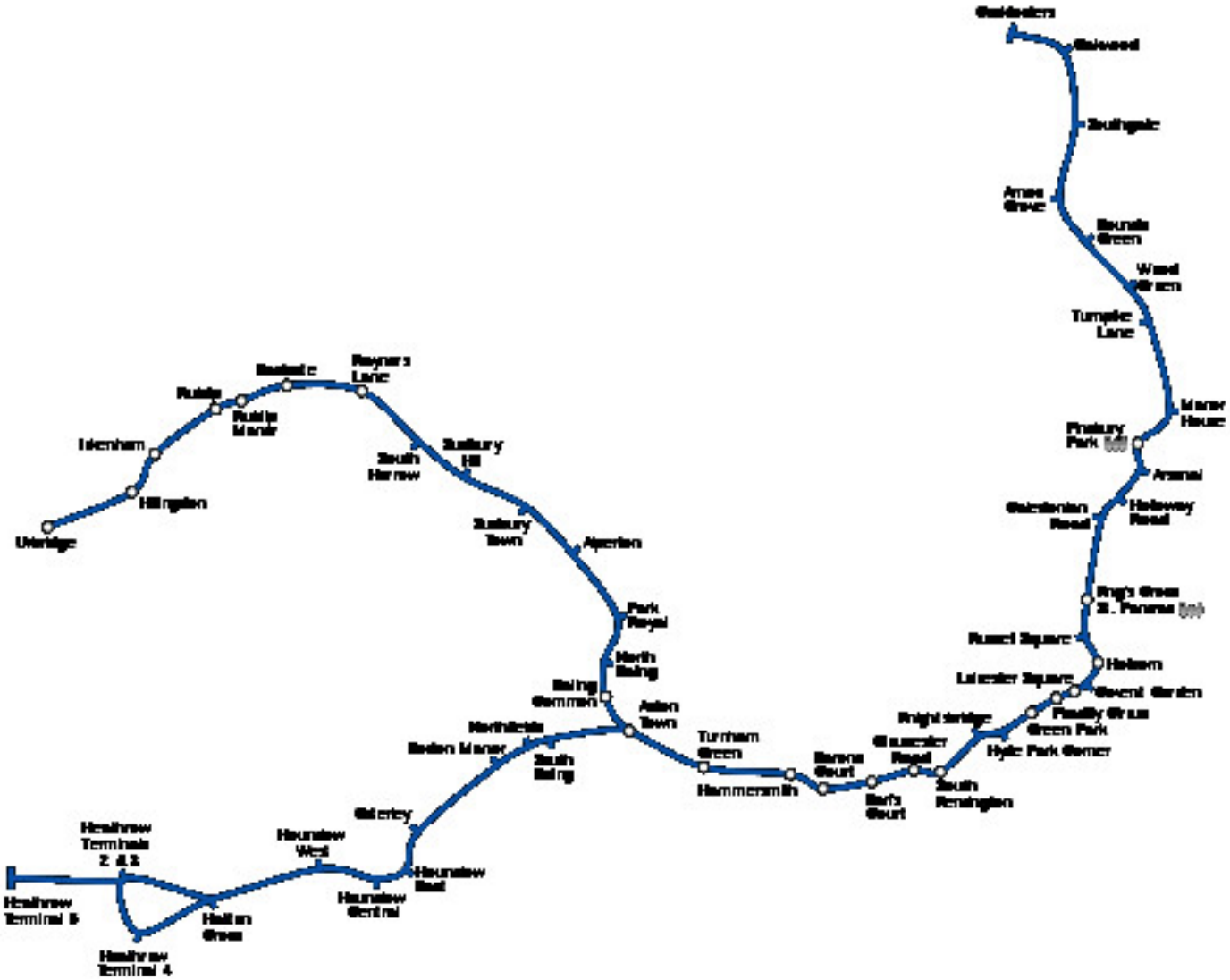
Properties of a simple closed loop:

- **No branches. No travel options.**
- **Start anywhere: end up there again.**
- **Definite direction (CW or CCW).**
- **An Inside and an Outside.**

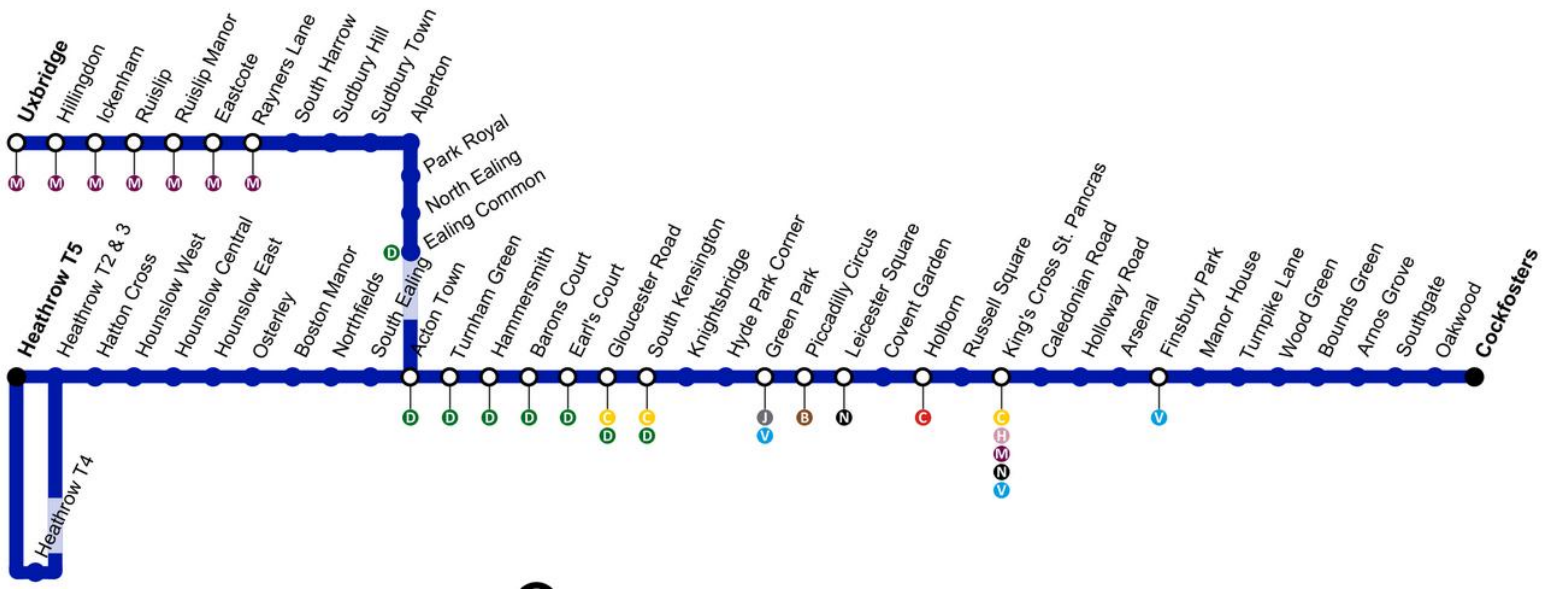
It is topologically equivalent to a circle.

Draw a (complicated) simple loop.

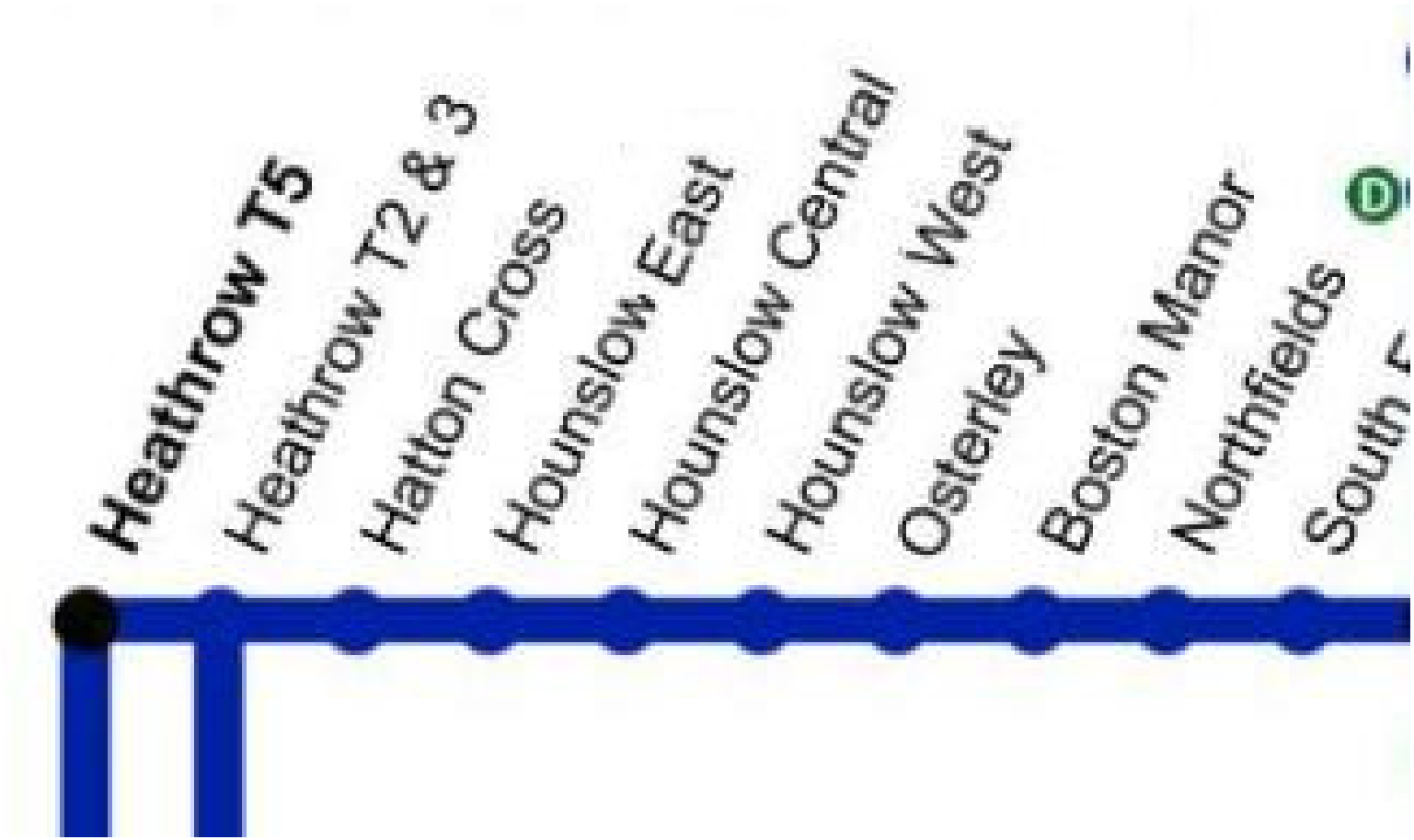
Piccadilly Line, Topographic



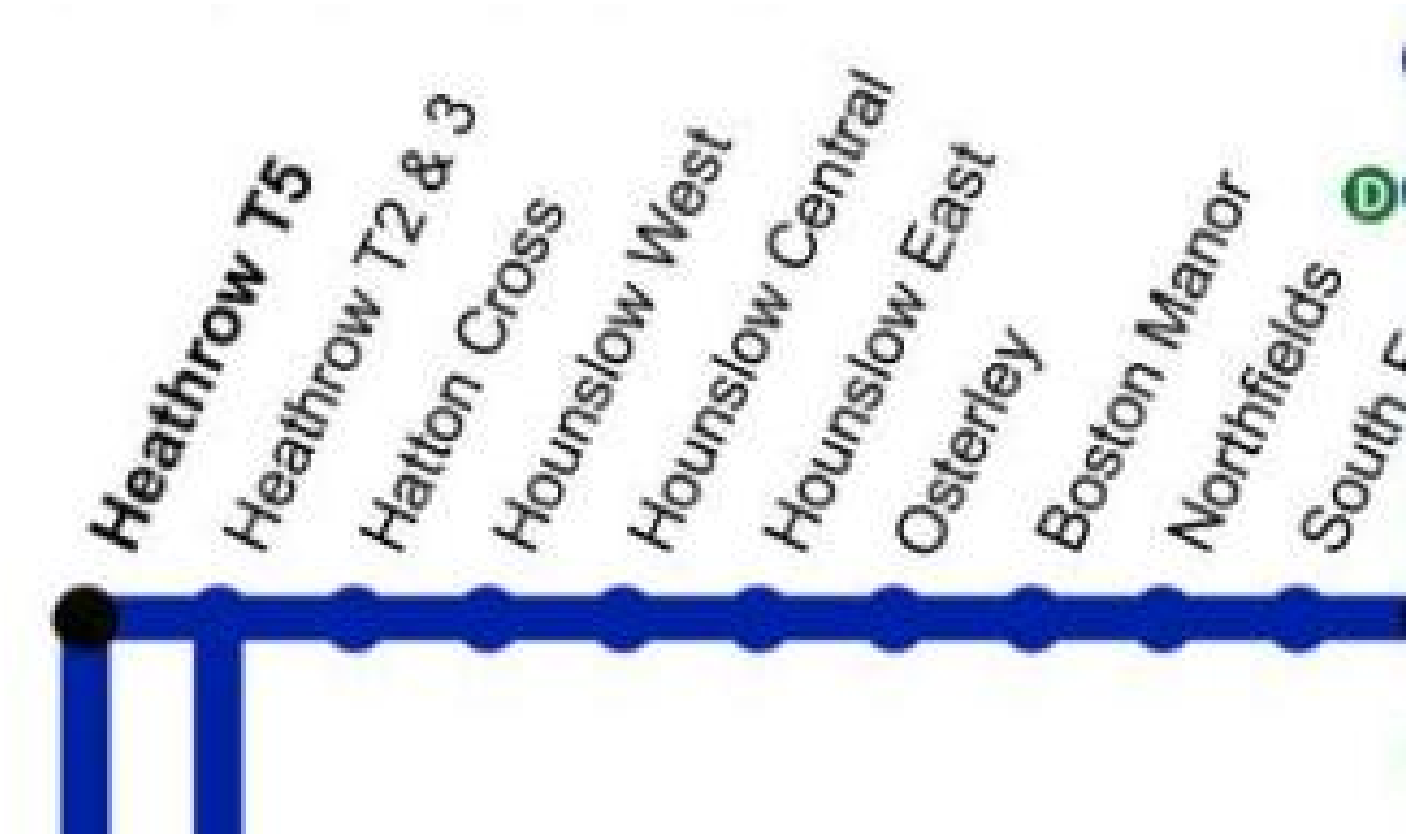
Piccadilly Line, Topological



Piccadilly Line, Detail

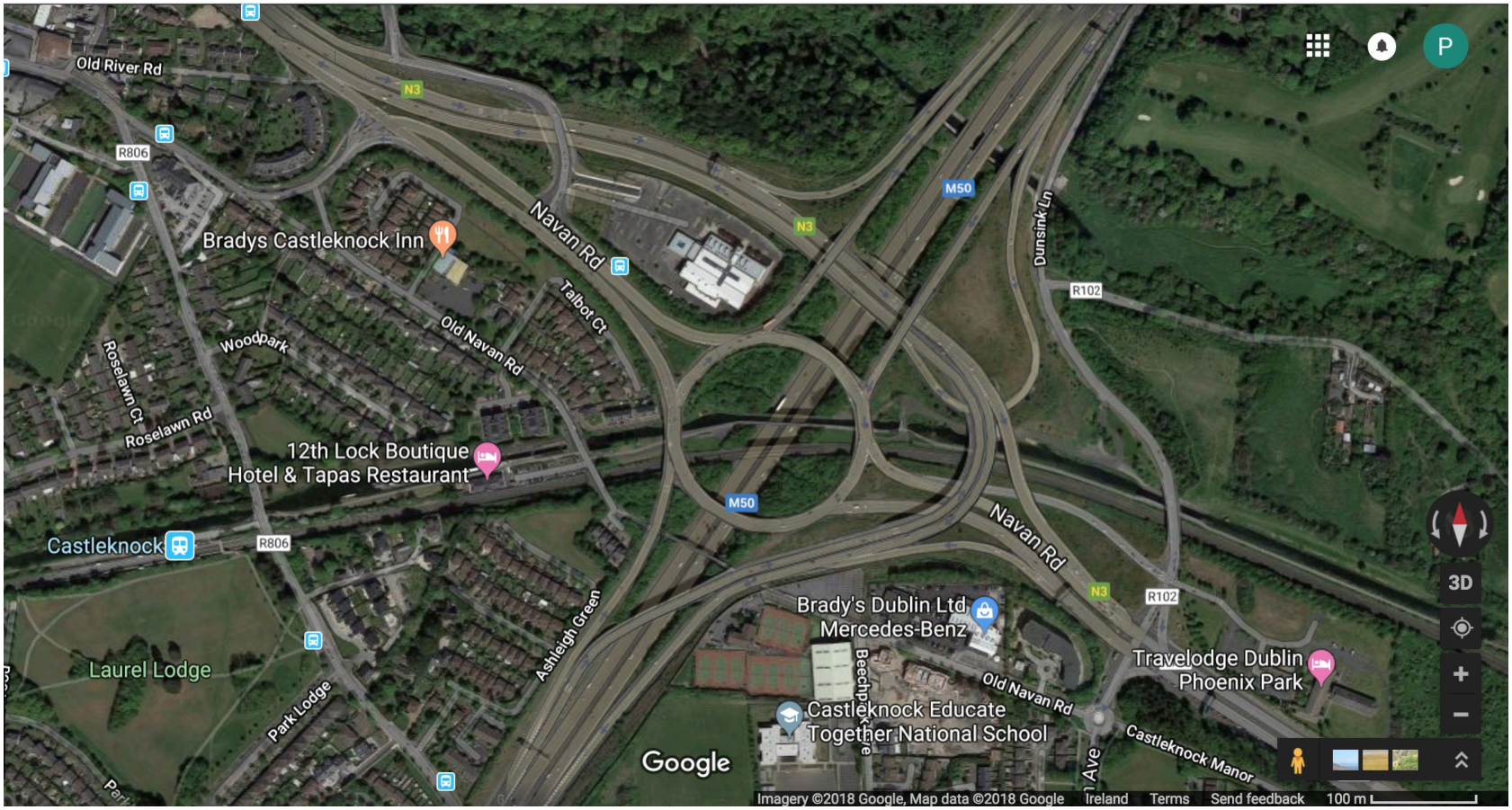


Piccadilly Line, Detail

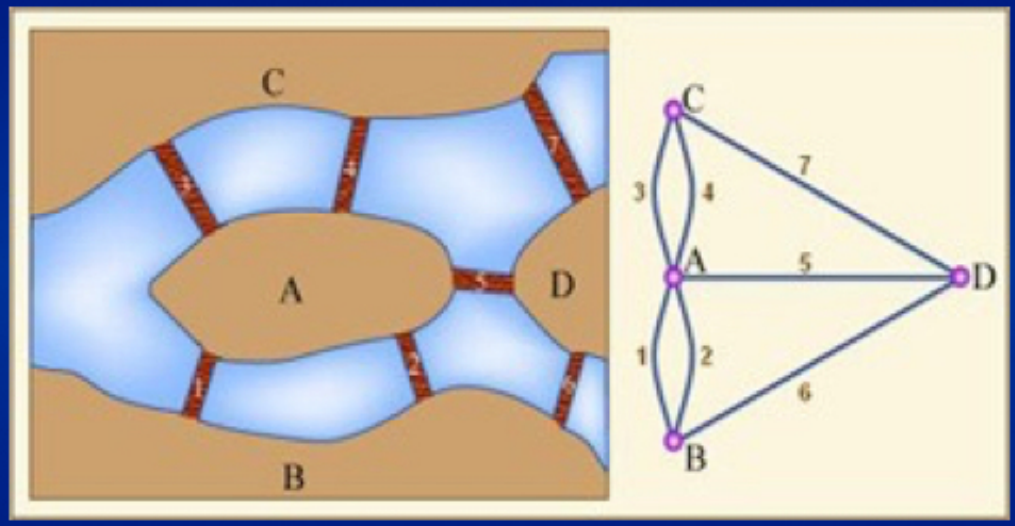


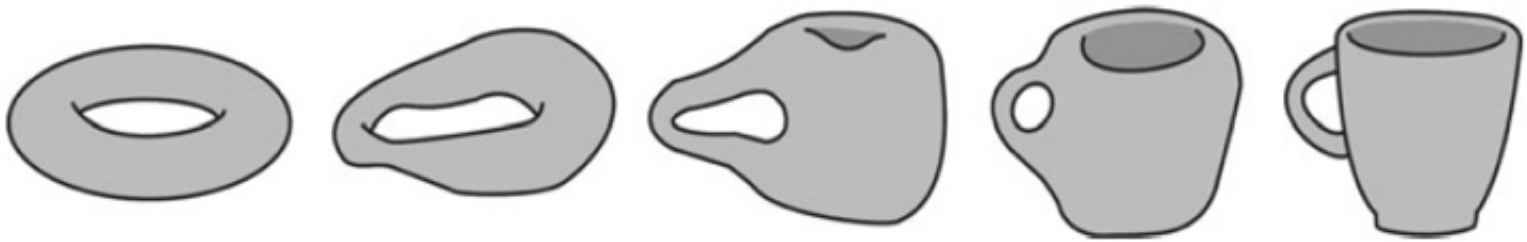
Spaghetti Junction on M50

Spaghetti Junction on M50



Topology
is often called
Rubber Sheet Geometry





Definition of a Topologist

Continuous distortion without tearing or glueing.

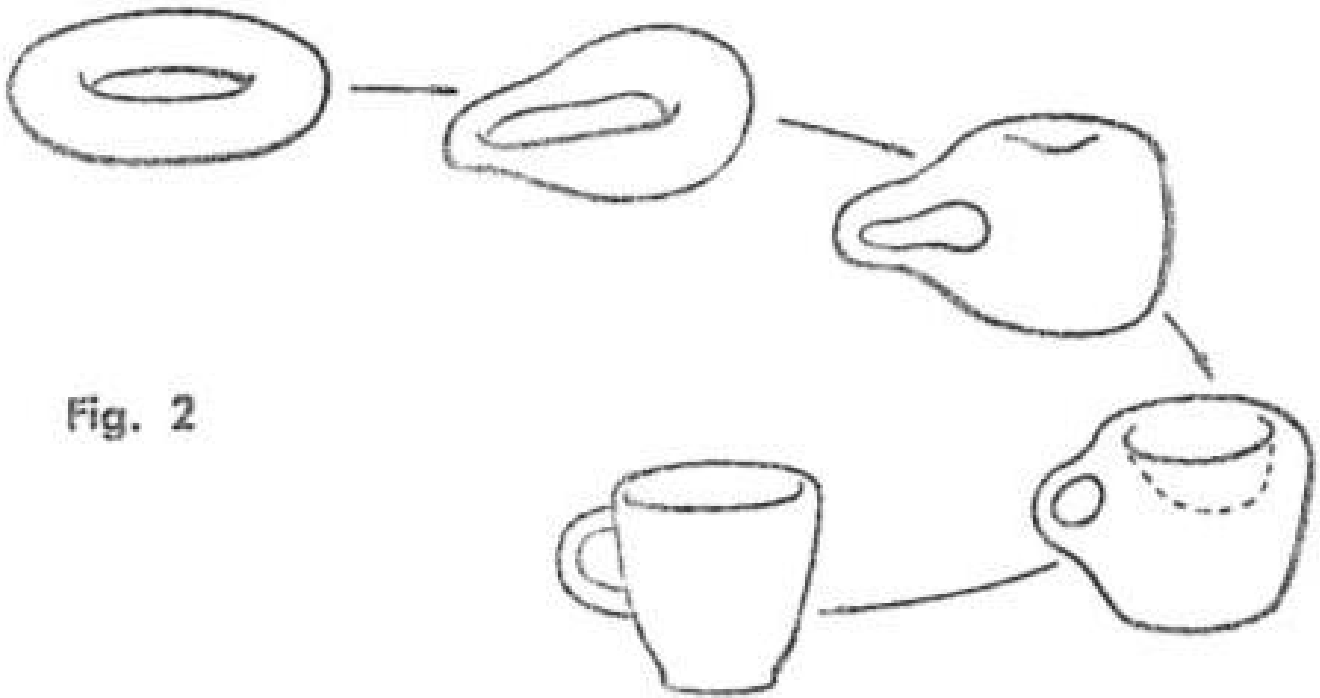


Figure: “A topologist is someone who doesn’t know the difference between a doughnut and a coffee-cup.” [Joke!]

Topological Invariance

Topology is about *Continuity* and *connectedness.*

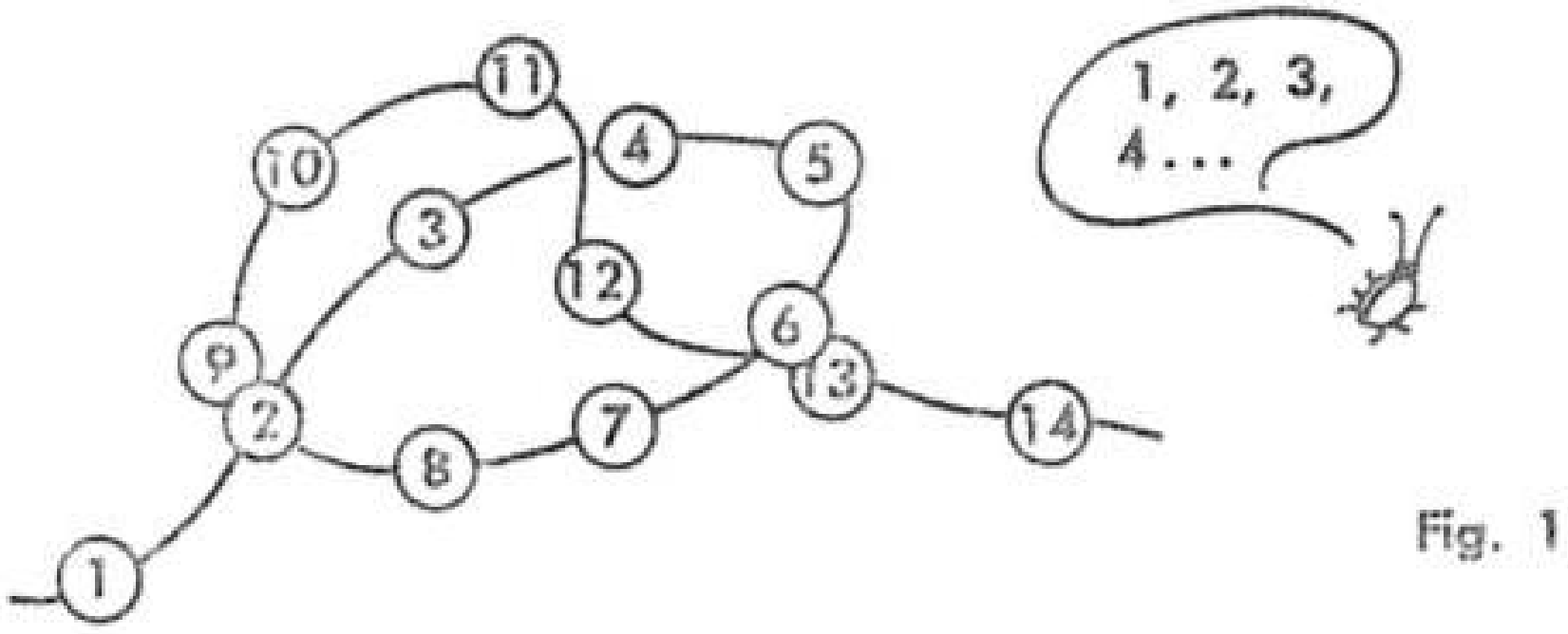


Figure: Order of points unchanged under distortion. A bug sees only the order of the points, not the shape of the curve.

Intrinsic and Extrinsic Properties

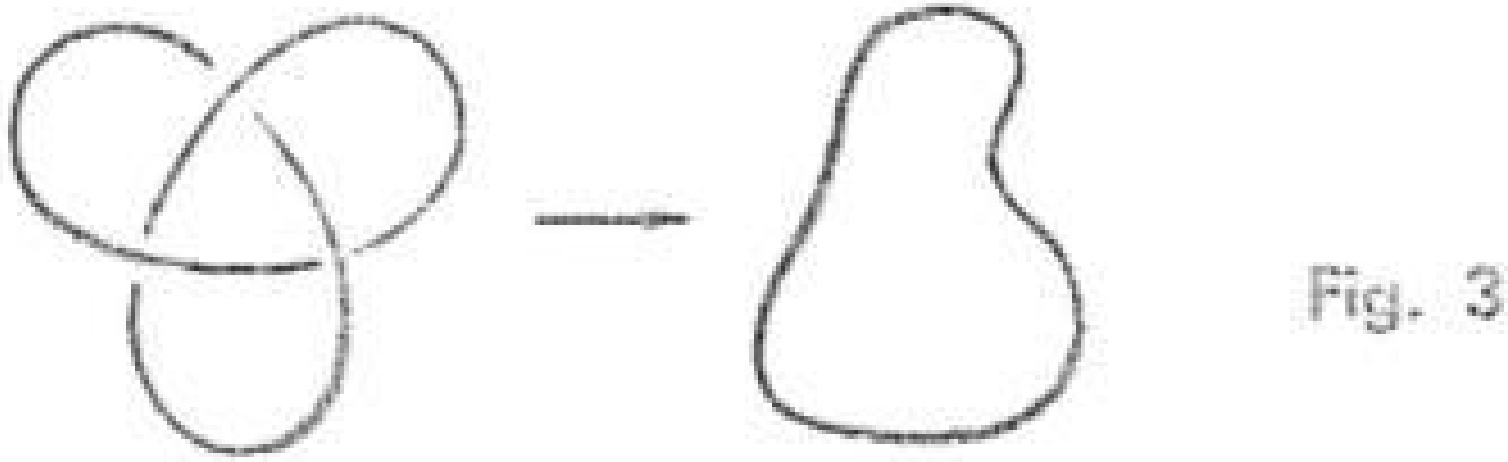


Figure: Knot and loop are topologically equivalent, but cannot be transformed to eachother without breaking and re-glueing.

Homeomorphism versus Ambient Isotopy.

A Jordan Curve

Figure: Simple closed curve divides plane into two regions.

A *Jordan curve* is equivalent to a circle.
It has an inside and an outside.

Closed Loops on a Torus

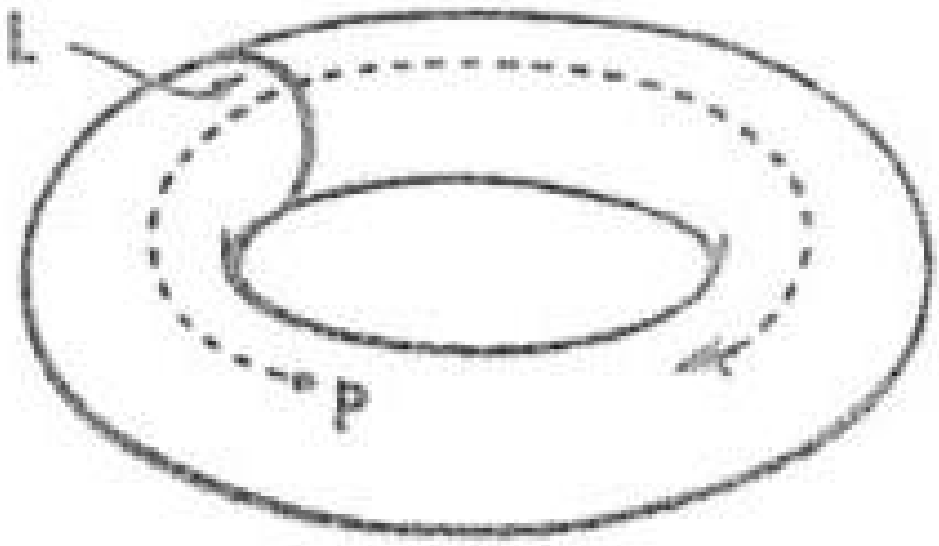


Fig. 6

Figure: Two circles that intersect at just one point.

Even with these two loops, there is only one region.

On a Torus, $V - E + F = 0$

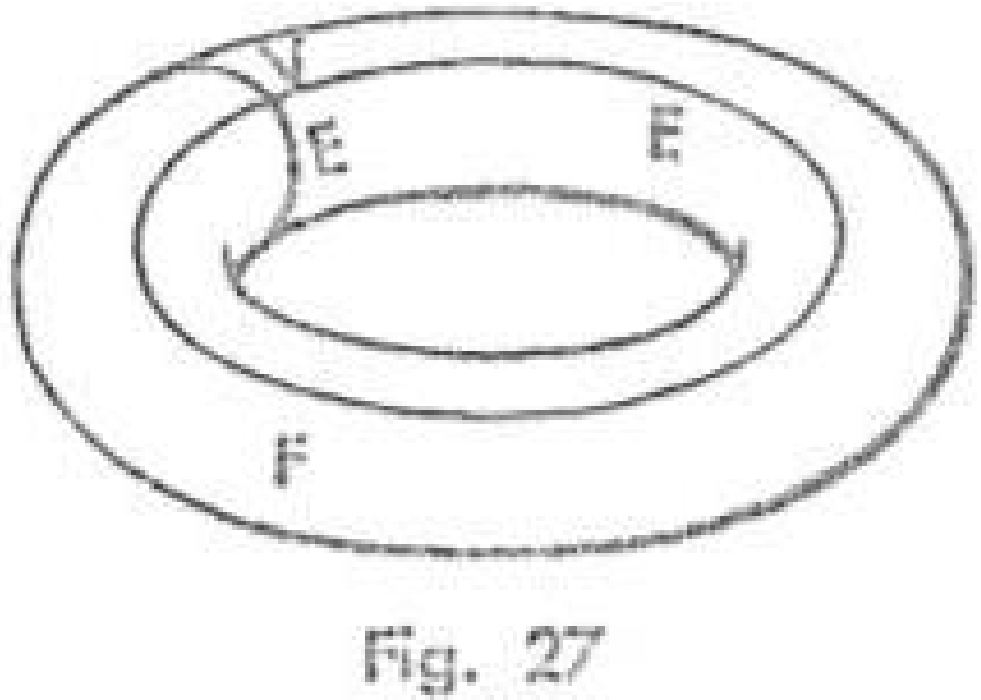
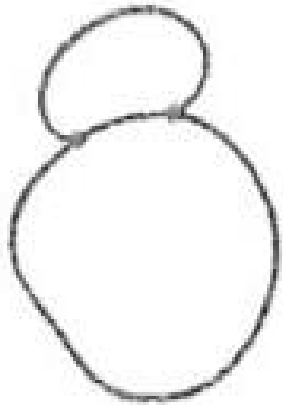


Fig. 27

Figure: Euler's Formula for a surface with a hole.

Connect Two Points on a Closed Loop



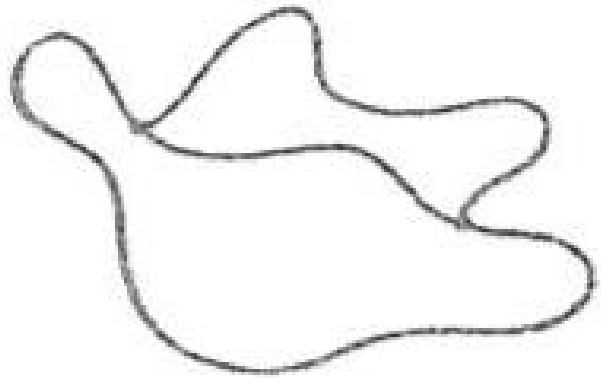


Fig. 7

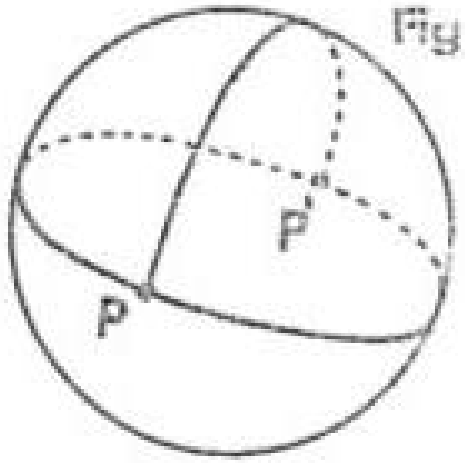


Fig. 8

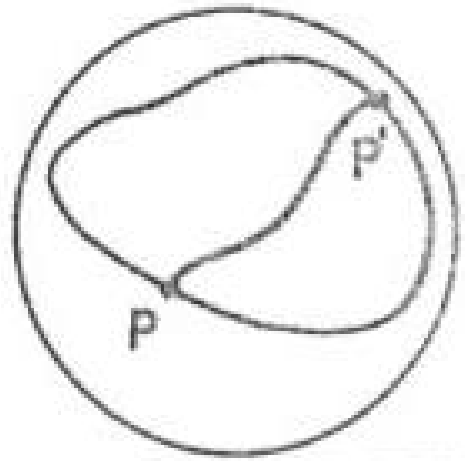


Fig. 9

Connect Two Points on a Closed Loop

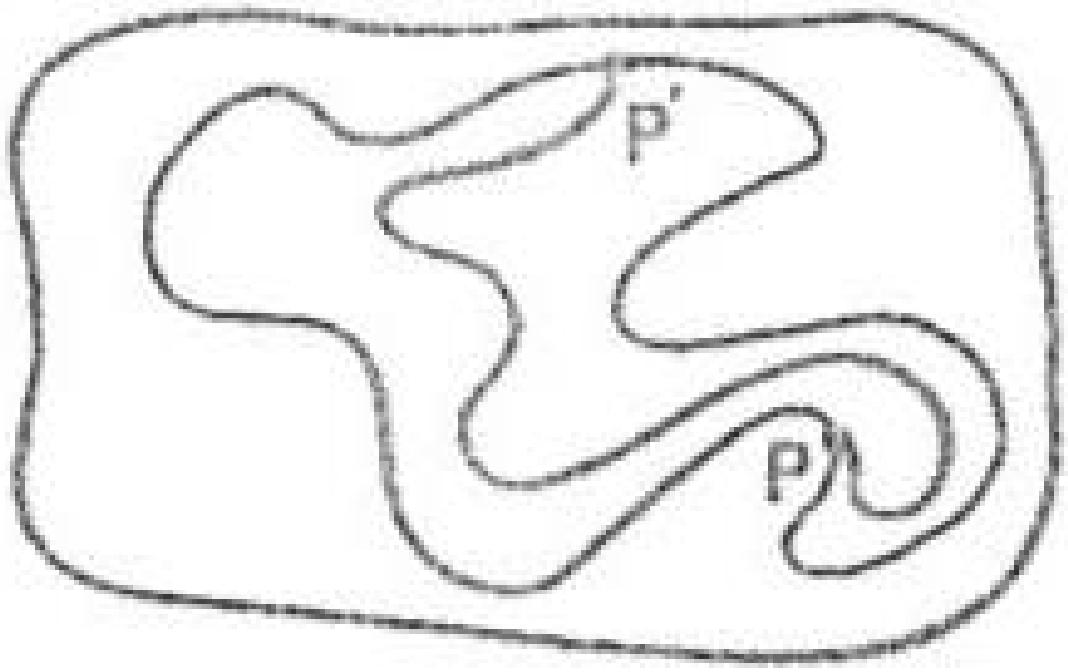
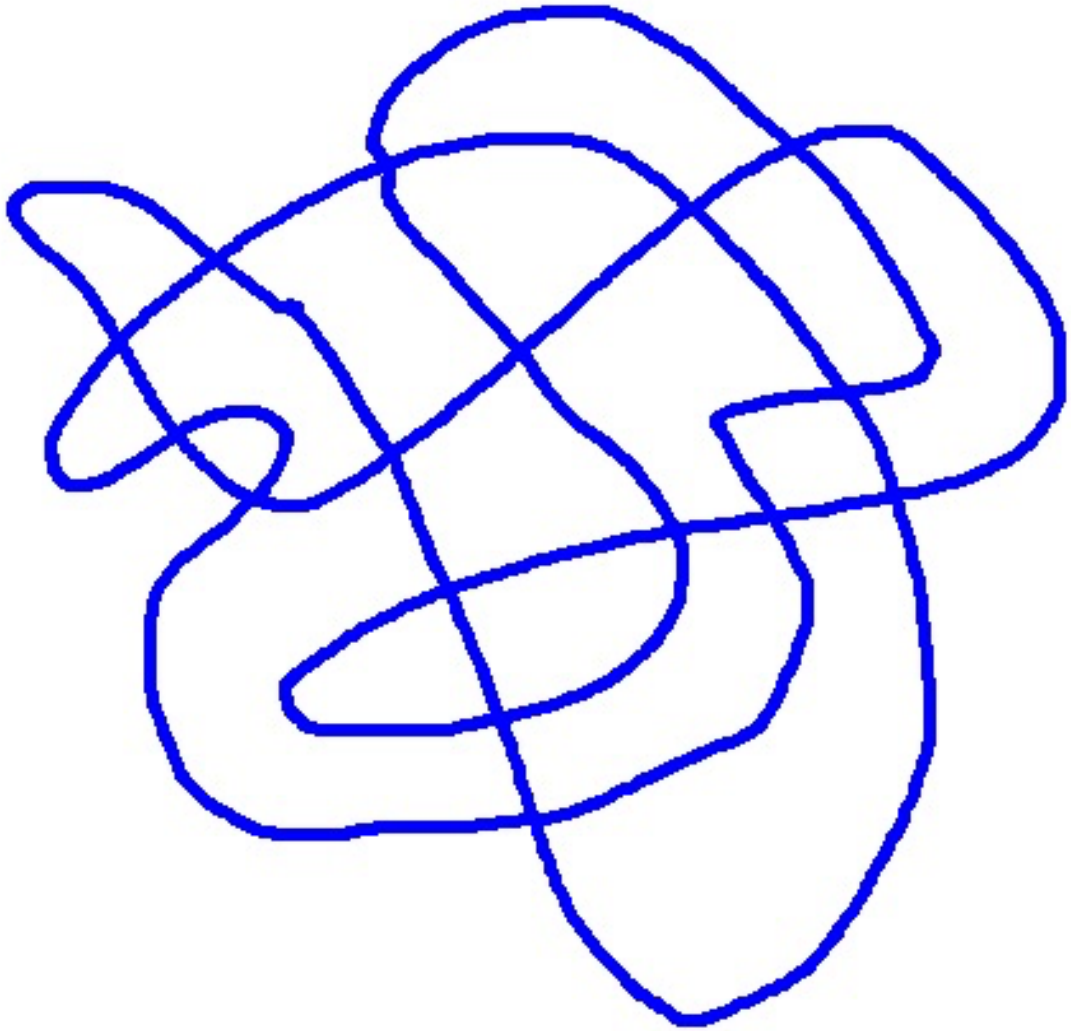


Fig. 10

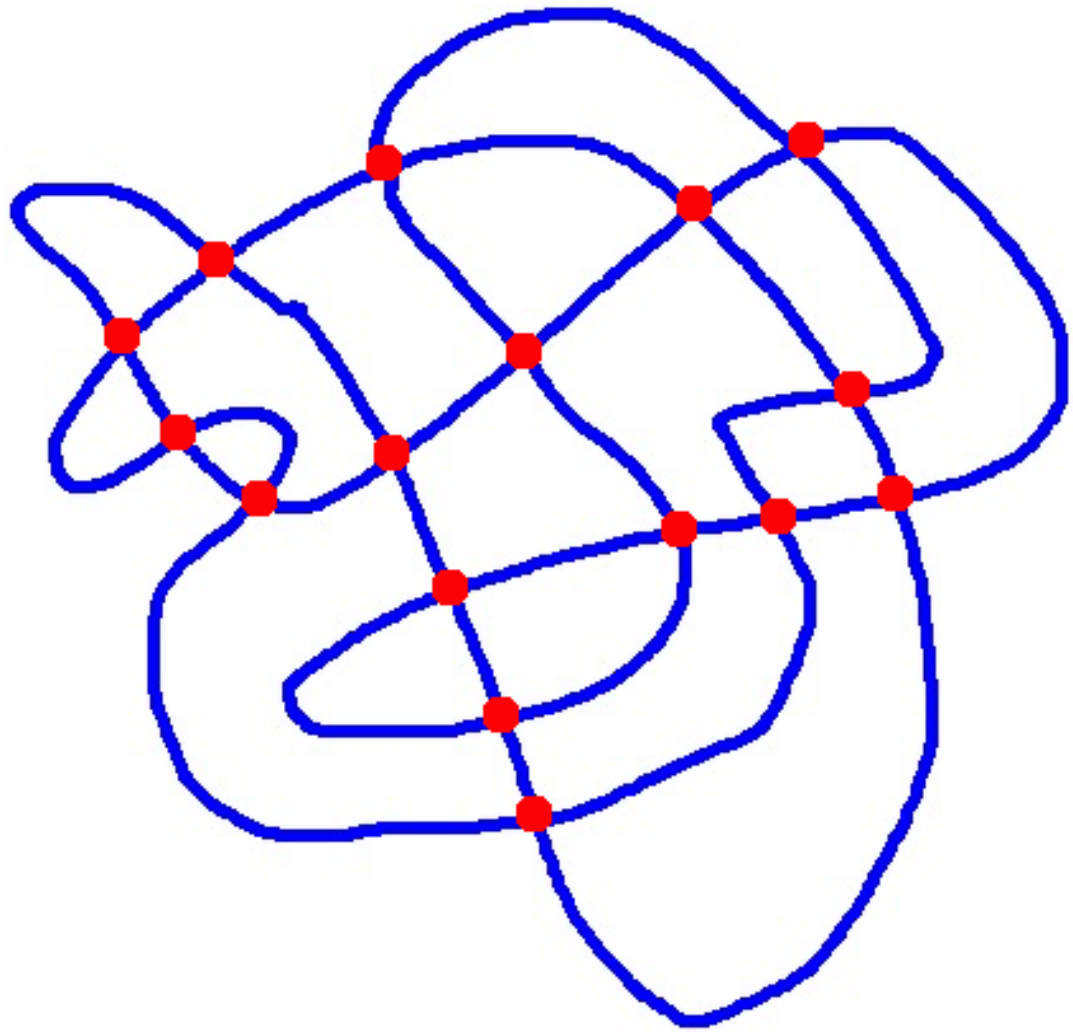
However distorted, there remain 2 joints, 3 edges and 3 regions.

$$V - E + F = 2.$$

The Doodle Theorem



The Doodle Theorem



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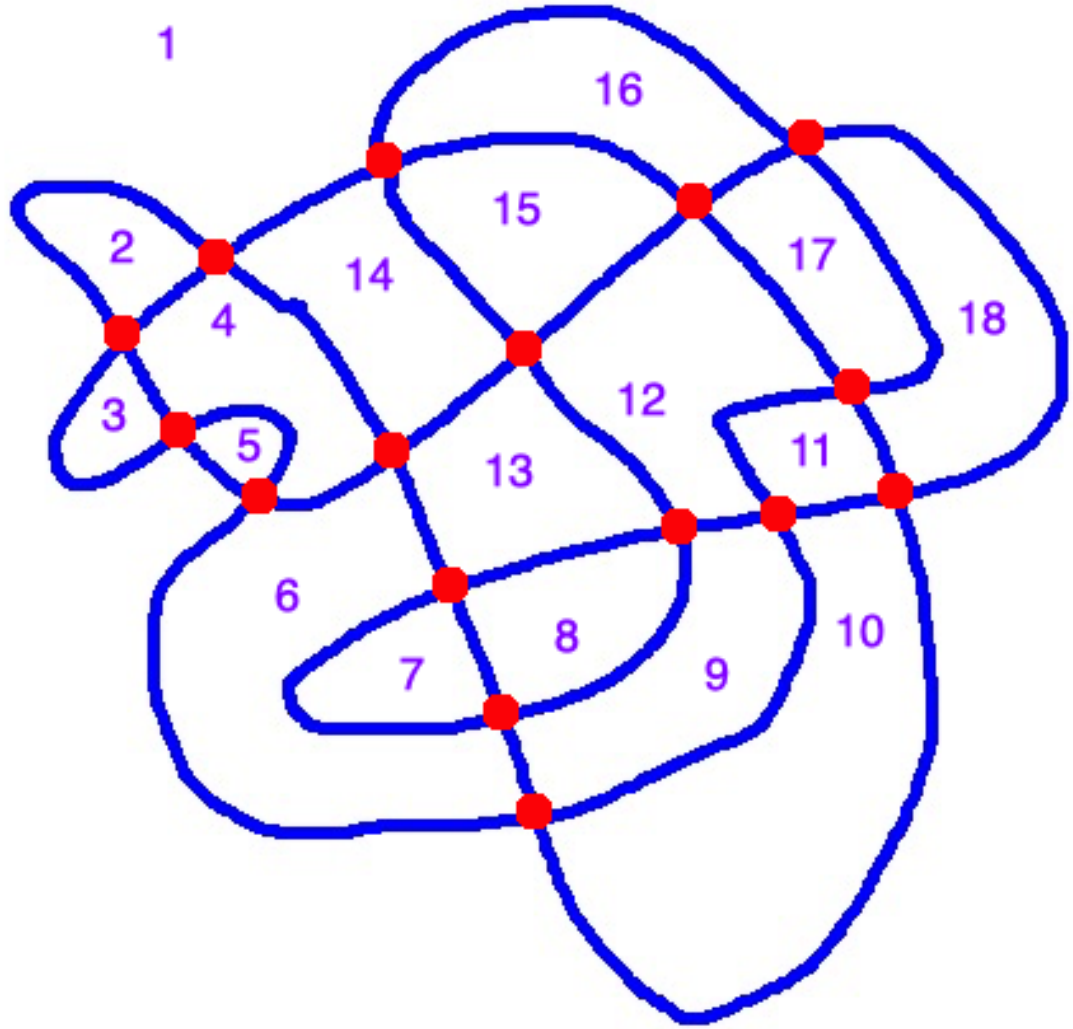
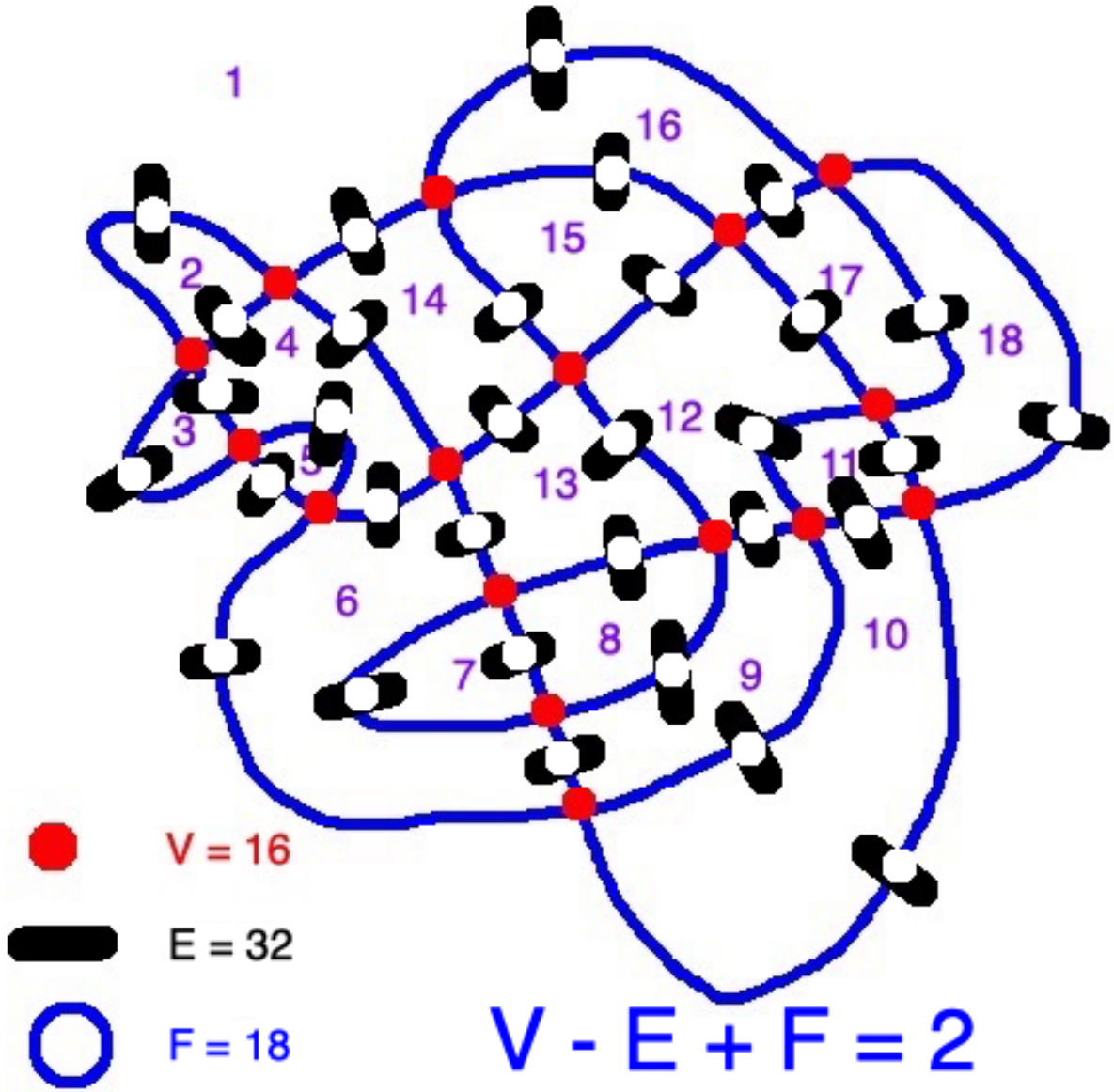


The Doodle Theorem



Euler's Formula for Polyhedra

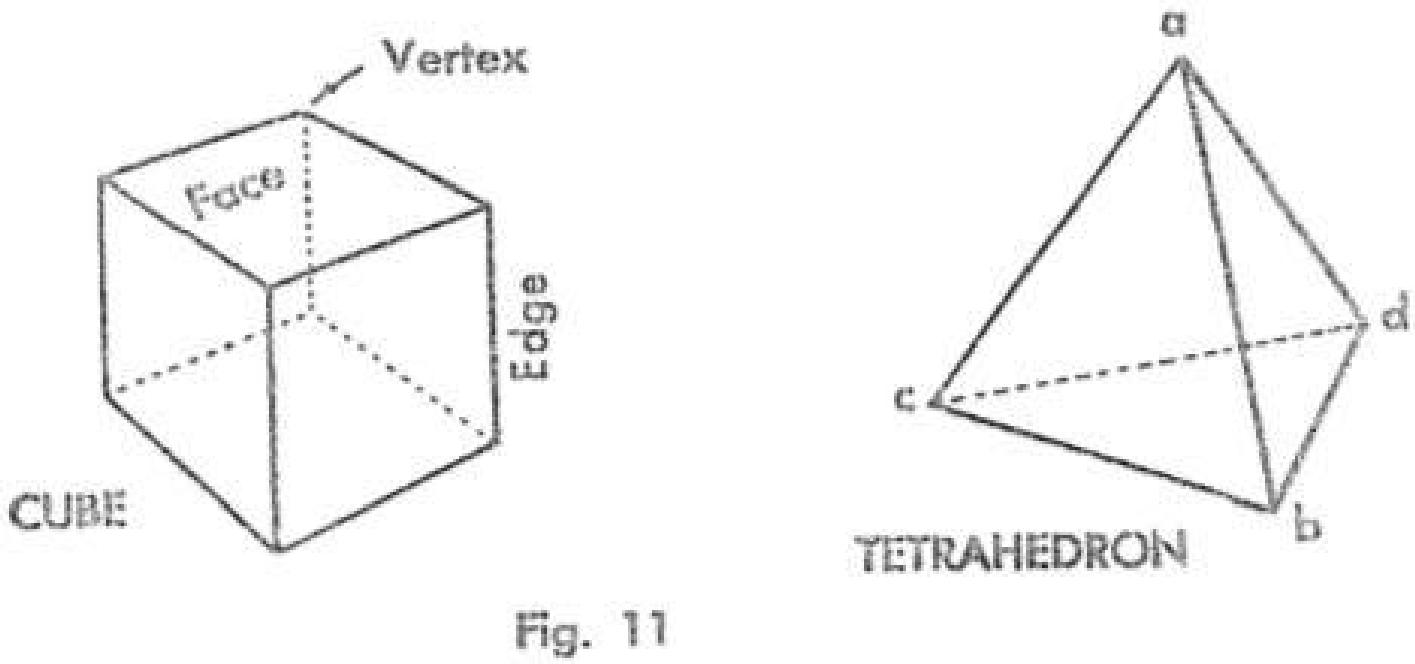


Fig. 11

Figure: Check that $V - E + F = 2$ holds.

Equator and Three Semi-Meridians

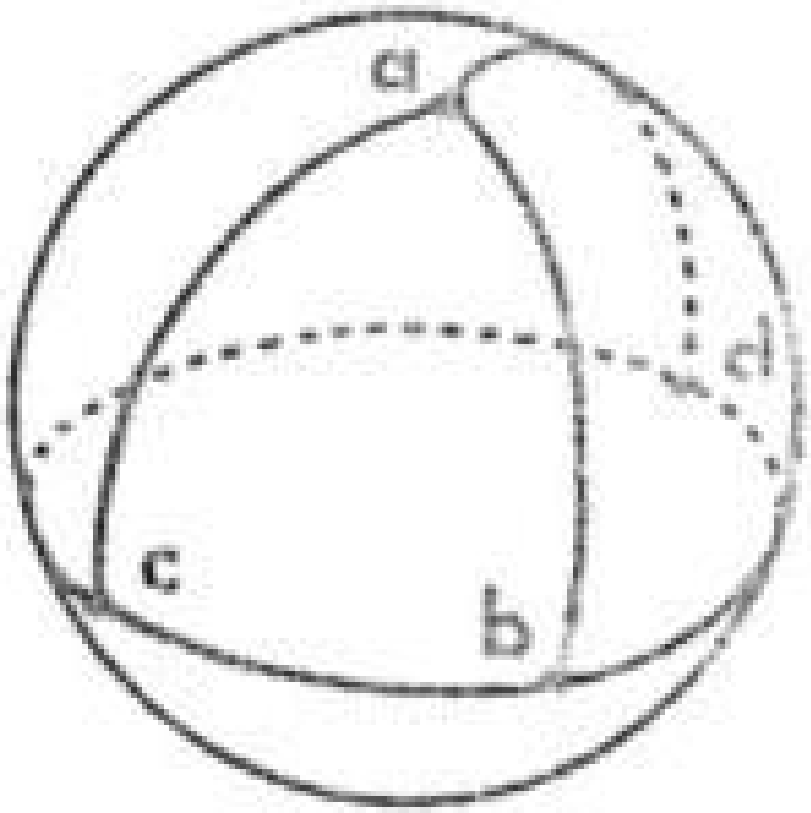


Fig. 12

$$V - E + F = 2$$

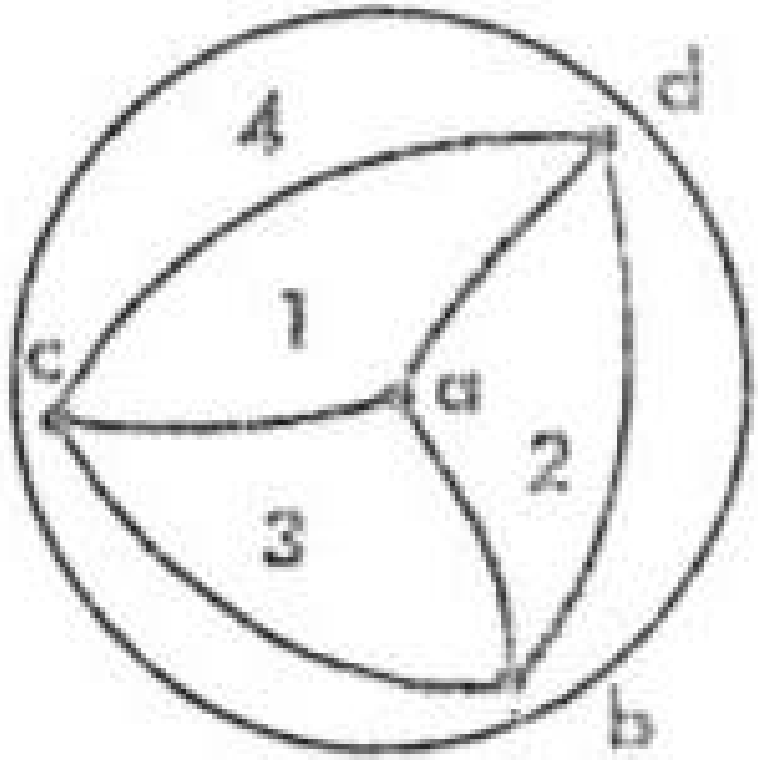
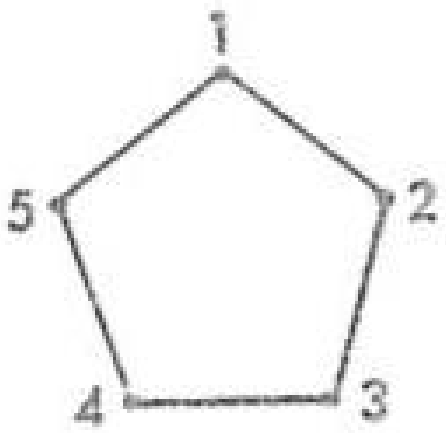


Fig. 13

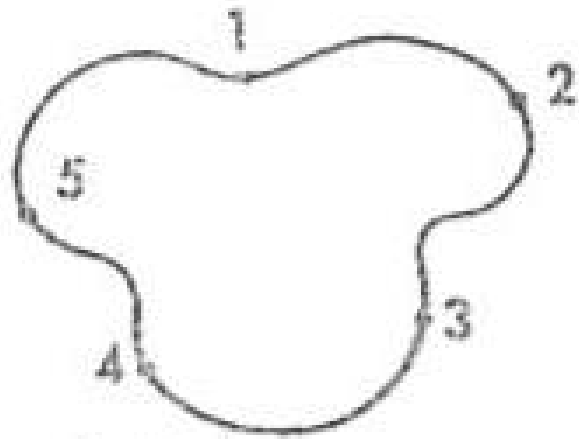
Figure: This is $\mathbf{K_4}$, the complete graph on 4 points.

Geometric and Topological Pentagons



Pentagon

Fig. 14



Topological Pentagon

Outline

The Greek Alphabet, Part 3

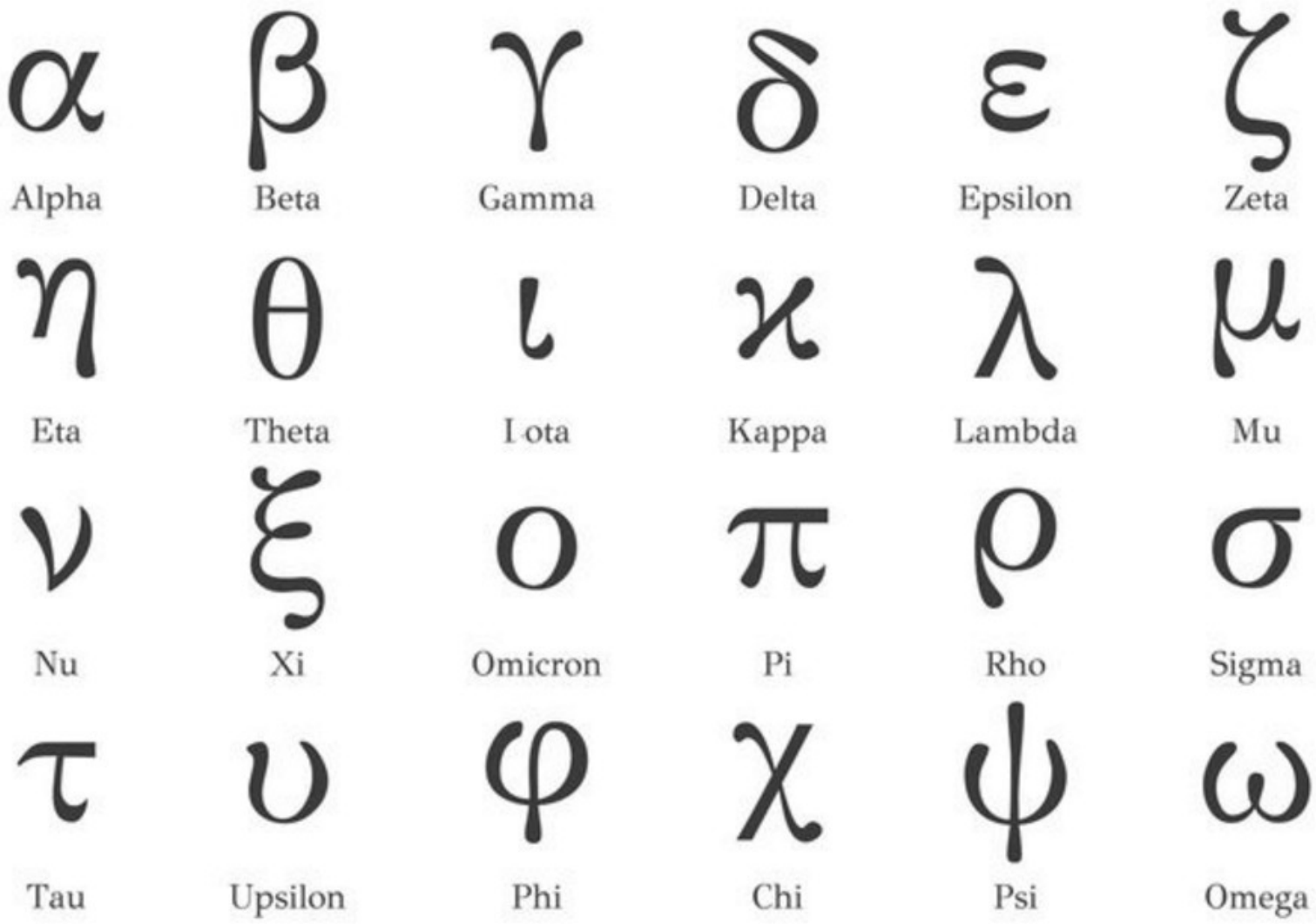


Figure: 24 beautiful letters

The Next Six Letters

We will consider the third group of six letters.

ν	ξ	o	π	ρ	σ
N	Ξ	O	Π	P	Σ

Let us focus first on the *small letters*
and come back to the big ones later.

$$\nu \qquad \xi \qquad o \qquad \pi \qquad \rho \qquad \sigma$$

Nu (ν) is in Planck's formula: $E = h\nu$.
Then ν is the frequency of a photon of light.

Xi (ξ) is the Greek X, as in $\kappa\lambda\iota\mu\alpha\xi$ or $\mathrm{K\Lambda IMA\Xi}$.

Omicron: Think of Oh-Micron, small Oh (not OMG).
Is there a large O, or Oh-Mega ?

Pi (π) is already very familiar to you all.

Rho (ρ) is Greek R, used for density.

Sigma (σ) is the Greek S. At the end of a word it is written ς.

Now we know eighteen letters. We're 75% done!

A Few Greek Words (for practice)

$\kappa\lambda\iota\mu\alpha\xi$
$\delta\rho\alpha\mu\alpha$
$\nu\epsilon\kappa\tau\alpha\rho$
$\kappa\omega\lambda o\nu$
$\kappa o\sigma\mu o\varsigma$

Climax: $\kappa\lambda\iota\mu\alpha\xi$
Drama: $\delta\rho\alpha\mu\alpha$
Nectar: $\nu\epsilon\kappa\tau\alpha\rho$
Colon: $\kappa\omega\lambda o\nu$
Cosmos: $\kappa o\sigma\mu o\varsigma$


Κέντρο
Center
Λιμάνι
Port

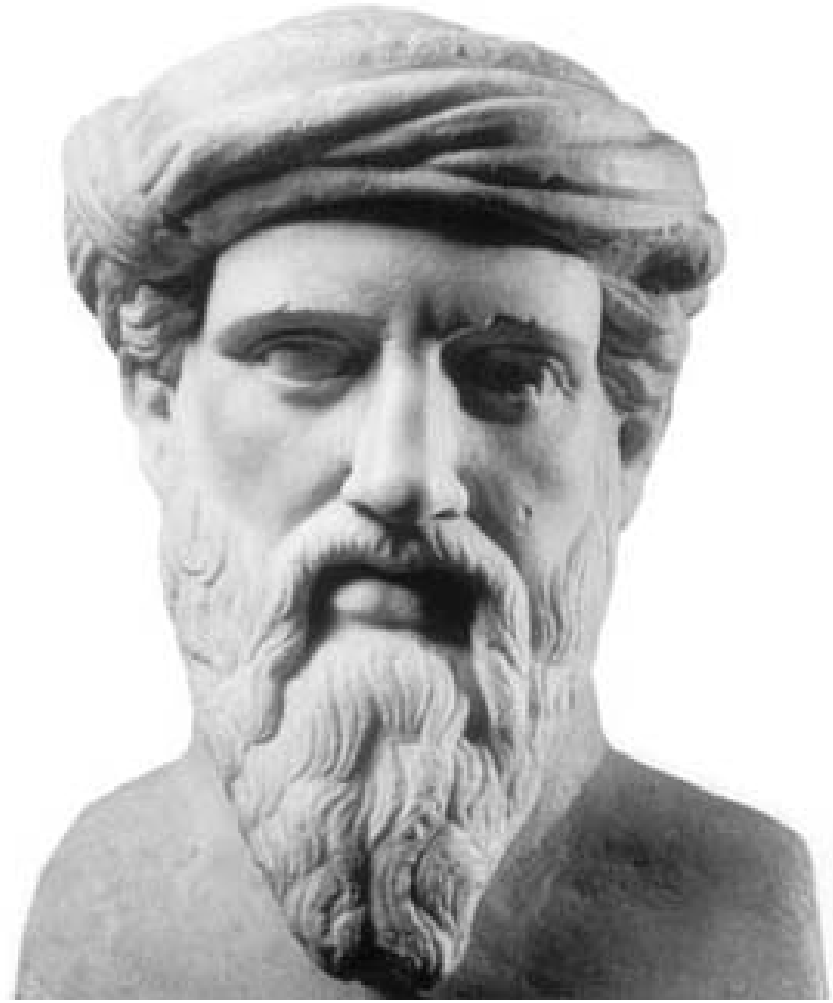

Μουσειο Ακρόπολης
Acropolis Museum


Αθήνα
Δελφοί
400M


ΤΑΞΙ

End of Greek 103

Outline



The Thallasic Age

The period from 800 BC to AD 800.

$\Theta\alpha\lambda\alpha\sigma\sigma\alpha$ **— the Sea.**

- **The first Olympic Games in 776 BC**
- **Homer and Hesiod lived around 700 BC**
- **Greek mathematics began to thrive**
- **First two major figures: Thales and Pythagoras.**

Pythagoras (c. 570–495 BC)

Pythagoras was

- Born on the island of Samos (off Turkey).
- Philosopher, mystic, prophet and religious leader.
- Contemporary with Confucius and Lao-Tzu.

Words philosophy (love of learning) and mathematics (that which is learned) attributed to Pythagoras.

May have been first person to imagine that natural phenomena can be understood through mathematics.

Pythagoras (c. 570–495 BC)

- No contemporary documents
- Myth, legend and tradition
- Second or third hand accounts
  often written centuries later
- Aristotle's biography no longer extant.

Hardly any statement about Pythagoras uncontested.

Difficult to separate history from myth and legend.

Pythagoras (c. 570–495 BC)

- Travelled to Egypt, Babylon and perhaps India
- Mathematics, astronomy and religious lore
- Theorem on right-angled triangles
- Result known to Babylonians 1000 years earlier
- No record of a proof by Pythagoras survives.

The Pythagoreans

Around 530 BC Pythagoras moved to Croton in Magna Graecia (now Southern Italy).

He established an organization or school (philosophical / religious / political).

Both men and women were members of "The Pythagoreans"

Adherents were very secretive:
Bound by an oath of allegiance

Led lives of temperance; observed strict moral codes.

Pythagorean Women

“Women were given equal opportunity to study as Pythagoreans, and learned practical domestic skills in addition to philosophy.

“Women were held to be different from men, sometimes in positive ways.

“The priestess, philosopher and mathematician Themistoclea is regarded as Pythagoras’ teacher; Theano, Damo and Melissa as female disciples.”

From the Wikipedia article: *The Pythagoreans.*

Pythagorean Quotes

- “I was *Euphorbus* at the siege of Troy.”
- “In anger, refrain from both speech and action.”
- “Educate the children and it won’t
  be necessary to punish the men.”
- “Abstain from beans!”

- “There is geometry in the humming of the strings,
  There is music in the spacing of the spheres.”
- “Number rules the universe.”

Harmony & Discord

By tradition, Pythagoras discovered the principles of *musical harmony.*

Stringed instruments produce harmonious sounds when string lengths are ratios of small numbers.

Extended this idea to the heavens: planets emit sounds according to their speed of movement

Concept of the “harmony of the spheres”.

Johannes Kepler: Harmonices Mundi

All is Number

The motto of the Pythagoreans: *All is Number.*

All natural phenomena in the universe can be expressed using whole numbers or ratios of them.

For the Pythagoreans, numbers were the essence and source of all things.

Modern physics holds that, at its deepest level, the universe is mathematical in nature.

This view is a topic of current serious discussion (*The Mathematical Universe*, by Max Tegmark).

Outline

Distraction: A Curious Year, AD 1089

What is so special about the year 1089?

- **Palmyra destroyed by an earthquake.**
- **First Cistercian monastery, Cîteaux Abbey, founded in southern France.**
- **The Council of Melfi issues decrees against simony and clerical marriage.**

Such vital information is obtained from Wikipedia.

Distraction: A Curious Number

Think of a three-digit number, for example 275**.**

Calculate the difference between this number and the number formed by reversing digits:

$$572 - 275 = 297$$

Now repeat the process, this time adding numbers:

$$297 + 792 = 1089$$

What is so special about the number 1089?

Distraction: A Curious Number

This “trick” nearly always works.

But it can fail in some cases.

Can you find the conditions for success?

See the Wikipedia page “1089 (number)”.

Thank you