Sum-Enchanted Evenings

The Fun and Joy of Mathematics

•

TASTER LECTURE

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School of Mathematics & Statistics
University College Dublin

Evening Course, UCD, Autumn 2018

Outline

Outline

WELCOME TO
Sum-Enchanted Evenings
The Fun and Joy of Mathematics

Taster Lecture

The course Sum-enchanted Evenings will run over ten (10) lectures from 24 September to 3 December.

The aim of the course is to show you

- The tremendous beauty of mathematics;
- Its great utility in our daily lives;
- The fun we can have studying maths.

Taster Lecture

Two years ago, I taught a course with the title

AweSums: The Majesty of Maths

It was well received, but the pace was
too fast for some of the participants.

Last year, I modified the content and renamed it

- Sum-enchanted Evenings.

That worked well, so this year the course
will be similar, but with much new material.

In this Taster Lecture I will give a sample
of some of the topics covered in the course.

Meaning and Content of Mathematics

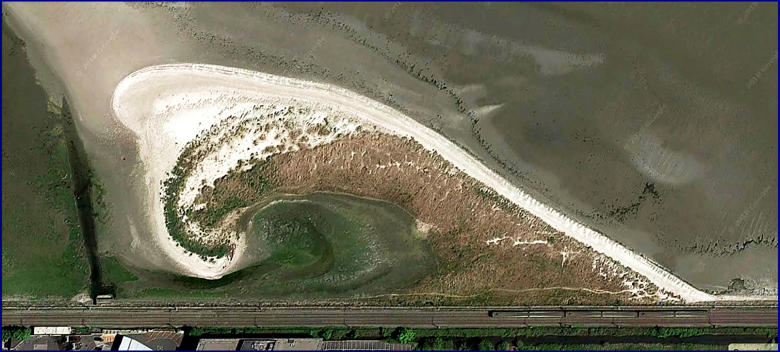
The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "study" or "learning".

It is the study of topics such as

- Quantity (numbers)
- Structure (patterns)
- Space (geometry)
- Change (analysis).

Outline

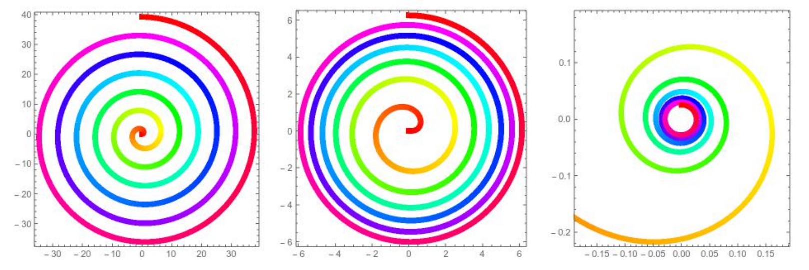
A Splendid Spiral in Booterstown



This sandbank, a beautiful spiral form, has slowly built up on the beach near Booterstown Station.

Spirals are found throughout the natural world.

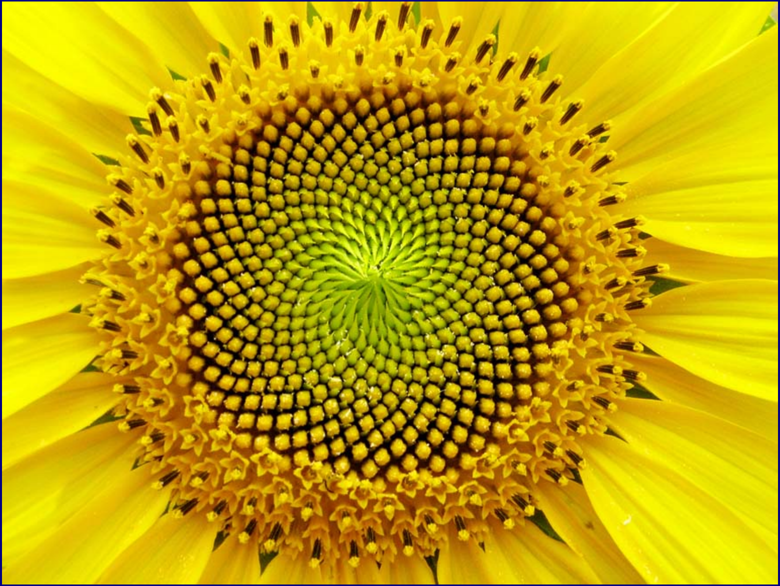
Some Mathematical Spirals



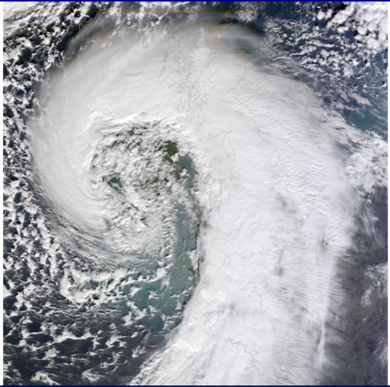
Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.

The Nautilus Shell: *a logarithmic Spiral.*

The Sunflower: Groups of Spirals



Spirals in the Physical World

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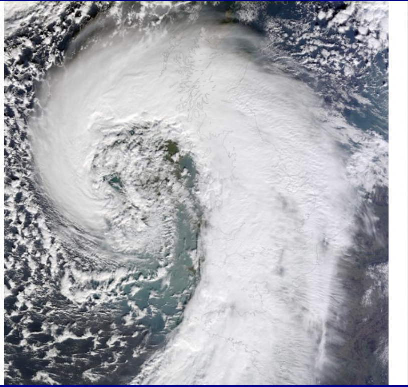

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https://thatsmaths.com/

Fibonacci Numbers

- Count the petals on a flower.
- Count leaves on a stem or
  bumps on an asparagus.
- Look at patterns on pineapples/pine-cones.
- Study the geometry of seeds on sunflowers.

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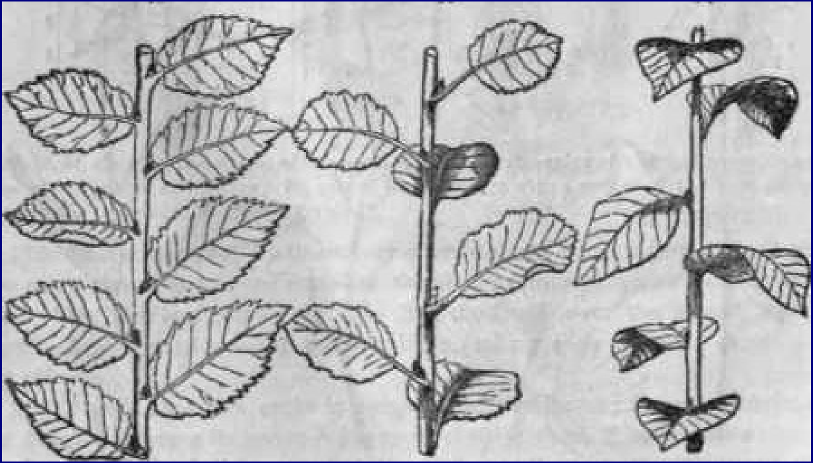
In all cases, we find numbers in the sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

This is the famous Fibonacci sequence.

Fibonacci and Phyllotaxis



Vi Hart's Videos

There are several mathematical videos on YouTube presented by Vi Hart.

Some of the ones on Fibonacci Numbers are at:

`https://www.youtube.com/watch?v=ahXIMUkSXX0`

It is *much easier* to go to Youtube and search for

"Vi Hart   Fibonacci"

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Let's take a peek!

Outline

Golden Ratio and Fibonacci Numbers

The Golden Ratio is a number defined as

$$\phi = \frac{1 + \sqrt{5}}{2} \approx 1.618.$$

It is intimately connected with
the Fibonacci Numbers.

Golden Rectangle

Ratio of breath to height is $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$.

Golden Rectangle in Your Pocket

Aspect ratio is about $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$.

Fibonacci Numbers

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where each number is the sum of the previous two.

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The explicit expression for the Fibonacci numbers is

$$F_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n - \frac{1}{\sqrt{5}}\left[\frac{1-\sqrt{5}}{2}\right]^n$$

Fibonacci Numbers

Let's consider the sequence of ratios of terms

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \ldots$$

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The ratios get closer and closer to the golden number:

$$\frac{F_{n+1}}{F_n} \to \phi \quad \textbf{as} \quad n \to \infty$$

Exotic Expressions for ϕ

We can write ϕ as a continued fraction

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

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$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}}$$

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These extraordinary expressions are actually quite easy to demonstrate!

Fibonacci Numbers in Nature

Look at post

Sunflowers and Fibonacci: Models of Efficiency

on the *ThatsMaths* blog:

thatsmaths.com

Outline

Ubiquity and Beauty of Symmetry

Symmetry is all around us.

- Many buildings are symmetric.
- Our bodies have bilateral symmetry.
- Crystals have great symmetry.
- Viruses can display stunning symmetries.
- At the sub-atomic scale, symmetry reigns.
- Galaxies have many symmetries.

The Taj Mahal

A Face with Symmetry: Halle Berry

Halle Berry Berry Halle

An Asymmetric Face: You know Who!

Symmetry and Group Theory

Symmetry is an essentially geometric concept.

The mathematical theory of symmetry is algebraic.

The key concept is that of a group.

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The mathematical theory of symmetry is algebraic.

The key concept is that of a group.

A group is a set of elements such that any two elements can be combined to produce another.

Instead of giving the mathematical definition, I will give an example to make things clear.

The *Dihedral Group* D_1

The group of symmetries of the human face and of all biological forms with bilateral symmetry. We could call D_1 the *Janus Group.*

I : The Identity transformation

R : Reflection about central line

Table: First Dihedral Group D_1.

	I	R
I	I	R
R	R	I

This is how we combine, or multiply transformations.

From 2 to 3 Dimensional Symmetry

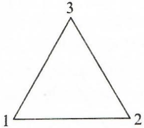


Fig. 4.6

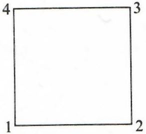
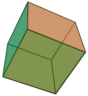
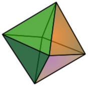
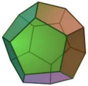
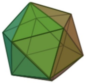


Fig. 4.7

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
 (Animation)	 (Animation)	 (Animation)	 (Animation)	 (Animation)

Outline

Mathematics and Art

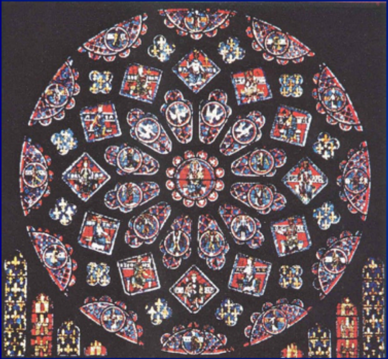
The link between maths and
art goes back thousands of years.

- Greek Architecture
- Renaissance Painting
- Gothic Cathedrals
- Oriental Carpets
- Islamic Mosaics


The Parthenon

Chartres cathedral

Rose window, Chartres

Raphael's School of Athens

Mosaics in the Alhambra

Persian Carpet

Alloy Wheels

Outline

I hope you agree that maths is Beautiful

But is it any use?

Useful: Maths is crucial for technology

Find us on: facebook

Useful

Maths is used in many aspects of our lives.

Searching for information: Google matrix [algebra].

Facebook & Twitter: Network analysis. Graph Theory.

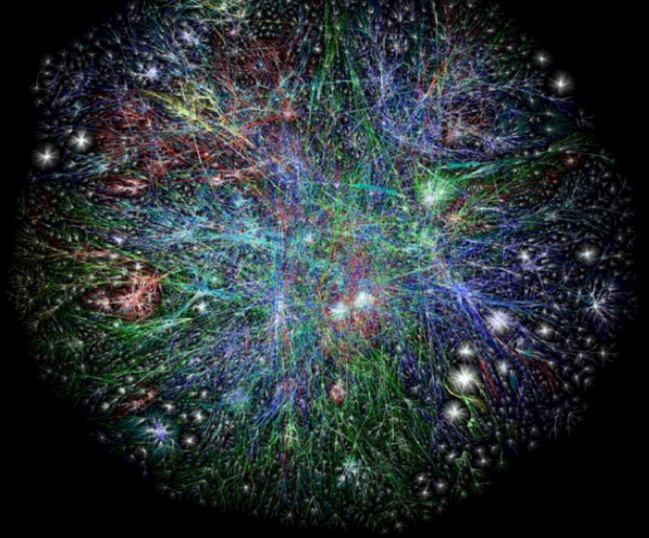
Download music or photos: Data compression [MP3,JPEG].

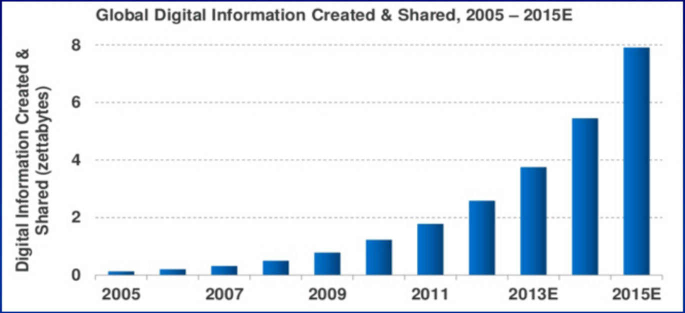
Commerce and Finance: Coding and Cryptography.

Biology and medicine. CAT Scans. Epidemiology.

Etc. etc. etc.

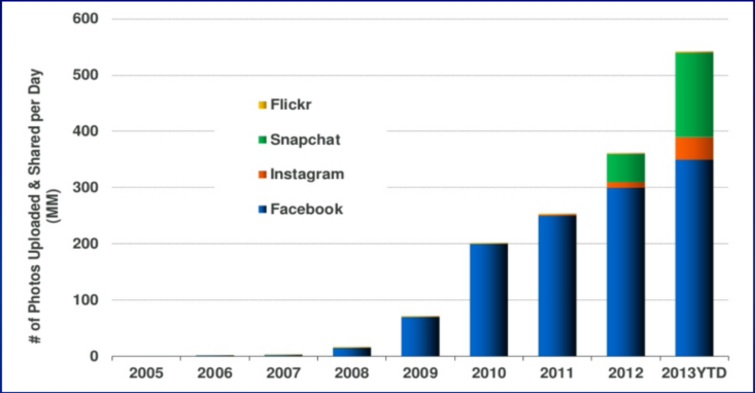
Useful: Maths is crucial for the Internet





Digital Information is growing exponentially:
> 3 Zbytes shared in 2013.

1 Zettabyte is
10^{21} = 1,000,000,000,000,000,000,000 bytes



500 million photos uploaded EVERY DAY.
That's half a billion !!!

Useful: Maths is crucial for Security

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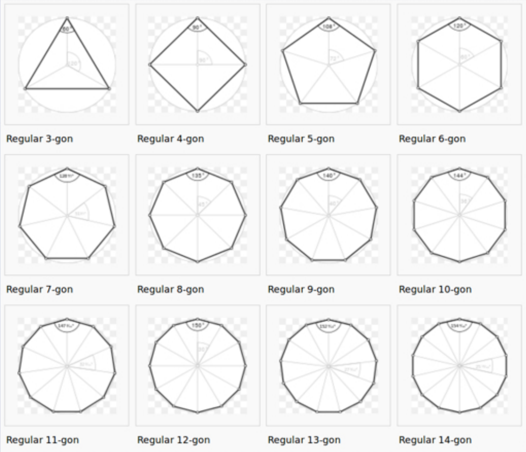
Outline

Euler's polyhedron formula.

Carving up the globe.

Regular Polygons



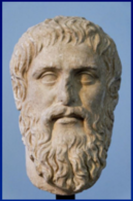
The Platonic Solids
(polyhedra)

Tetrahedron (four faces)	Cube or hexahedron (six faces)	Octahedron (eight faces)	Dodecahedron (twelve faces)	Icosahedron (twenty faces)
				

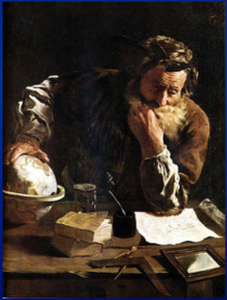
These five regular polyhedra were discovered in ancient Greece, perhaps by Pythagoras.

Plato used them as models of the universe.

They are analysed in Book XIII of Euclid's Elements.

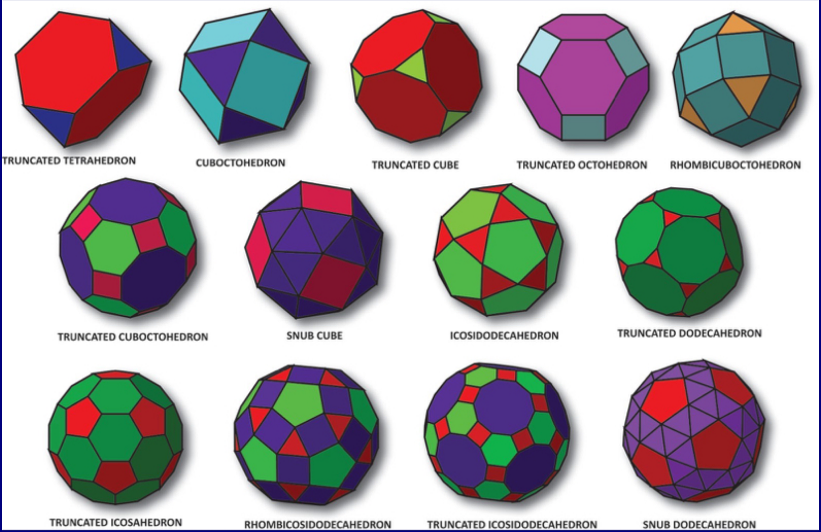


There are only five **Platonic** solids.



But **Archimedes** found, using different types of polygons, that he could construct 13 new solids.

The Thirteen Archimedean Solids



Euler's Polyhedron Formula

The great Swiss mathematician, **Leonard Euler**, noticed that, for all (convex) polyhedra,

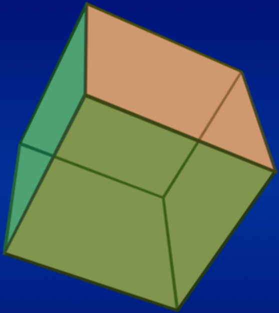
$$V - E + F = 2$$

where

- V = Number of vertices
- E = Number of edges
- F = Number of faces

Mnemonic: *Very Easy Formula*

For example, a Cube

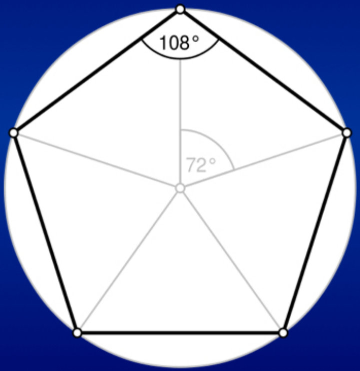


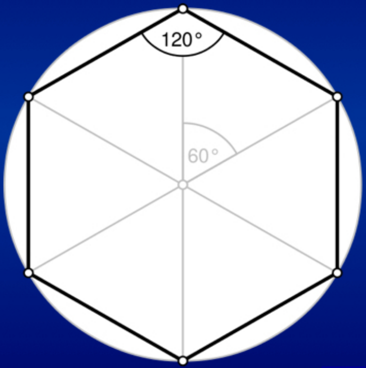
Number of vertices: V = 8
Number of edges: E = 12
Number of faces: F = 6

$$(V - E + F) = (8 - 12 + 6) = 2$$

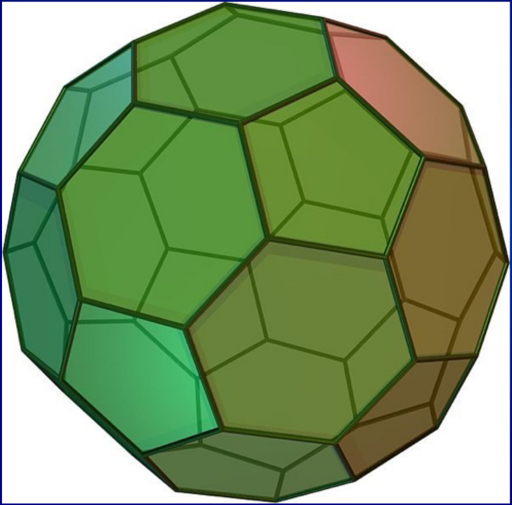
Mnemonic: *Very Easy Formula*

Pentagons and Hexagons





The Truncated Icosahedron

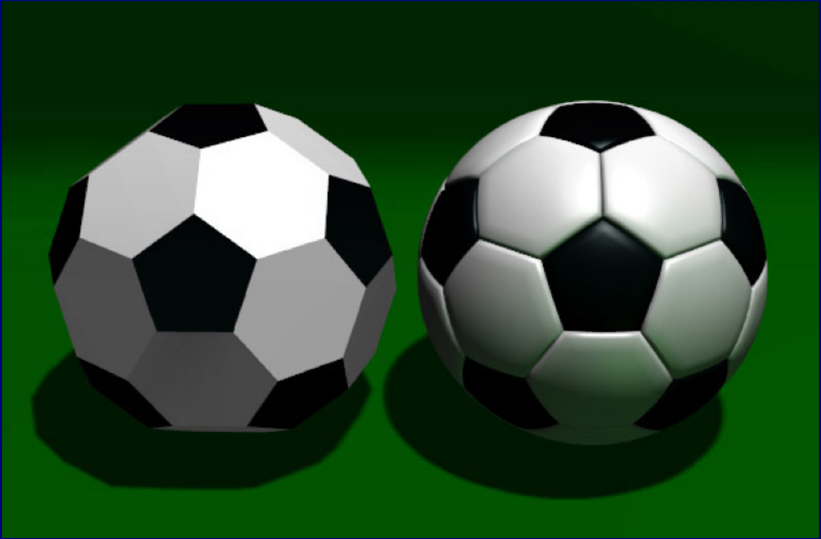


An Archimedean solid
with
pentagonal and
hexagonal faces.

The Truncated Icosahedron

Whare have you seen this before?

The Truncated Icosahedron



The "**Buckyball**", introduced at the 1970 World Cup Finals in Mexico.

It has 32 panels: 20 hexagons and 12 pentagons.

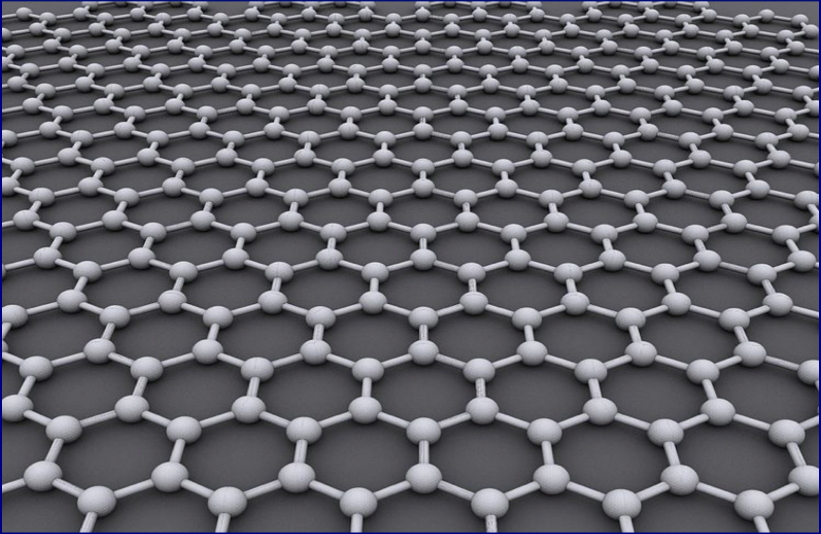
A Geodesic Dome designed by the American architect Richard Buckminster "Bucky" Fuller.

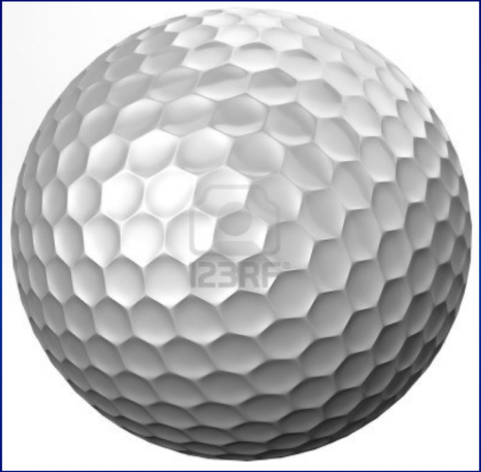
Buckminsterfullerene is a molecule with formula C_{60}

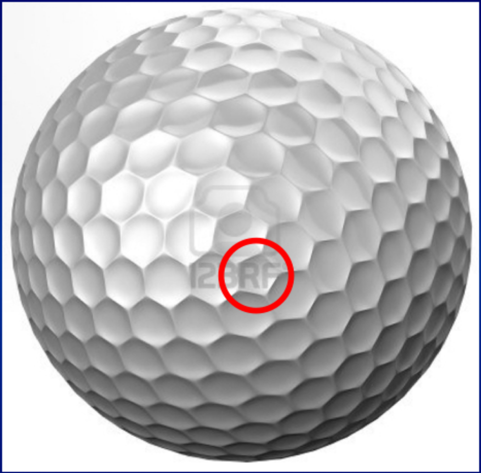
It was first synthesized in 1985.

Graphene

A hexagonal pattern of carbon one atom thick



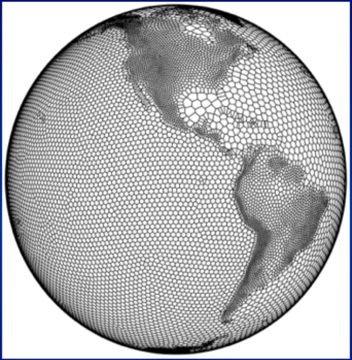

123RF



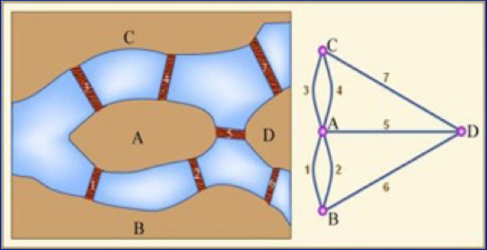
Euler's Polyhedron Formula

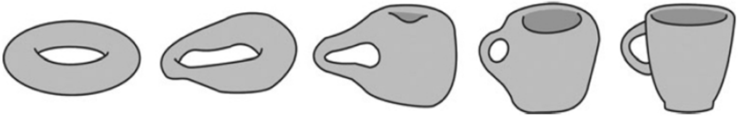
$$V - E + F = 2$$

still holds.



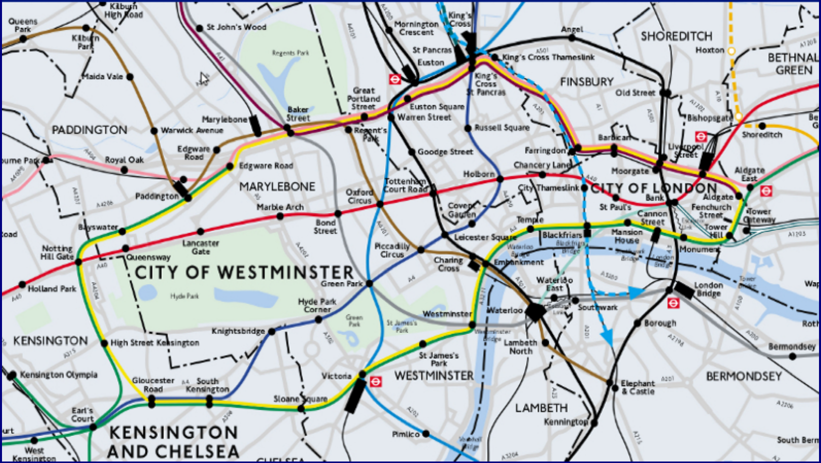
Topology
is often called
Rubber Sheet Geometry



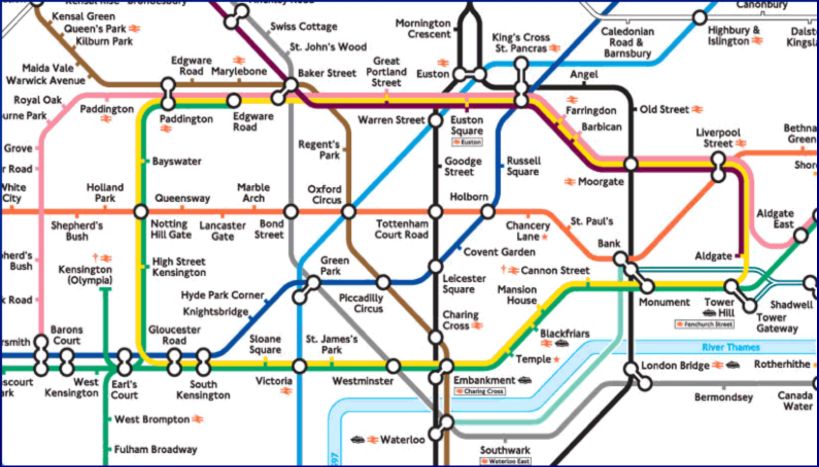


Topology and the London Underground

Topographical Map



Topology and the London Underground
Topological Map



Outline

Who is this?

Who is this?

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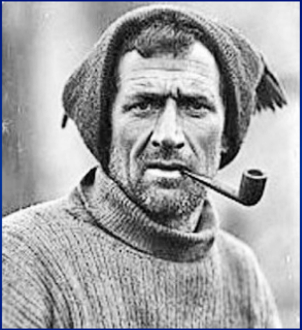
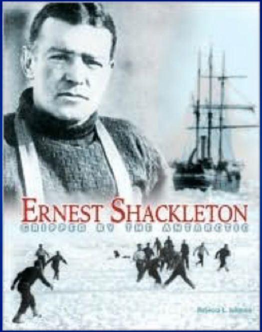
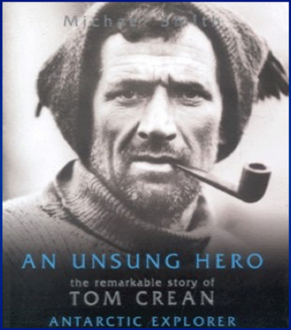


Ernest Shackleton   Tom Crean





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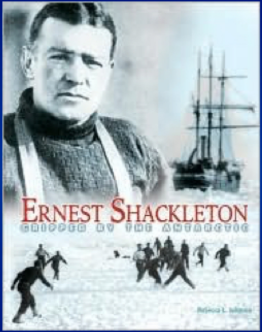
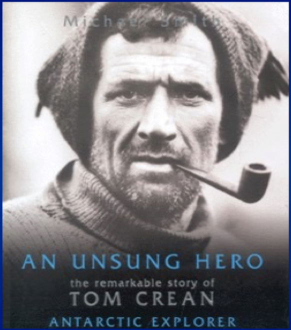




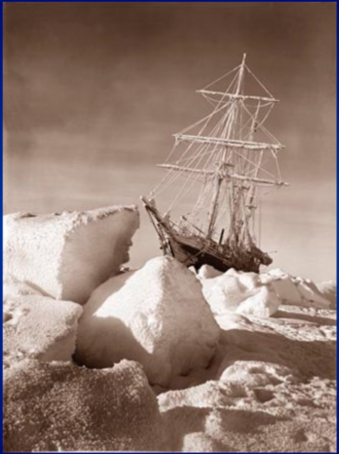
Two great Antarctic explorers, both born in Ireland

Shackleton's Imperial Trans-Antarctic Expedition (1914)

Shackleton's Imperial Trans-Antarctic Expedition (1914)

Shackleton's Imperial Trans-Antarctic Expedition (1914)

Endurance is Icebound



Shackleton's Imperial Trans-Antarctic Expedition (1914)

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Six men sailed 800 miles across the Southern Ocean to South Georgia.

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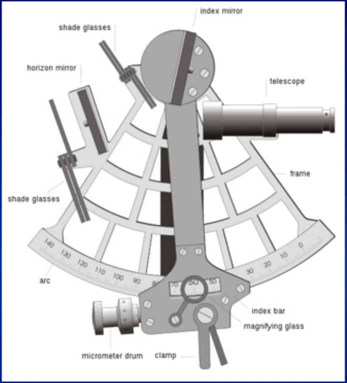
How did they find their way?

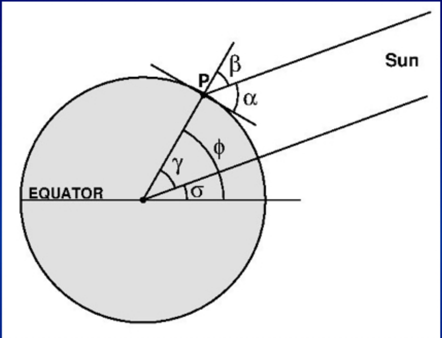
Six men sailed 800 miles across the Southern Ocean to South Georgia.

How did they find their way?

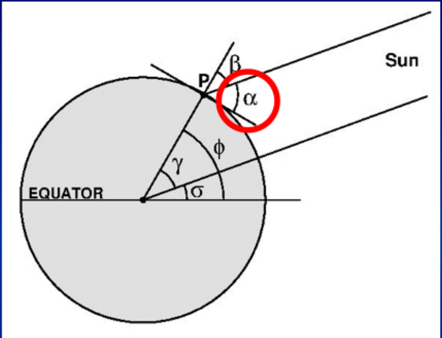
MATHEMATICS !!!



A sextant, used to determine latitude.



Angles used to calculate the latitude.



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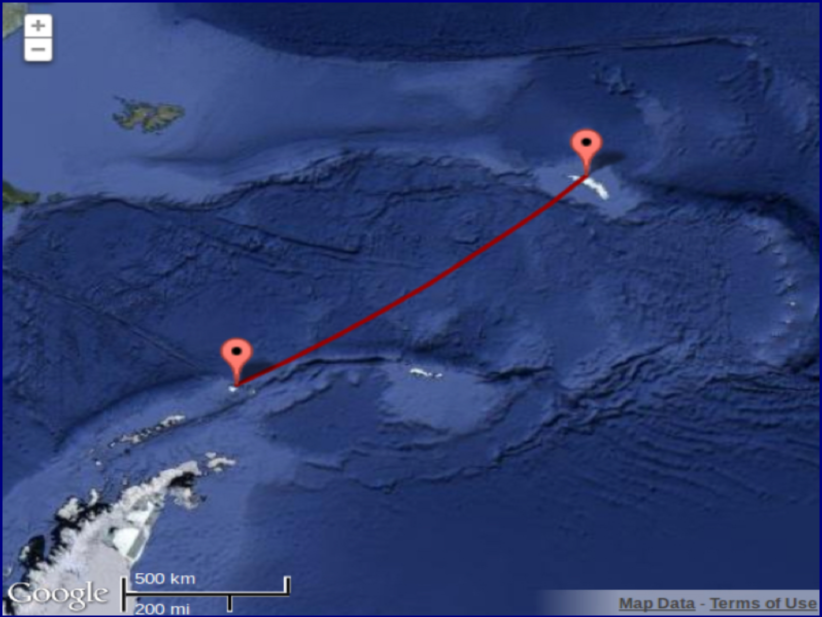
Google
500 km
200 mi
Map Data - Terms of Use

The boat journey to South Georgia was a spectacular feat of navigation.

It resulted in the saving of 28 lives.

This was possible thanks to elementary geometry.

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That's Maths!

Outline

Recreational Mathematics

Recreational mathematics puts the focus on **insight, imagination and beauty.**

Recreational Maths includes the study of

- The culture of mathematics,
- Its relevance to art, music and literature,
- Its role in technology,
- Mathematical games and puzzles,
- The lives of the great mathematicians.

Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine Plus:

`https://plus.maths.org/.`

All past content is available and is a valuable resource for school students and teachers.

Content of an Earlier Course

Lecture	*Content*
1	Outline of Course. Emergence of Numbers.
2	Georg Cantor. Set Theory.
3	Pythagoras. Irrational Numbers.
4	Hilbert. Gauss. The Real Number Line
5	Powers. Logarithms. Prime Numbers.
6	Functions. Archimedes. Natural Logs.
7	Exponential Growth. Euler. Sequences & Series.
8	Trigonometry. Taylor Series.
9	Basel Problem. Complex Numbers. Euler's Formula.
10	Prime Number Theorem. Riemann Hypothesis.

This year's course will be different.
If you want to know how, come along!

Thank you