

Sum-Enchanted Evenings

The Fun and Joy of Mathematics



TASTER LECTURE

Peter Lynch

**School of Mathematics & Statistics
University College Dublin**

Evening Course, UCD, Autumn 2018



Outline

Introduction

Beautiful Spirals

The Golden Ratio

Symmetry

Beautiful Symmetry

The Utility of Mathematics

Euler's Gem

Shackleton's Rescue Voyage

Recreational Mathematics



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WELCOME TO

Sum-Enchanted Evenings

The Fun and Joy of Mathematics



Taster Lecture

The course Sum-enchanted Evenings will run over ten (10) lectures from 24 September to 3 December.

The aim of the course is to show you

- ▶ **The tremendous *beauty* of mathematics;**
- ▶ **Its great *utility* in our daily lives;**
- ▶ **The *fun* we can have studying maths.**



Taster Lecture

Two years ago, I taught a course with the title

AweSums: The Majesty of Maths

It was well received, but the pace was too fast for some of the participants.

Last year, I modified the content and renamed it

- ▶ Sum-enchanted Evenings.

That worked well, so this year the course will be similar, but with much new material.

In this *Taster Lecture* I will give a sample of some of the topics covered in the course.



Meaning and Content of Mathematics

The word **Mathematics** comes from Greek *μαθημα* (*máthéma*), meaning “knowledge” or “study” or “learning”.

It is the study of topics such as

- ▶ Quantity (numbers)
- ▶ Structure (patterns)
- ▶ Space (geometry)
- ▶ Change (analysis).



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A Splendid Spiral in Booterstown

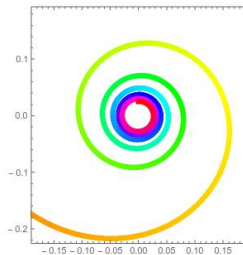
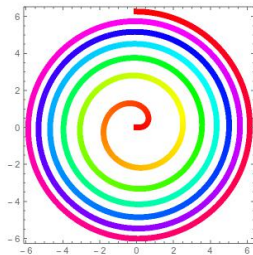
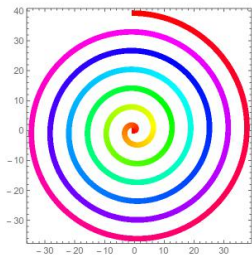


This sandbank, a beautiful spiral form, has slowly built up on the beach near Booterstown Station.

Spirals are found throughout the natural world.



Some Mathematical Spirals



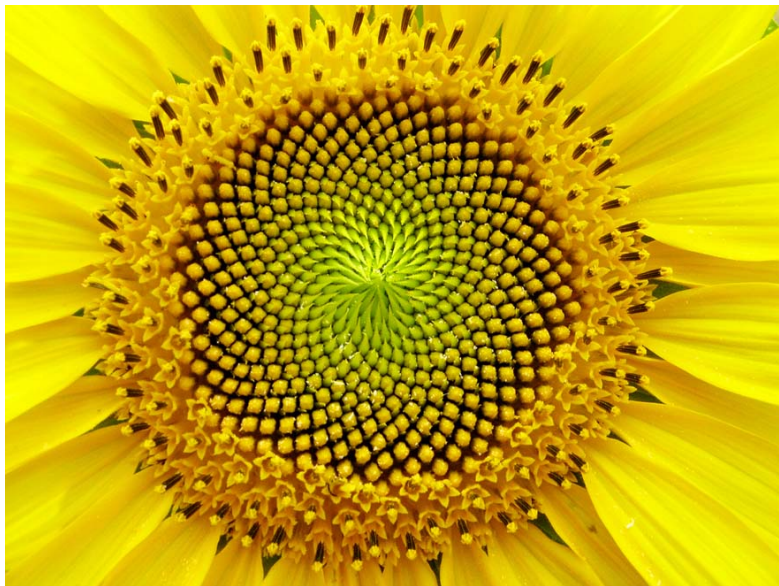
Archimedes Spiral. Fermat Spiral. Hyperbolic Spiral.



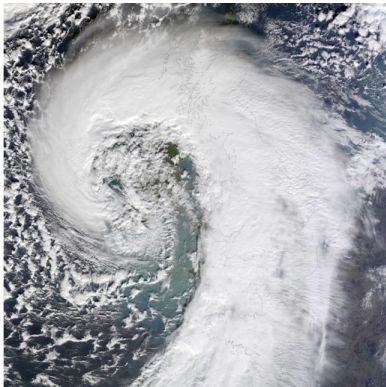
The Nautilus Shell: *a logarithmic Spiral.*



The Sunflower: Groups of Spirals



Spirals in the Physical World



★ ★ ★

<https://thatmaths.com/>



Fibonacci Numbers

- ▶ Count the petals on a flower.
- ▶ Count leaves on a stem or bumps on an asparagus.
- ▶ Look at patterns on pineapples/pine-cones.
- ▶ Study the geometry of seeds on sunflowers.

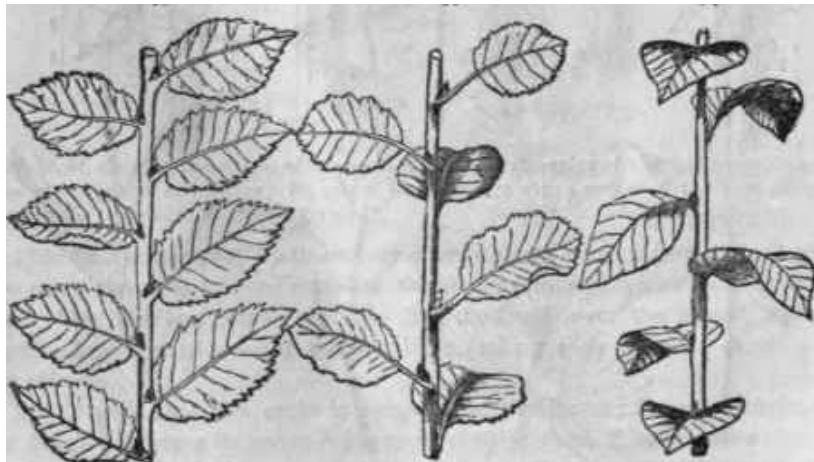
In all cases, we find numbers in the sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

This is the famous Fibonacci sequence.



Fibonacci and Phyllotaxis



Vi Hart's Videos

There are several mathematical videos on YouTube presented by Vi Hart.

Some of the ones on Fibonacci Numbers are at:

<https://www.youtube.com/watch?v=ahXIMUkSXX0>

It is *much easier* to go to Youtube and search for

“Vi Hart Fibonacci”

Let's take a peek!



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Golden Ratio and Fibonacci Numbers

The Golden Ratio is a number defined as

$$\phi = \frac{1 + \sqrt{5}}{2} \approx 1.618.$$

It is intimately connected with
the *Fibonacci Numbers*.



Golden Rectangle



Ratio of breath to height is $\phi = \frac{1+\sqrt{5}}{2} \approx 1.6$.



Golden Rectangle in Your Pocket



Aspect ratio is about $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$.



Fibonacci Numbers

The Fibonacci sequence is the sequence

$$\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \dots\}$$

where *each number is the sum of the previous two*.

The Fibonacci numbers obey a recurrence relation

$$F_{n+1} = F_n + F_{n-1}$$

with the *starting values* $F_0 = 0$ and $F_1 = 1$.

The explicit expression for the Fibonacci numbers is

$$F_n = \frac{1}{\sqrt{5}} \left[\frac{1 + \sqrt{5}}{2} \right]^n - \frac{1}{\sqrt{5}} \left[\frac{1 - \sqrt{5}}{2} \right]^n$$



Fibonacci Numbers

Let's consider the sequence of ratios of terms

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \dots$$

The ratios get closer and closer to the golden number:

$$\frac{F_{n+1}}{F_n} \rightarrow \phi \quad \text{as} \quad n \rightarrow \infty$$



Exotic Expressions for ϕ

We can write ϕ as a *continued fraction*

$$\phi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

We can also write it as a *continued root*

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}}$$

These extraordinary expressions are actually quite easy to demonstrate!



Fibonacci Numbers in Nature

Look at post

Sunflowers and Fibonacci: Models of Efficiency
on the *ThatsMaths* blog:

thatsmaths.com



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Ubiquity and Beauty of Symmetry

Symmetry is all around us.

- ▶ Many buildings are symmetric.
- ▶ Our bodies have bilateral symmetry.
- ▶ Crystals have great symmetry.
- ▶ Viruses can display stunning symmetries.
- ▶ At the sub-atomic scale, symmetry reigns.
- ▶ Galaxies have many symmetries.



The Taj Mahal



A Face with Symmetry: Halle Berry



Halle Berry

Berry Halle



An Asymmetric Face: You know Who!



Symmetry and Group Theory

Symmetry is an essentially *geometric* concept.

The mathematical theory of symmetry is *algebraic*.

The key concept is that of a group.

A group is a *set of elements* such that any two elements can be combined to produce another.

Instead of giving the mathematical definition, I will give an example to make things clear.



The *Dihedral Group* D_1

The group of symmetries of the human face and of all biological forms with bilateral symmetry. We could call D_1 the *Janus Group*.

I : The Identity transformation

R : Reflection about central line

Table : First Dihedral Group D_1 .

	I	R
I	I	R
R	R	I

This is how we combine, or *multiply* transformations.



From 2 to 3 Dimensional Symmetry

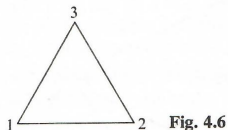


Fig. 4.6

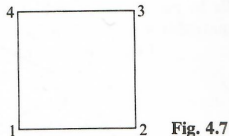







Fig. 4.7

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
				
(Animation)	(Animation)	(Animation)	(Animation)	(Animation)



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Mathematics and Art

The link between maths and art goes back thousands of years.

- Greek Architecture
- Renaissance Painting
- Gothic Cathedrals
- Oriental Carpets
- Islamic Mosaics



The Parthenon



[Intro](#)

[Spirals](#)

[Phi](#)

[Symmetry](#)

[Beauty](#)

[Useful](#)

[EG](#)

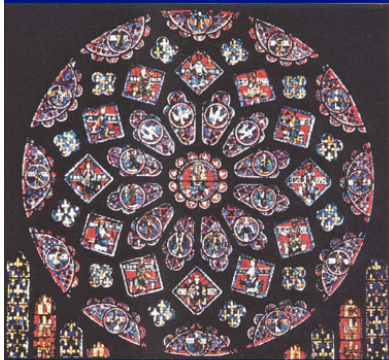
[TomCrean](#)

[RecMath](#)

Chartres cathedral



Rose window, Chartres



Raphael's School of Athens



Mosaics in the Alhambra



Persian Carpet



Alloy Wheels



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I hope you agree that
maths is Beautiful

But is it any use?



Useful: Maths is crucial for technology



Find us on:
facebook



Useful

Maths is used in many aspects of our lives.

Searching for information: Google matrix [algebra].

Facebook & Twitter: Network analysis. Graph Theory.

Download music or photos: Data compression [MP3,JPEG].

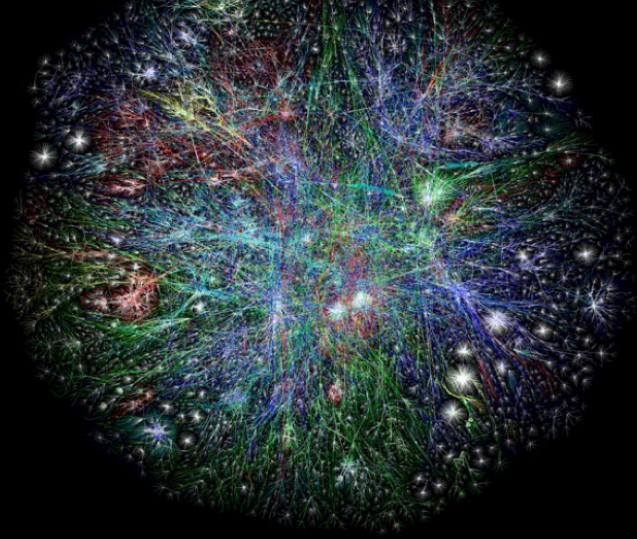
Commerce and Finance: Coding and Cryptography.

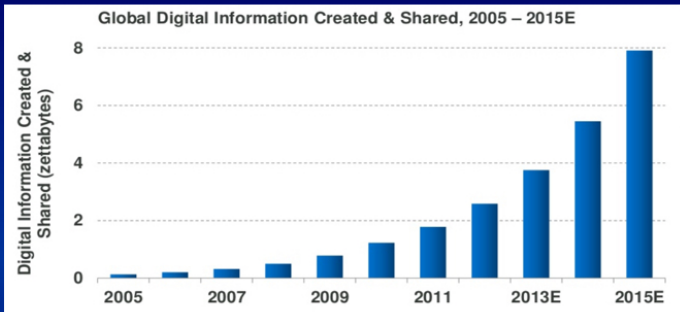
Biology and medicine. CAT Scans. Epidemiology.

Etc. etc. etc.



Useful: Maths is crucial for the Internet

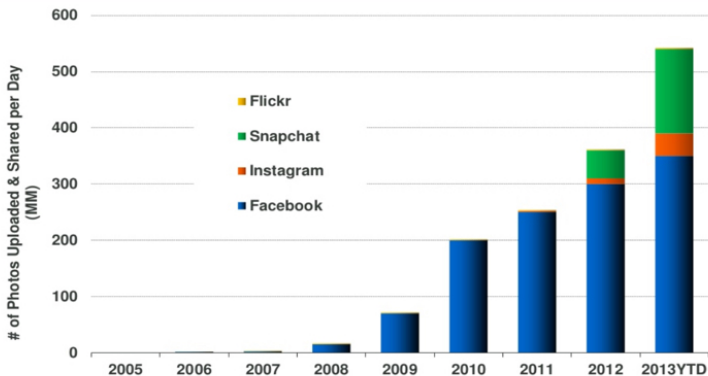




Digital Information is growing exponentially:
> 3 Zbytes shared in 2013.

1 Zettabyte is
 $10^{21} = 1,000,000,000,000,000,000,000$ bytes





500 million photos uploaded EVERY DAY.
That's half a billion !!!



Useful: Maths is crucial for Security



Maths Week 2013



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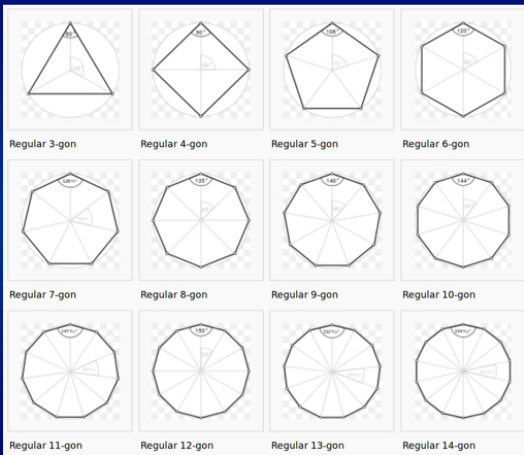


Euler's polyhedron formula.






Carving up the globe.



Regular Polygons



The Platonic Solids (polyhedra)

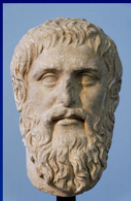
Tetrahedron (four faces)	Cube or hexahedron (six faces)	Octahedron (eight faces)	Dodecahedron (twelve faces)	Icosahedron (twenty faces)
				

These five regular polyhedra were discovered in ancient Greece, perhaps by **Pythagoras**.

Plato used them as models of the universe.

They are analysed in Book XIII of **Euclid's Elements**.



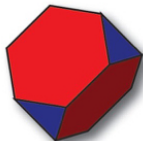


There are only five **Platonic** solids.

But **Archimedes** found, using different types of polygons, that he could construct 13 new solids.



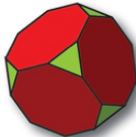
The Thirteen Archimedean Solids



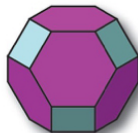
TRUNCATED TETRAHEDRON



CUBOCTAHEDRON



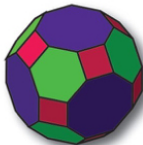
TRUNCATED CUBE



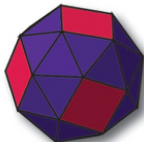
TRUNCATED OCTAHEDRON



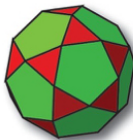
RHOMBICUBOCTAHEDRON



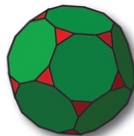
TRUNCATED CUBOCTAHEDRON



SNUB CUBE



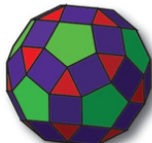
ICOSIDODECAHEDRON



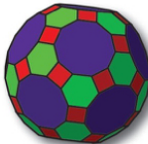
TRUNCATED DODECAHEDRON



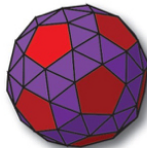
TRUNCATED ICOSAHEDRON



RHOMBICOSIDODECAHEDRON



TRUNCATED ICOSIDODECAHEDRON



SNUB DODECAHEDRON



Euler's Polyhedron Formula

The great Swiss mathematician, **Leonard Euler**, noticed that, for all (convex) polyhedra,

$$V - E + F = 2$$

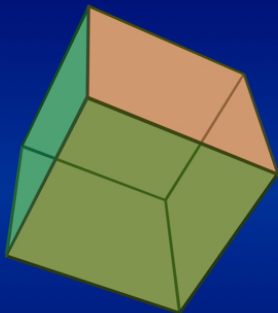
where

- **V** = Number of vertices
- **E** = Number of edges
- **F** = Number of faces

Mnemonic: Very Easy Formula



For example, a Cube



Number of vertices: $V = 8$

Number of edges: $E = 12$

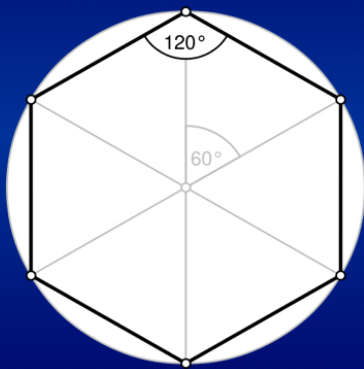
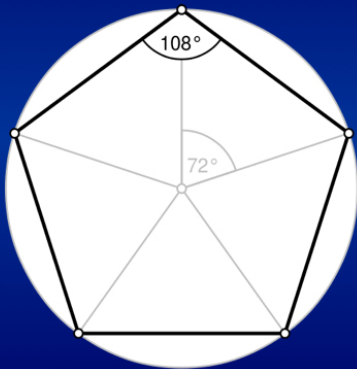
Number of faces: $F = 6$

$$(V - E + F) = (8 - 12 + 6) = 2$$

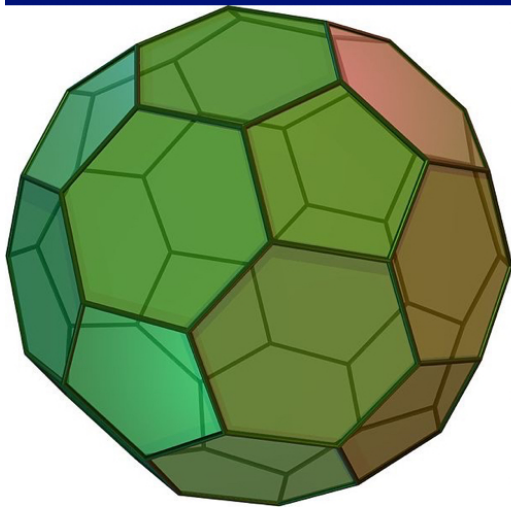
Mnemonic: Very Easy Formula



Pentagons and Hexagons



The Truncated Icosahedron



**An Archimedean solid
with
pentagonal and
hexagonal faces.**



The Truncated Icosahedron



Where have
you seen this
before?



The Truncated Icosahedron





The "**Buckyball**", introduced at the 1970 World Cup Finals in Mexico.

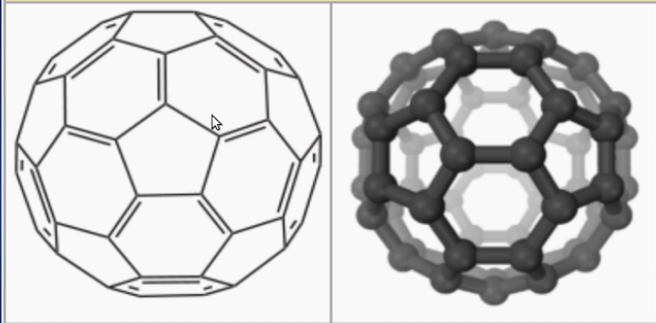
It has 32 panels: 20 hexagons and 12 pentagons.



A Geodesic Dome designed by the American architect Richard Buckminster "Bucky" Fuller.



Buckminsterfullerene



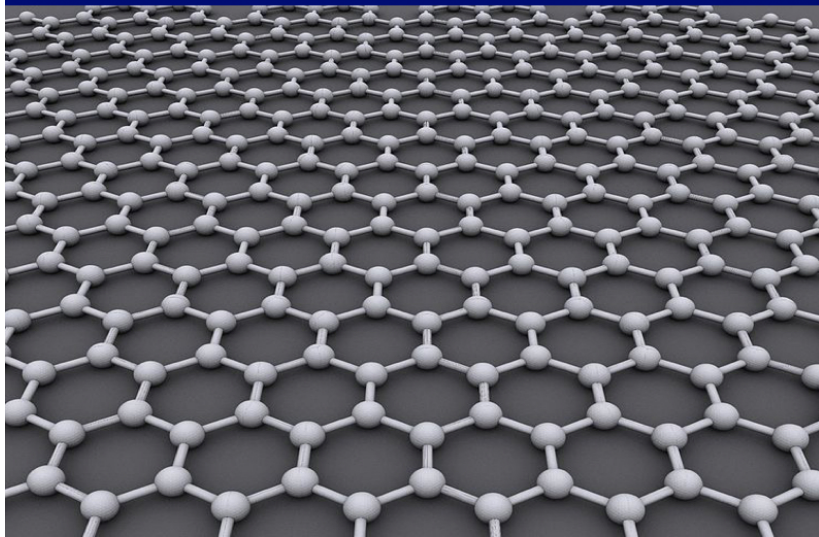
Buckminsterfullerene is a molecule with formula C_{60}

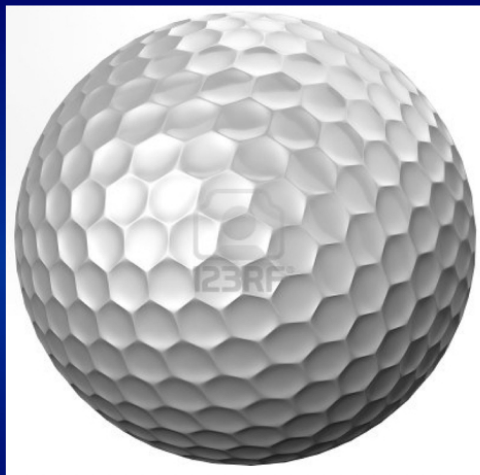
It was first synthesized in 1985.

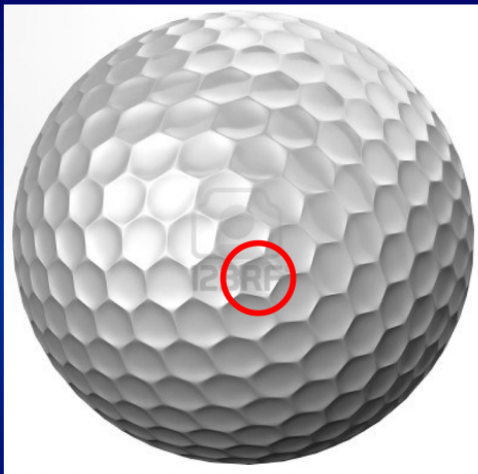


Graphene

A hexagonal pattern of carbon one atom thick



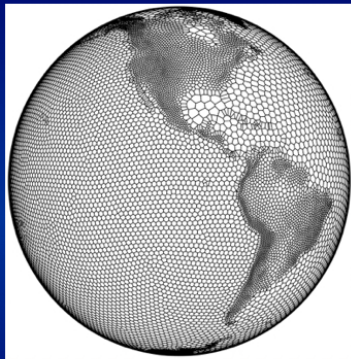




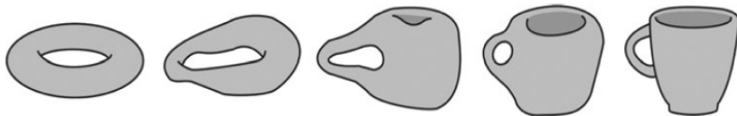
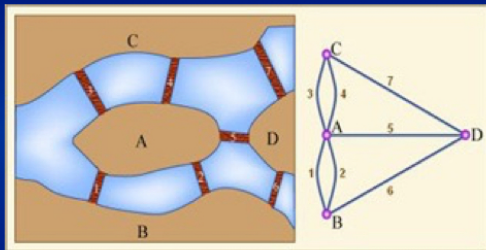
Euler's Polyhedron Formula

$$V - E + F = 2$$

still holds.

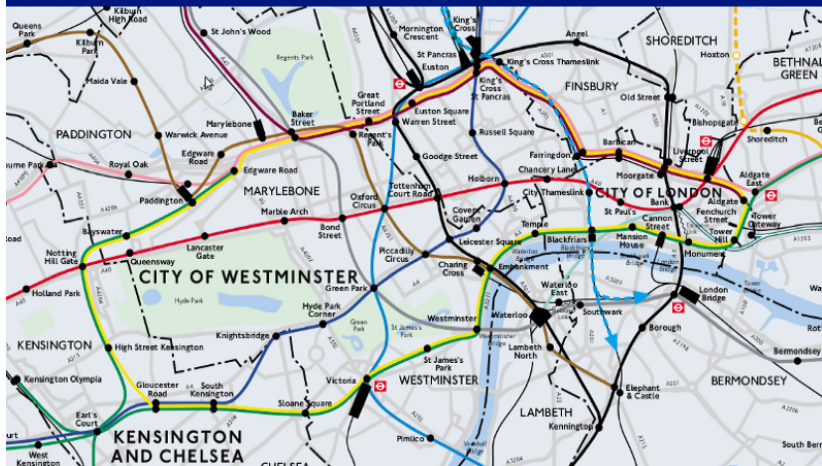


Topology is often called Rubber Sheet Geometry



Topology and the London Underground

Topographical Map



Topology and the London Underground

Topological Map



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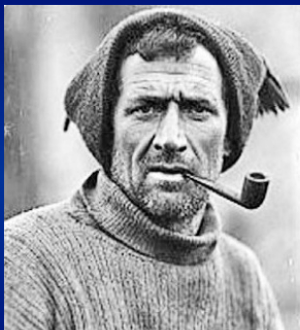
Who is this?



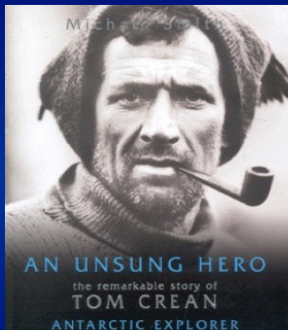
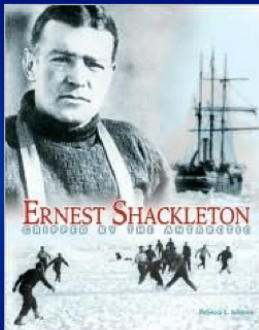
Who is this?



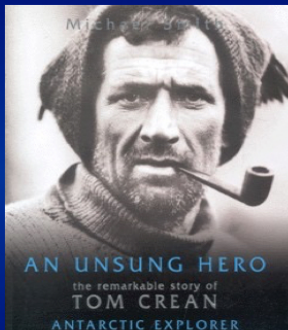
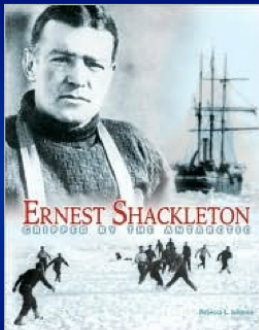
Who is this?



Ernest Shackleton Tom Crean



Ernest Shackleton Tom Crean



Two great Antarctic explorers, both born in Ireland



Shackleton's Imperial Trans-Antarctic Expedition (1914)



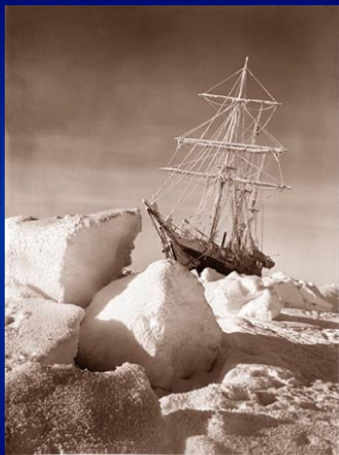
Shackleton's Imperial Trans-Antarctic Expedition (1914)



Shackleton's Imperial Trans-Antarctic Expedition (1914)



Endurance is Icebound



Shackleton's Imperial Trans-Antarctic Expedition (1914)



Shackleton's Imperial Trans-Antarctic Expedition (1914)





Six men sailed 800 miles across the Southern Ocean to South Georgia.



Six men sailed 800 miles across the Southern Ocean to South Georgia.

How did they find their way?

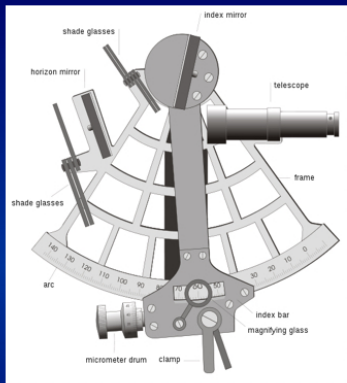


**Six men sailed 800 miles across the
Southern Ocean to South Georgia.**

How did they find their way?

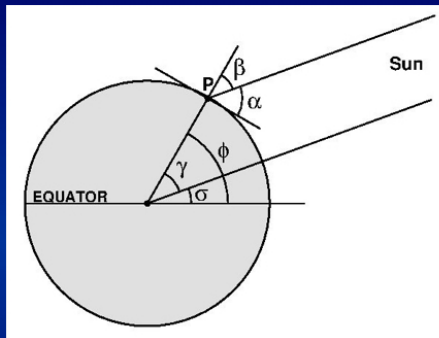
MATHEMATICS !!!





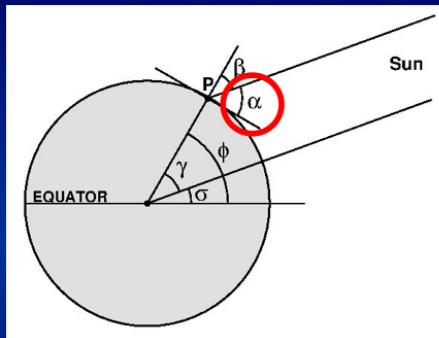
A sextant, used to determine latitude.





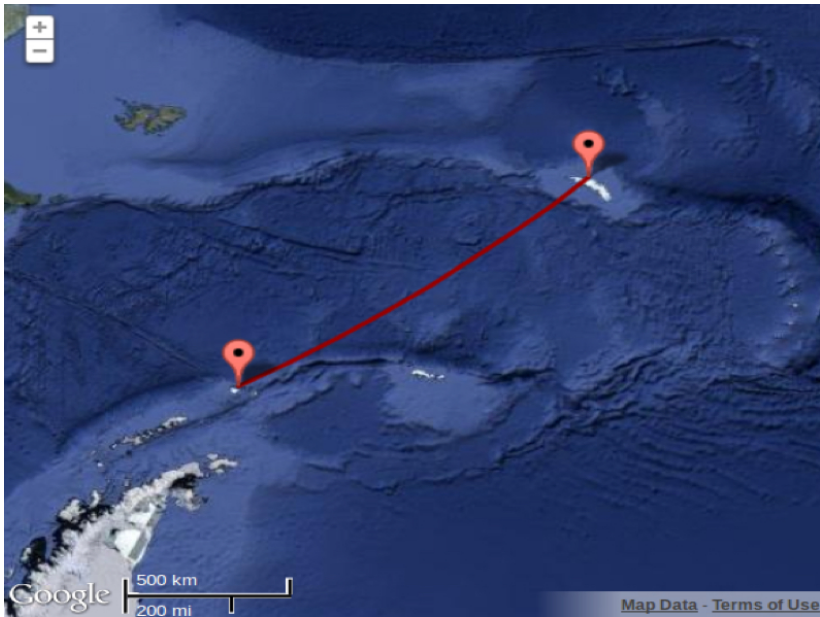
Angles used to calculate the latitude.





Angles used to calculate the latitude.





**The boat journey to South Georgia
was a spectacular feat of navigation.**

It resulted in the saving of 28 lives.

**This was possible thanks to
elementary geometry.**



**The boat journey to South Georgia
was a spectacular feat of navigation.**

It resulted in the saving of 28 lives.

**This was possible thanks to
elementary geometry.**

That's Maths!



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Recreational Mathematics

Recreational mathematics puts the focus on insight, imagination and beauty.

Recreational Maths includes the study of

- ▶ **The culture of mathematics,**
- ▶ **Its relevance to art, music and literature,**
- ▶ **Its role in technology,**
- ▶ **Mathematical games and puzzles,**
- ▶ **The lives of the great mathematicians.**



Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine Plus:

`https://plus.maths.org/`

All past content is available and is a valuable resource for school students and teachers.



Content of an Earlier Course

Lecture	Content
1	Outline of Course. Emergence of Numbers.
2	Georg Cantor. Set Theory.
3	Pythagoras. Irrational Numbers.
4	Hilbert. Gauss. The Real Number Line
5	Powers. Logarithms. Prime Numbers.
6	Functions. Archimedes. Natural Logs.
7	Exponential Growth. Euler. Sequences & Series.
8	Trigonometry. Taylor Series.
9	Basel Problem. Complex Numbers. Euler's Formula.
10	Prime Number Theorem. Riemann Hypothesis.

**This year's course will be different.
If you want to know how, come along!**



Thank you

