

AweSums

Marvels and Mysteries of Mathematics



LECTURE 1

Peter Lynch

**School of Mathematics & Statistics
University College Dublin**

Evening Course, UCD, Autumn 2019



Outline

Introduction

Overview

Visual Maths 1

Distraction 1: A Piem

The Beginnings

Babylonian Numeration Game

Summary of Lecture



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Aim of the Course

AweSums

The course will run over ten (10) lectures, from 30 September to 9 December.

No lecture on 28th October.
So, the course splits into 4+6.

The **aim of the course** is to show you

- ▶ The great **beauty** of mathematics;
- ▶ Its tremendous **utility** in our daily lives;
- ▶ The **fun** we can have studying maths.



Meaning and Content of Mathematics

The word **Mathematics** comes from Greek *μαθημα* (*máthéma*), meaning “knowledge” or “study” or “learning”.

It is the study of topics such as

- ▶ Quantity
- ▶ Structure
- ▶ Space
- ▶ Change



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It is the study of topics such as

- ▶ Quantity: [Numbers. Arithmetic]
- ▶ Structure: [Patterns. Algebra]
- ▶ Space: [Geometry. Topology]
- ▶ Change: [Analysis. Calculus]



Tom Lehrer: Mathematician, Musician and Comic Genius

ThatsMaths article in *The Irish Times* in
September 2018 about mathematician
and comic genius **Tom Lehrer**.

(You can find articles using the Search Box.)

<https://thatsmaths.com/>



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<https://thatsmaths.com/>

- ▶ `/Users/peter/Dropbox/Music/Videos.html`
- ▶ Run Video (vsn 1)
- ▶ List Keywords
- ▶ Run Video (vsn 2) later.



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Names for the Course

- ▶ Maths for Everyone
- ▶ The Fun of Maths
- ▶ Recreational Maths
- ▶ **Sum-Enchanted Evenings**
- ▶ Our Mathematical World
- ▶ The History and Development of Maths
- ▶ Mathematics: Beautiful, Useful & Fun



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- ▶ The History and Development of Maths
- ▶ Mathematics: Beautiful, Useful & Fun

All of these titles have advantages, but this year I have returned to:

AweSums



Notes and Slides

- ▶ **All the slides will be available online:**
<http://mathsci.ucd.ie/~plynch/AweSums>
[just Google for "Peter Lynch UCD"]
- ▶ ***No notes* are to be provided.**
Why Not? See next slide.
- ▶ **Additional Reading Recommendations.**
- ▶ **Optional Exercises and Problems.**



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- ▶ **No Assignments!**
- ▶ **No Assessments!**
- ▶ **No Examinations!**



Why No Notes?

- ▶ **Maths is NOT a Spectator Sport**
- ▶ **Active engagement is essential to understanding.**
- ▶ **You should take your own notes:**
 - ▶ **This involves repetition of what you hear.**
 - ▶ **This involves repetition of what you see.**
 - ▶ **What you write passes through your mind!**
 - ▶ **This process is a great help to memory.**



Lectures

- ▶ **Classes run from 7pm to 9pm.**
- ▶ **120 minutes intensive lecturing too long (both for you and for me).**
- ▶ **Educational Theory:**
 - ▶ **Attention & retention both decrease with time.**
- ▶ **Class will be broken into short sections.**



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If you cannot attend a class:

- ▶ **Please do not bother to email me.**
- ▶ **There is no need to give any reasons.**
- ▶ **The presentation slides will be available.**



Communications

In the unlikely event that a class has to be cancelled,

- ▶ I will notify UCD “Lifelong Learning”.
- ▶ If you wish to supply an email, I will also try to email you directly.
- ▶ You may wish to form a **WhatsApp** group.

I will also tell you about other mathematical events in Dublin if I hear about them.



“Typical” Structure of a Class

1. **Problem: Background and Theory (30 min)**
2. **Distraction (10 min)**
3. **Some History of the problem (30 min)**
4. **Another Distraction (10 min)**
5. **Exercises, Puzzles, History (20 min)**
6. **Questions & Discussion (20 min)**

Total duration: about 120 minutes.



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Total duration: about 120 minutes.

I will (normally) be available after classes to answer questions or offer clarifications.



Some Distractions

- ▶ **Visual Awareness: Maths Everywhere**
- ▶ **Puzzles: E.g. Watermelon Puzzle**
- ▶ **Sieve of Eratosthenes**
- ▶ **The Greek Alphabet**
- ▶ **Lateral Thinking in Maths**
- ▶ *Lecture sans paroles*
- ▶ **How Cubic and Quartic Equations were cracked**
- ▶ **Four-colour Theorem**
- ▶ **Online Encyclopedia of Integer Sequences**

Please ask me if you have a favorite topic!



It's Your Course

I expect a group with a wide range of knowledge and “mathematical maturity”.

Everybody should benefit from the course.

If anything is unclear, **SHOUT OUT!** or whisper!

If something is missing, let me know.

Feedback on the course is very welcome.



It's Your Course

Classes begin at 7 pm. and run till 9 pm.

Pi Restaruant (downstairs) closes at 8:00.



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- ▶ **Break at 7:50 for 15–20 minutes.**
- ▶ **Don't break at all !!!**

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Let's have a show of hands: Who wants a break?



Popular Mathematics Books

1. John H Conway and Richard K Guy, 1996:
The Book of Numbers. Copernicus, New York.
2. ♥ ⇒ John Darbyshire, 2004:
Prime Obsession. Plume Publishing.
3. ♥ ⇒ William Dunham, 1991:
Journey through Genius. Penguin Books.
4. Marcus Du Sautoy, 2004:
The Music of the Primes. Harper Perennial.
5. ♥ ⇒ Richard Elwes, 2010:
Mathematics 1001. Firefly Books.
6. Peter Lynch, 2016: *That's Maths*.
Gill Books. Published in October 2016.



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To Begin: An Optical Illusion

A cautionary tale:

In maths we often use pictures to prove things.

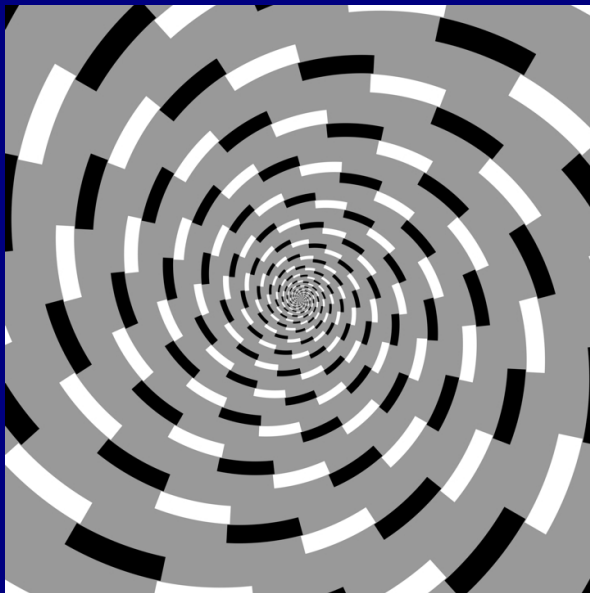
This is usually very helpful.

However, it can sometimes mislead us.

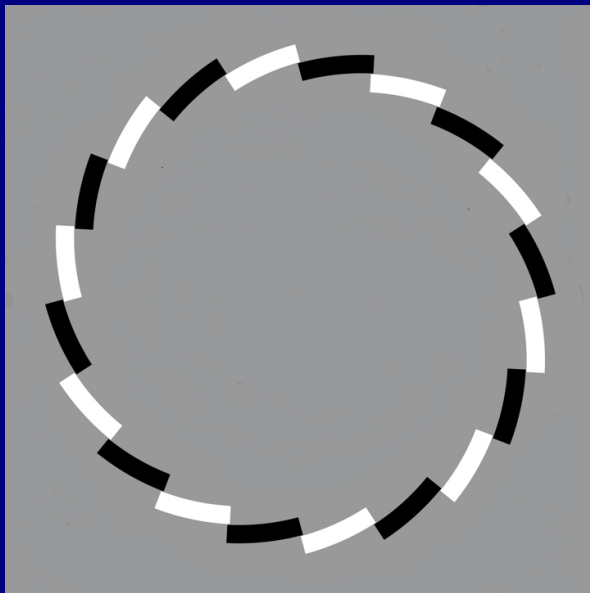
Let us look at the [Fraser Spiral](#).



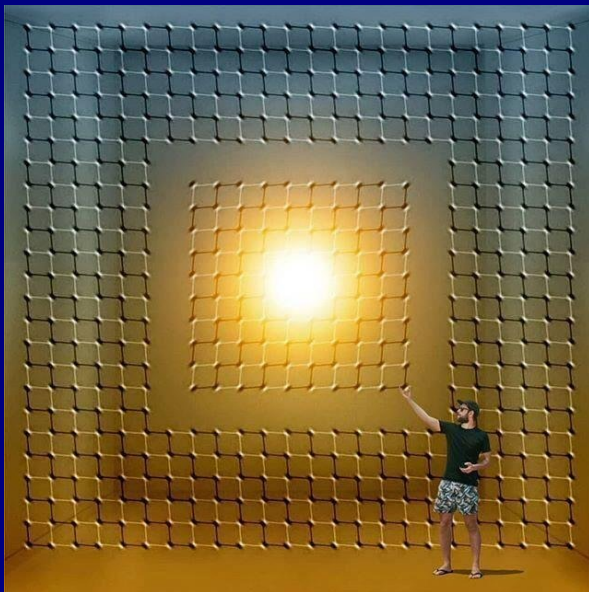
Fraser Spiral



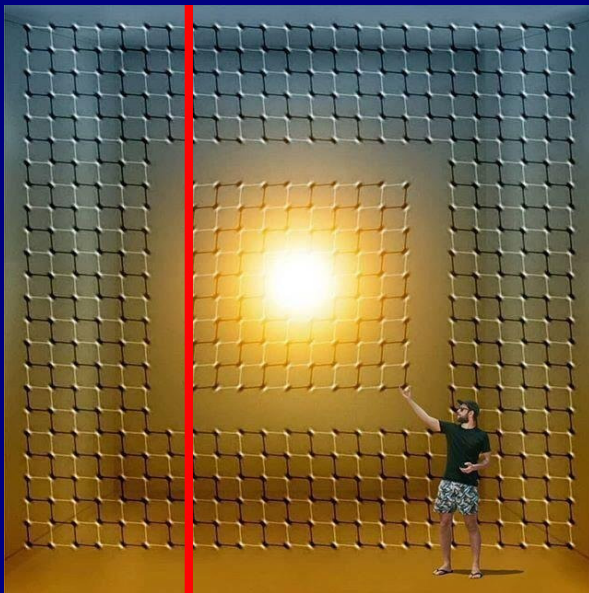
Fraser Spiral



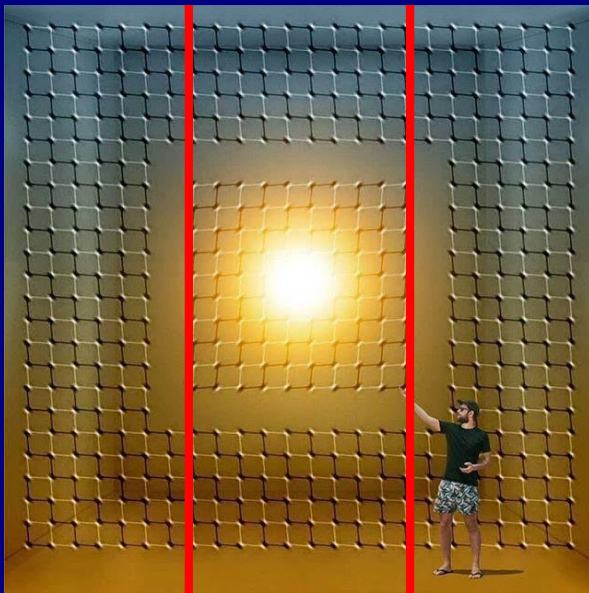
<https://pbs.twimg.com/media/DI5lmeIU8AEpXB7.jpg>



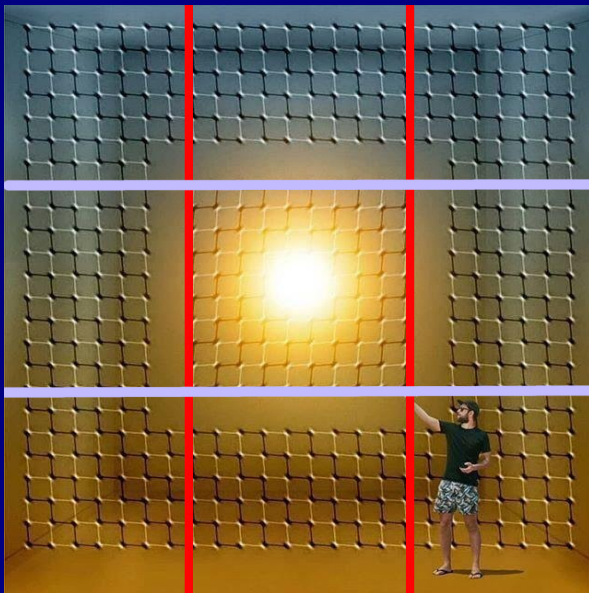
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Visual Maths Proofs

Can the sum of an infinite number of quantities have a finite value?

Let's look at the infinite series

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$



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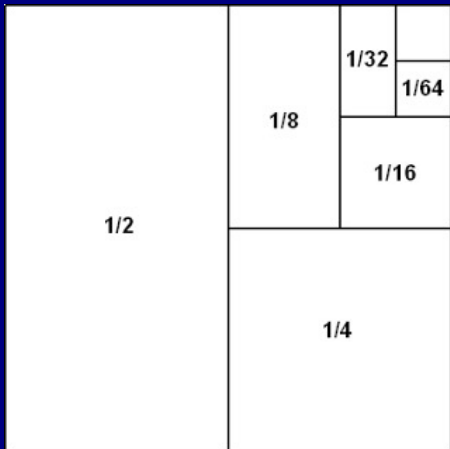
$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

Each term is half the size of the preceding one.

The terms are getting smaller but it is **not obvious** that the series **converges**.



A picture makes everything clear:



Unit Square: At each stage, we add half the remainder of the square.



Conclusion

The infinite series

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

has a **finite** sum:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$$

The terms are getting smaller quickly enough
for the series to be **convergent**.



Another Simple Proof

What is the sum of the first n odd numbers?

$$1 = 1^2 \quad (1 + 3) = 4 = 2^2 \quad (1 + 3 + 5) = 9 = 3^2$$

Is this pattern continued? Can we prove it?

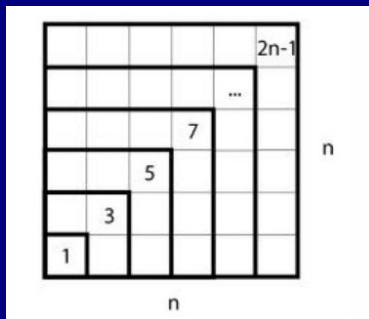


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$$[1 + 3 + 5 + 7 + \dots + (2n - 1)] = n^2$$



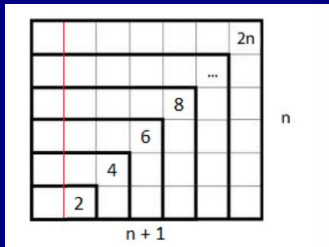
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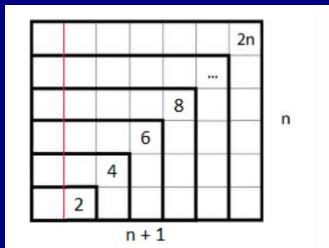
We just add a column on the left. This increases each term of the sequence of odd numbers by 1.

$$[2 + 4 + 6 + \dots + 2n] = n(n + 1)$$



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$$S = 2 + 4 + 6 + 8 + \dots + 2n$$



We just add a column on the left. This increases each term of the sequence of odd numbers by 1.

$$[2 + 4 + 6 + \dots + 2n] = n(n + 1)$$

Now divide both sides by 2 to get:

$$[1 + 2 + 3 + \dots + n] = \frac{1}{2}n(n + 1)$$



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Distraction 1: Remember π

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*How I want a drink,
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Repeat: To Remember π

To 15-figure accuracy, π is equal to

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Distraction 1: Remember $1/\pi$

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Can I remember the reciprocal?



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3 1 8 3 10



Distraction 1: Remember $1/\pi$

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Now you know π and $1/\pi$ to an accuracy
greater than you are ever likely to need!



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The Ancient Origins of Mathematics

Basic social living was possible without numbers

... but ...

elementary **comparisons** and **measures** are needed to ensure fairness and avoid conflicts.



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The need for mathematical thinking arose in problems like fair division of food.

Problem: How do you divide a woolly mammoth?



Division of Food

To divide a collection of apples, the idea of a **one-to-one correspondence** arose.

There was no direct need for **numbers** yet: the apples did not need to be counted, just broken into batches.



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The problem of dividing up a slaughtered animal is more tricky: The forequarters and hindquarters of a woolly mammoth are not the same!



Fair Division: Main Idea

- ▶ Divide a set of goods or resources between several people.
- ▶ Each person should receive his/her due share.
- ▶ Each person should be satisfied **after the division** (this is an **envy-free solution**).



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This problem arises in various real-world settings: auctions, divorce settlements, electronic spectrum and frequency allocation, airport traffic management.

It is an active research area in Mathematics, Economics, Conflict Resolution, and more.



I Cut and You Choose

For two people or two families, the familiar technique “I cut and you choose” should keep everyone happy.

This is the method used by children to divide a cake.

It works even for an inhomogeneous cake, say, half chocolate and half lemon sponge.



To divide fairly between all members of a family is **much more difficult** (as many of you know!).

Exercise: Try to devise a generalization of the “cut-and-choose” method that works for three people . . . and one that works for four people.

This is a difficult problem



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Consider the partition of Berlin



Partition of Berlin (Potsdam Agreement, 1945)

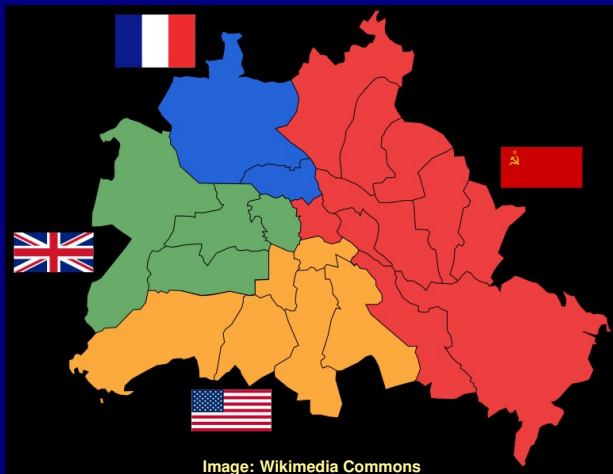
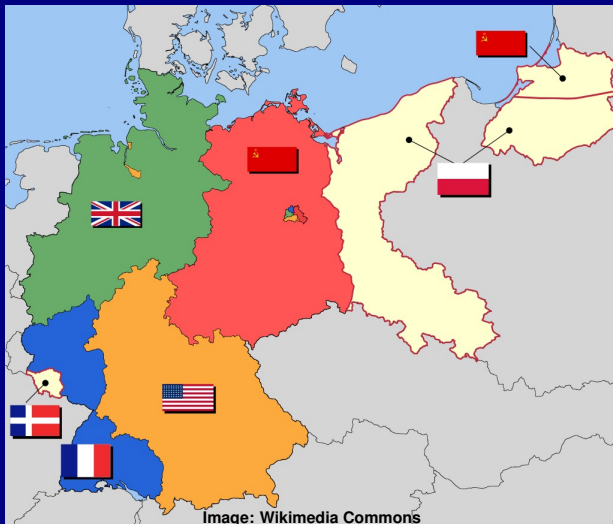


Image: Wikimedia Commons

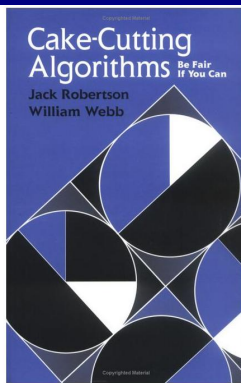
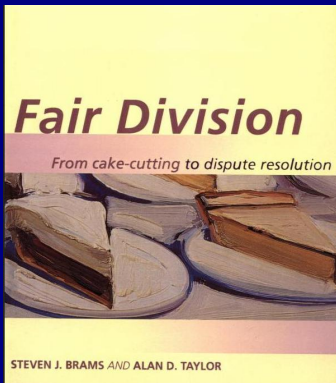


Partition of Germany (Potsdam Agreement, 1945)



Books on Fair Division

Two books devoted exclusively to this problem and its variations



Tally Sticks

Keeping an account of sheep and such animals was done using a tally stick.

The number of notches corresponds to the number of sheep.

Again, for small flocks, no concept of **actual numbers** was essential.

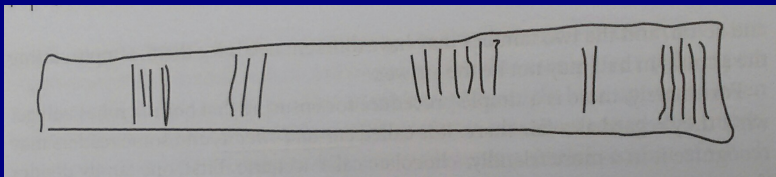


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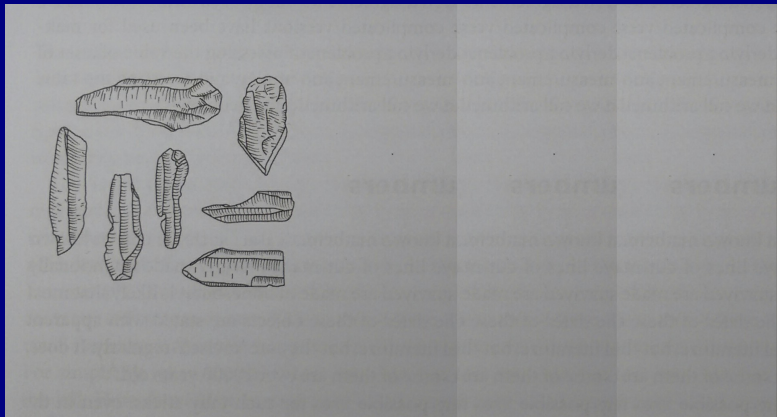
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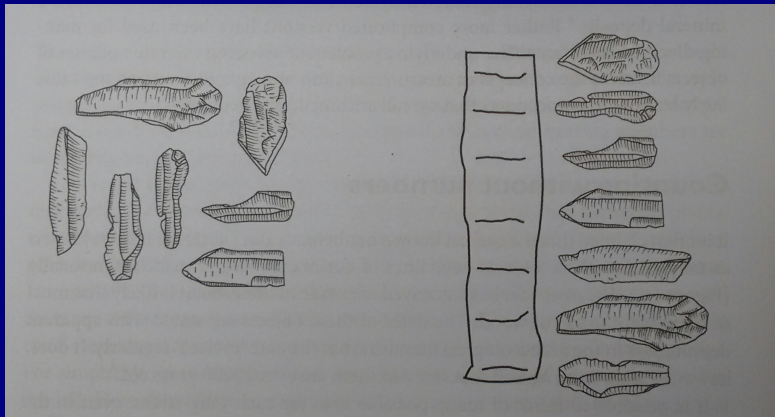
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Keeping Stock without Counting



Keeping Stock without Counting



The origin of the number line ???



Numbers

At some stage, the general notion of a number arose. Even in considering the fingers of a hand, numbers up to five would arise.



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Gradually the idea of **five as a concept** would emerge. Placing two hands together immediately gives us the idea of a **one-to-one correspondence**:

Both hands have five fingers.



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Both hands have five fingers.

Through repetition and familiarity, the concept of five would become natural. Any set of objects that are in **one-to-one correspondence** with the fingers of the hand must have five elements.



Numerals

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at least up to about 10, came into use.**



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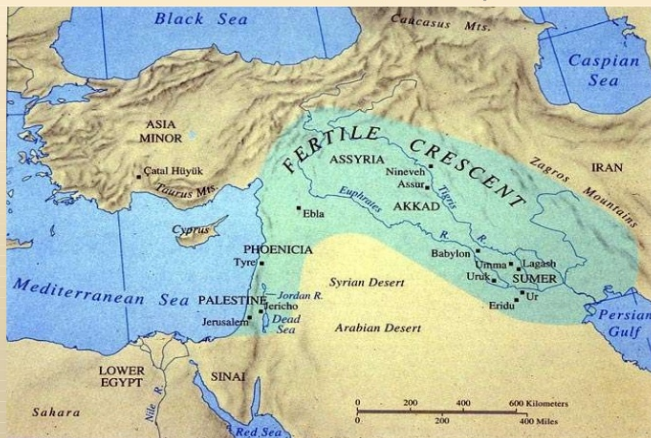
Eventually, numerals, or symbols for the numbers, emerged.

Much numerical material is found in writings from Mesopotamia and from Ancient Egypt.



The Fertile Crescent

The Fertile Crescent/Mesopotamia



Mesopotamia

Loosely called the Babylonian civilisation.

A vast number of cuneiform tablets survive.



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**WE WILL RETURN TO BABYLON PRESENTLY
AND READ A CUNEIFORM TABLET!**



Bartering & Money

One group might have surplus **fish**
while another group have excess **fruit**.
Both gain by agreeing to an **exchange**.



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Exercise: Discuss the opinion of Aristotle in his **Ethics**: “With money we can measure everything.”



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Reading a Tablet

On the next slide we will see a cuneiform tablet. It was discovered in the Sumerian city of Nippur (in modern-day Iraq), and dates to around 1500 BC.

We're not completely sure what this is, but most scholars suspect that it is a **homework exercise**.

It is not preserved perfectly, and dealing with this is part of the challenge (and part of the fun).

If you study the picture closely, you should be able to discover a lot about **Babylonian numerals**.



The Nippur Tablet



The Nippur Tablet Challenge



1. How do Babylonian numerals work?
2. Describe the maths on this tablet.
3. Write the number 72 in Babylonian numerals.



The Nippur Tablet Challenge



1. How do Babylonian numerals work?
2. Describe the maths on this tablet.
3. Write the number 72 in Babylonian numerals.

Does this seem impossible? Have faith in yourself!



Pause to Decode the Nippur Tablet



The Sexagesimal System

The Babylonian numerical system used 60 as its base. **Why?**



The Sexagesimal System

The Babylonian numerical system used 60 as its base. **Why?**

It is uncertain why, but reasonable to speculate that, since there are about 360 days in a year 60 was chosen to facilitate astronomical calculations.



The Babylonian Numerals

𐀀 1	𐀁 11	𐀂 21	𐀃 31	𐀄 41	𐀅 51
𐀆 2	𐀇 12	𐀈 22	𐀉 32	𐀊 42	𐀋 52
𐀌 3	𐀍 13	𐀎 23	𐀏 33	𐀐 43	𐀑 53
𐀒 4	𐀓 14	𐀔 24	𐀕 34	𐀖 44	𐀗 54
𐀘 5	𐀙 15	𐀚 25	𐀛 35	𐀜 45	𐀝 55
𐀟 6	𐀠 16	𐀡 26	𐀢 36	𐀣 46	𐀤 56
𐀥 7	𐀦 17	𐀧 27	𐀨 37	𐀩 47	𐀪 57
𐀬 8	𐀭 18	𐀮 28	𐀯 38	𐀰 48	𐀱 58
𐀳 9	𐀴 19	𐀵 29	𐀶 39	𐀷 49	𐀸 59
𐀹 10	𐁀 20	𐁁 30	𐁂 40	𐁃 50	



The Sexagesimal System

**The great advantage is that 60 has many divisors:
1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30.**

This obviously facilitates all the division problems.



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In Babylon, they wrote $70 = [1 \mid 10]$ and $254 = [4 \mid 14]$

We can add these: $324 = [5 \mid 24]$.

Thus, basic arithmetic is possible with this system.



Time Measurement

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Exercise: Find out how high the **Spire** is. Using public web-cams, could it be used as a time-piece?



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There is a hangover from the sexagesimal system in our 'modern' units:

We have 60 seconds in a minute and 60 minutes in an hour.

We have 360 degrees in a circle so our latitude and longitude are influenced by Babylonian mathematics.



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Can you think of any other examples?



Outline

Introduction

Overview

Visual Maths 1

Distraction 1: A Piem

The Beginnings

Babylonian Numeration Game

Summary of Lecture



Summary of Lecture 1

- ▶ **Overview of Course**
- ▶ **Visual Proofs**
- ▶ **Value of π**
- ▶ **Origin of Numbers**
- ▶ **Fair Division**
- ▶ **Babylonian Numeration**



Thank you

