Sum-Enchanted Evenings

The Fun and Joy of Mathematics

•

LECTURE 4

Peter Lynch
School of Mathematics & Statistics
University College Dublin

Evening Course, UCD, Autumn 2017

Outline

Outline

Meaning and Content of Mathematics

The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning "knowledge" or "study" or "learning".

It is the study of topics such as

- Quantity (numbers)
- Structure (patterns)
- Space (geometry)
- Change (analysis).

Outline

Set Theory Puzzle

In a small Canadian village, everyone speaks either English or French, or both.

80% of the people speak French

60% of the people speak English

What percentage speak both English and French?

Set Theory Puzzle

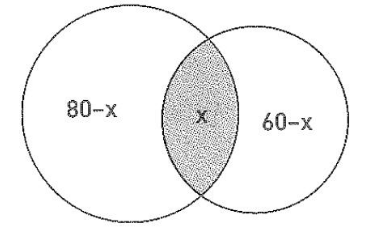
In a small Canadian village, everyone speaks either English or French, or both.

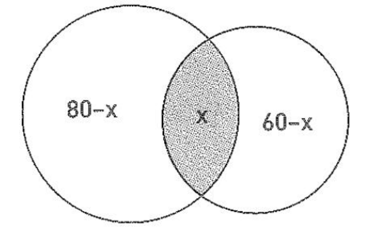
80% of the people speak French

60% of the people speak English

What percentage speak both English and French?

Answer next week!

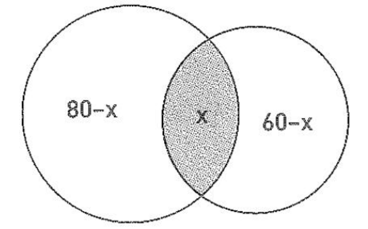

80−x
x
60−x



$$(80 - x) + x + (60 - x) = 100\,.$$

Therefore

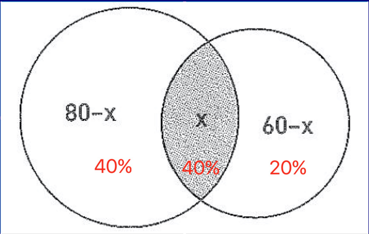
$$140 - x = 100 \qquad \textbf{or} \qquad x = 40\,.$$



$$(80 - x) + x + (60 - x) = 100\,.$$

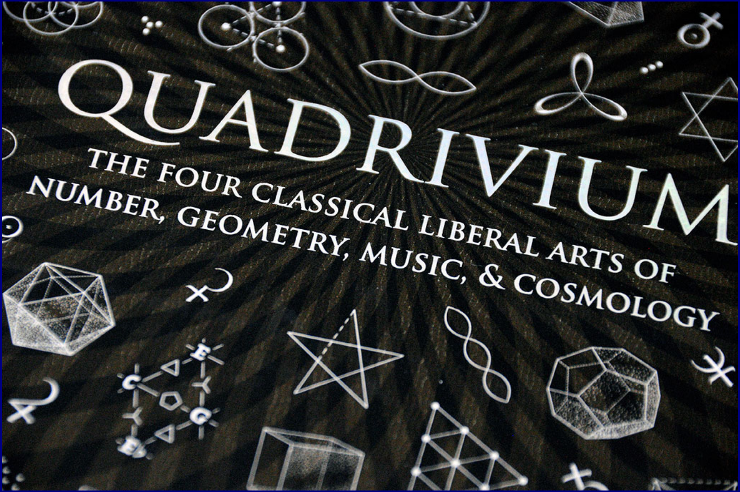
Therefore

$$140 - x = 100 \qquad \textbf{or} \qquad x = 40\,.$$



Outline

The Quadrivium



The Quadrivium

The Quadrivium originated with the Pythagoreans around 500 BC.

The Pythagoreans' quest was to find the eternal laws of the Universe, and they organized their studies into the scheme later known as the Quadrivium.

It comprised four disciplines:

- Arithmetic
- Geometry
- Music
- Astronomy

The Quadrivium

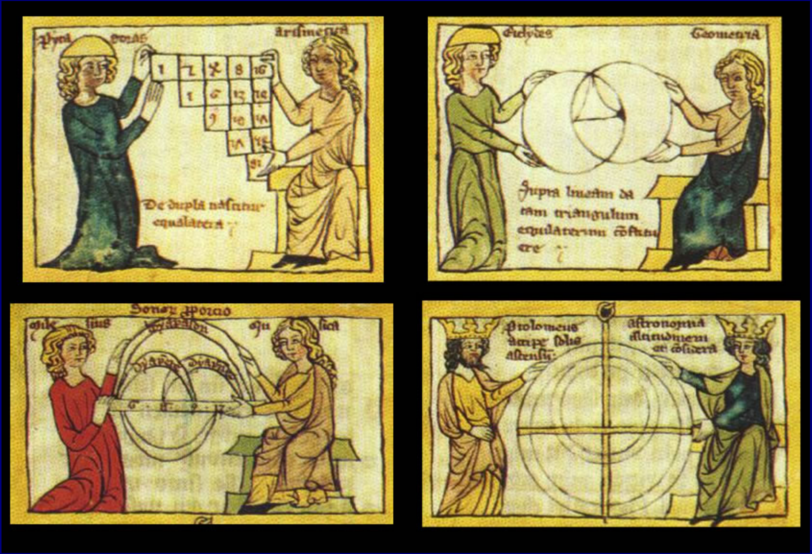
First comes Arithmetic, concerned with the infinite linear array of numbers.

Moving beyond the line to the plane and 3D space, we have Geometry.

The third discipline is Music, which is an application of the science of numbers.

Fourth comes Astronomy, the application of Geometry to the world of space.

The Quadrivium



Static/Dynamic. Pure/Applied

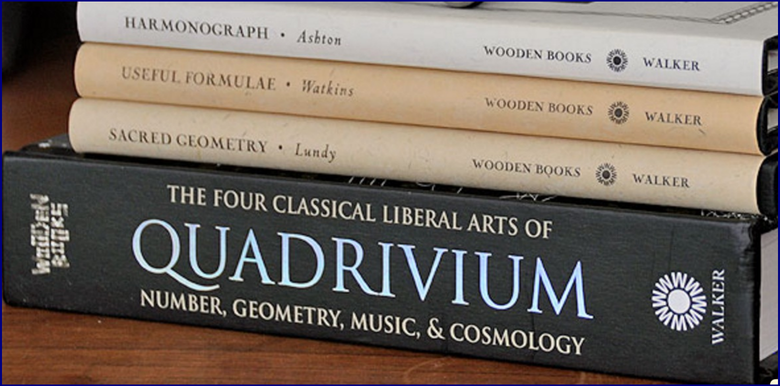
- Arithmetic (static number)
- Music (moving number)
- Geometry (measurement of static Earth)
- Astronomy (measurement of moving Heavens)

Arithmetic represents numbers at rest,
Geometry is magnitudes at rest,

Music is numbers in motion and
Astronomy is geometry in motion.

The first two are pure in nature,
while the last two are applied.

The Quadrivium



The Pythagoreans

Pythagoras distinguished between **quantity** and **magnitude.**

Objects that can be counted yield quantities or numbers.

Substances that are measured provide magnitudes.

Thus, cattle are counted whereas milk is measured.

The Pythagoreans

Pythagoras distinguished between quantity and magnitude.

Objects that can be counted yield quantities or numbers.

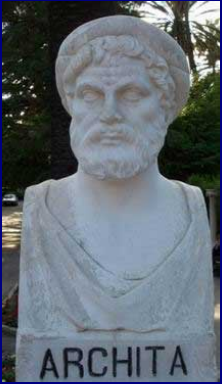
Substances that are measured provide magnitudes.

Thus, cattle are counted whereas milk is measured.

Arithmetic studies quantities or numbers and
Music involves the relationship between numbers
and their evolution in time.

Geometry deals with magnitudes, and
Astronomy with their distribution in space.

Archytas (428–350 BC): *APXΥTAΣ*



$A\rho\chi\upsilon\tau\alpha\varsigma$.

Born in Tarentum, son of Hestiaeus.

Mathematician and philosopher.

Pythagorean, student of Philolaus.

Provided a solution for the Delian problem of doubling the cube.

Said to have tutored Plato in mathematics(?)

Archytas (428–350 BC)

Archytas lived in Tarentum (now in Southern Italy).

One of the last scholars of the Pythagorean School and was a good friend of Plato.

The designation of the four disciplines of the Quadrivium was ascribed to Archytas.

His views were to dominate pedagogical thought for over two millennia.

Partly due to Archytas, mathematics has played a prominent role in education ever since.

Plato's Academy

According to Plato, mathematical knowledge was essential for an understanding of the Universe. The curriculum was outlined in Plato's **Republic.**

Inscription over the entrance to Plato's Academy:

"Let None But Geometers Enter Here".

This indicated that the Quadrivium was a prerequisite for the study of philosophy in ancient Greece.

Boethius (AD 480–524)

The Western Roman Empire was in many ways static for centuries.

No new mathematics between the conquest of Greece and the fall of the Roman Empire in AD 476.

Boethius, the 6th century Roman philosopher, was one of the last great scholars of antiquity.

The organization of the Quadrivium was formalized by Boethius.

It was the mainstay of the medieval monastic system of education.

The Quadrivium

The Liberal Arts

The seven liberal arts comprised
the Trivium and the Quadrivium.

The Trivium was centred on three arts of language:

- Grammar: proper structure of language.
- Logic: for arriving at the truth.
- Rhetoric: the beautiful use of language.

Aim of the Trivium: Goodness, Truth and Beauty.

Aristotle traced the origin of the Trivium back to Zeno.

The Ultimate Goal

The goal of studying the Quadrivium was
an insight into the nature of reality,
an understanding of the Universe.

The Quadrivium offered the seeker of wisdom
an understanding of the integral nature of
the Universe and the role of humankind within it.

In the medieval era, it preceded the study of theology.

Outline

Theorem of Pythagoras

The Theorem of Pythagoras is of fundamental importance in Euclidean geometry

It encapsulates the structure of space.

In the BBC series, The Ascent of Man, Jacob Bronowski said

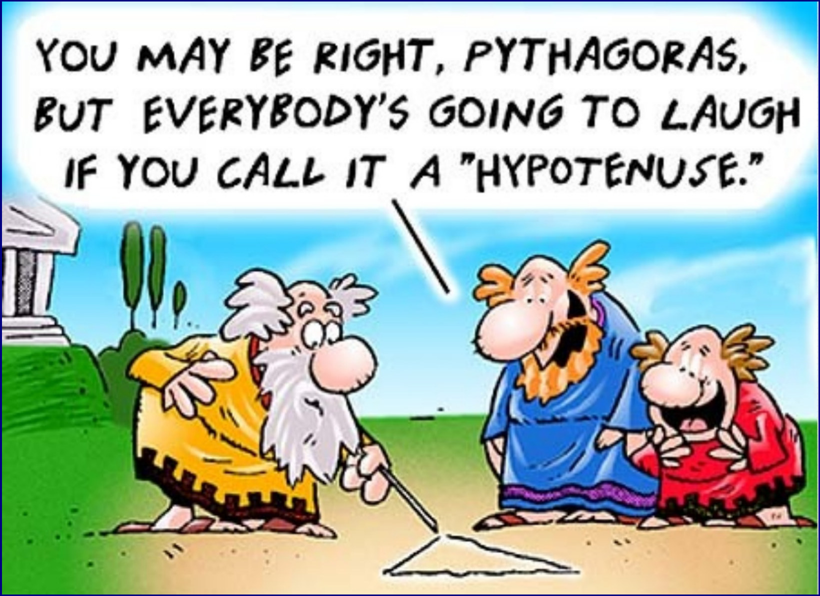
"The theorem of Pythagoras remains the most important single theorem in mathematics."

Theorem of Pythagoras

YouTube video with Danny Kaye

Google search for
"Danny Kaye Hypotenuse"

https:
//www.youtube.com/watch?v=oeRCsAGQVy8


YOU MAY BE RIGHT, PYTHAGORAS, BUT EVERYBODY'S GOING TO LAUGH IF YOU CALL IT A "HYPOTENUSE."

Hypotenuse

The side of a right triangle opposite to the right angle.

1570s, from Late Latin hypotenusa, from Greek hypoteinousa “stretching under" (the right angle).

Fem. present participle of hypoteinein, from hypo- “under" + teinein “to stretch"

From Online Etymology Dictionary: `http://www.etymonline.com/`

Mathigon.org

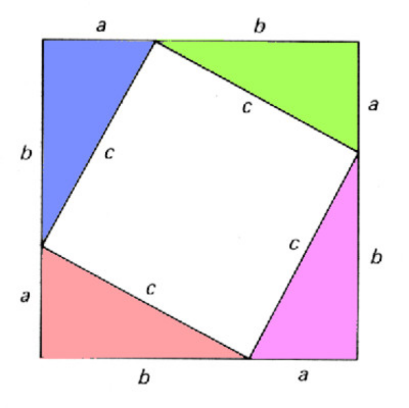
Mathigon.org video on Proofs without Formulas:

- What is the sum of the angles in a triangle?
- What is the sum of the angles in a polygon?
- What is the area of a triangle?
- How does Pythagoras' Theorem work?

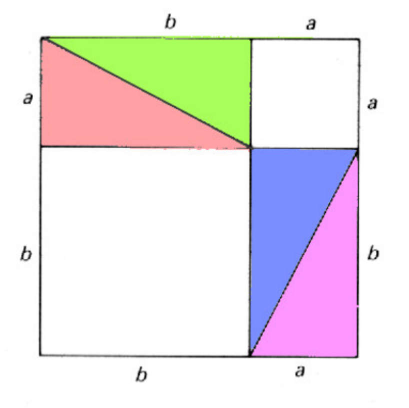
In the video below, these and other important concepts are explained in only two minutes using nothing but graphics.

https://youtu.be/IUCK8bk0xPo

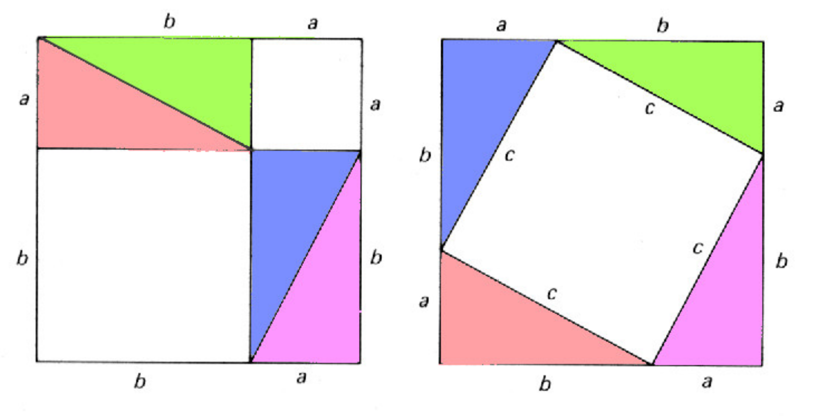
Proof without Formulae



Proof without Formulae



Proof without Formulae



$$a^2 + b^2 = c^2$$

Why is this Important / Interesting?

Squares on the sides of triangles don't seem much.

But the theorem gives us distances.

Why is this Important / Interesting?

Squares on the sides of triangles don't seem much.

But the theorem gives us distances.

If one point is at $(0, 0)$ and another at (x, y), the theorem gives the distance:

$$r^2 = x^2 + y^2 \qquad \text{or} \qquad r = \sqrt{x^2 + y^2}$$

Why is this Important / Interesting?

Squares on the sides of triangles don't seem much.

But the theorem gives us distances.

If one point is at $(0, 0)$ and another at (x, y), the theorem gives the distance:

$$r^2 = x^2 + y^2 \qquad \text{or} \qquad r = \sqrt{x^2 + y^2}$$

This tells us about the structure of space.

I should expand on this topic (e.g., SAR)

Outline

The Greek Alphabet, Part 3

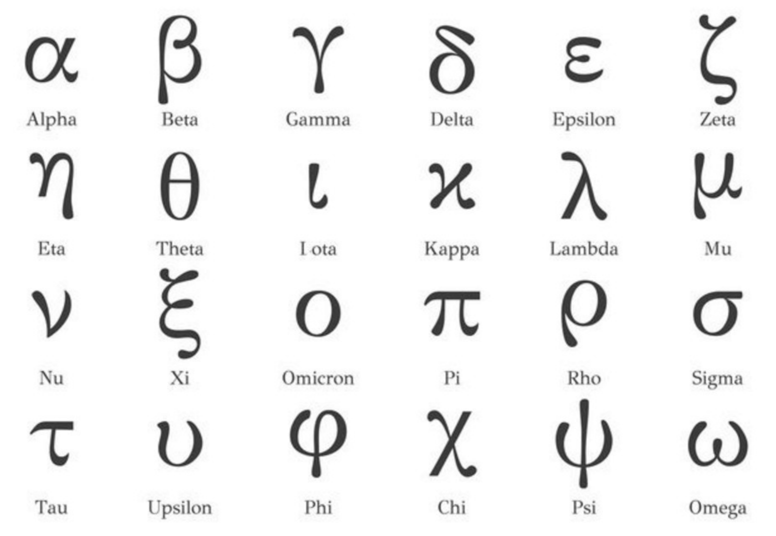


Figure : 24 beautiful letters

The Next Six Letters

We will consider the third group of six letters.

ν	ξ	o	π	ρ	σ
N	Ξ	O	Π	P	Σ

Let us focus first on the **small letters**
and come back to the big ones later.

$\nu \quad \xi \quad o \quad \pi \quad \rho \quad \sigma$

Nu (ν) is in Planck's formula: $E = h\nu$.
Then ν is the frequency of a photon of light.

Xi (ξ) is the Greek X, as in $\kappa\lambda\iota\mu\alpha\xi$ or $\mathrm{K\Lambda IMAX}$.

Omicron: Think of Oh-Micron, small Oh (not OMG).
Is there a large O, or Oh-Mega ?

Pi (π) is already very familiar to you all.

Rho (ρ) is Greek R, used for density.

Sigma (σ) is the Greek S. At the end of a word it is written ς.

Now we know eighteen letters. We're 75% done!

A Few Greek Words (for practice)

$\kappa\lambda\iota\mu\alpha\xi$
$\delta\rho\alpha\mu\alpha$
$\nu\epsilon\kappa\tau\alpha\rho$
$\kappa\omega\lambda o\nu$
$\kappa o\sigma\mu o\varsigma$

A Few Greek Words (for practice)

$\kappa\lambda\iota\mu\alpha\xi$
$\delta\rho\alpha\mu\alpha$
$\nu\epsilon\kappa\tau\alpha\rho$
$\kappa\omega\lambda o\nu$
$\kappa o\sigma\mu o\varsigma$

Climax: $\kappa\lambda\iota\mu\alpha\xi$
Drama: $\delta\rho\alpha\mu\alpha$
Nectar: $\nu\epsilon\kappa\tau\alpha\rho$
Colon: $\kappa\omega\lambda o\nu$
Cosmos: $\kappa o\sigma\mu o\varsigma$


Κέντρο
Center
Λιμάνι
Port


Μουσειο Ακρόπολης
Acropolis Museum

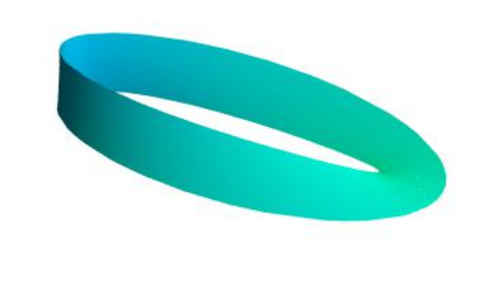

Αθήνα
Δελφοί
400M


ΤΑΞΙ

End of Greek 103

Outline

The Möbius Band



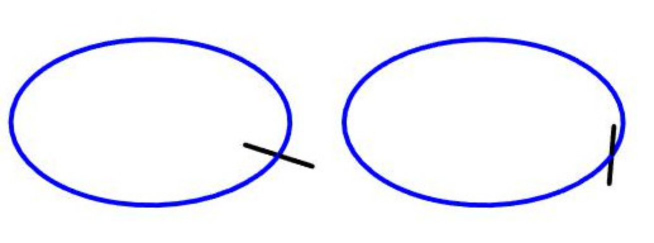
You may be familiar with the Möbius strip or Möbius band. It has one side and one edge.

It was discovered independently by August Möbius and Johann Listing in the same year, 1858.

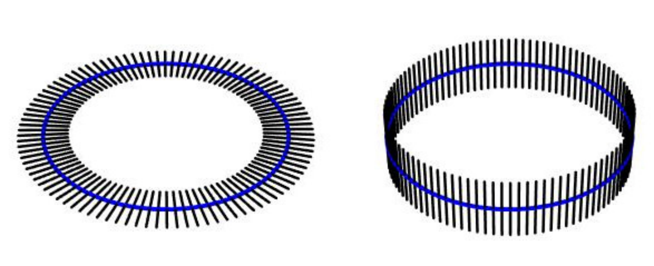
Building the Band

It is easy to make a Möbius band from a paper strip.

For a geometrical construction, we start with a circle and a small line segment with centre on this circle.



Now move the line segment around the circle:



To show the boundary of the surface, we color one end of the line segment red and the other magenta.

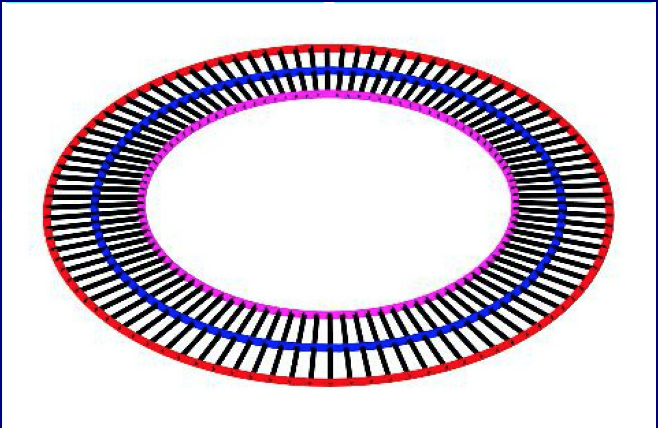


Figure : The boundary comprises two unlinked circles

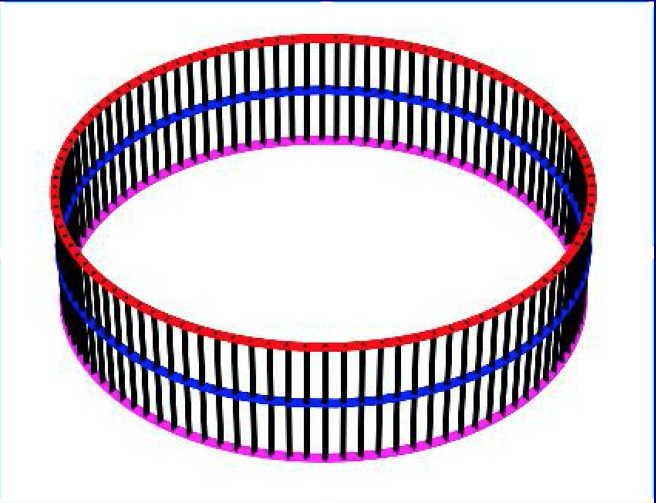
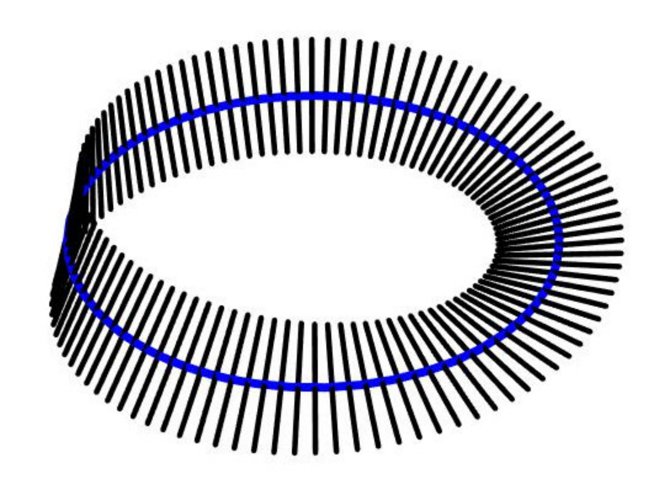


Figure : The boundary comprises two unlinked circles

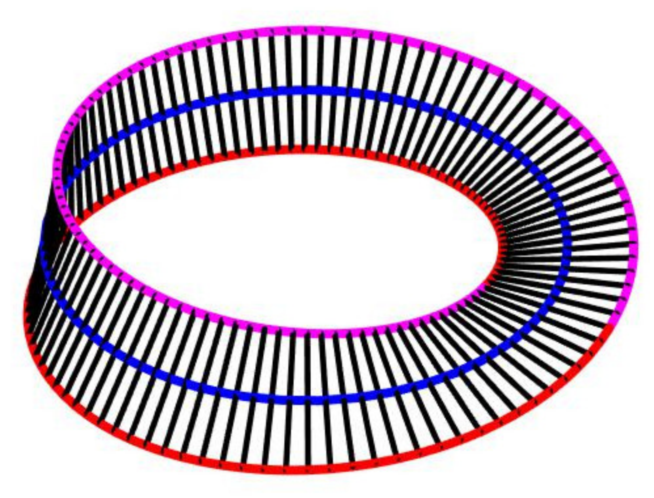
The Möbius Band

Now, as the line moves, we give it a half-twist:



The Möbius Band

The two boundary curves now join up to become one:



The Möbius Band

The Möbius Band has only one side.

It is possible to get from any point on the surface to any other point without crossing the edge.

The surface also has just one edge.

Band with a Full Twist

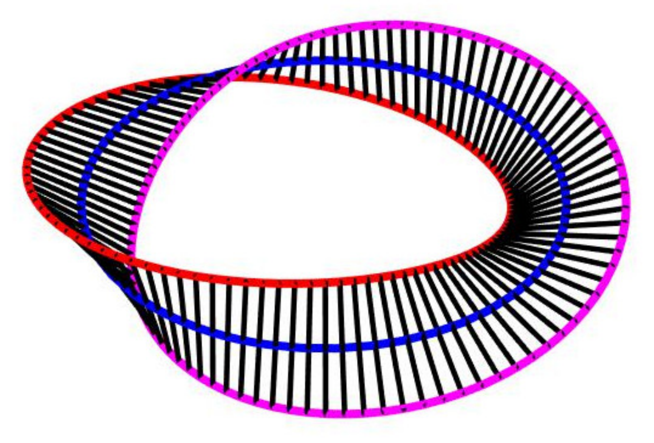


Figure : The boundary comprises two linked circles

Band with Three Half-twists

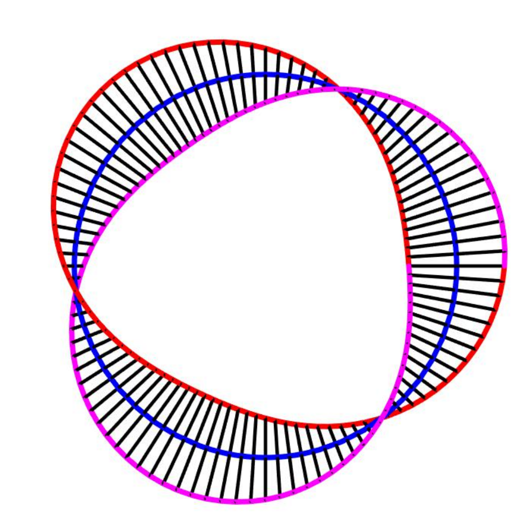
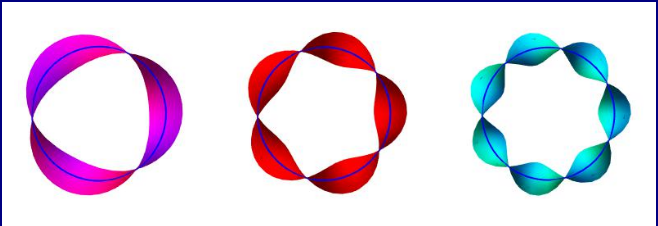
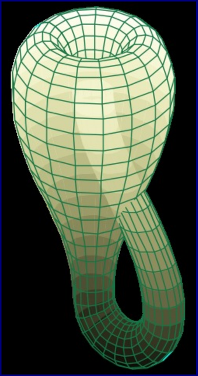


Figure : The boundary is a knot, a trefoil curve



Two Möbius Bands make a Klein Bottle



A mathematician named Klein
Thought the Möbius band was divine.
    Said he: "If you glue
    The edges of two,
You'll get a weird bottle like mine."

Equations for the Möbius Band

The process of moving the line segment around the circle leads us to the equations for the Möbius band.

In cylindrical polar coordinates the circle is $(r, \theta, z) = (a, \theta, 0)$**.**

The tip of the segment, relative to its centre, is

$$(r, \theta, z) = (b \cos \phi, 0, b \sin \phi)$$

where $b = \frac{1}{2}\ell$ **is half the segment length and** $\phi = \alpha\theta$**, with** α **determining the amount of twist.**

The tip of the line has $(r, z) = (a + b \cos \alpha\theta, b \sin \alpha\theta)$**.**

Equations for the Möbius Band

In cartesian coordinates, the equations become

$$\begin{aligned} x &= (a + b\cos\alpha\theta)\cos\theta \\ y &= (a + b\cos\alpha\theta)\sin\theta \\ z &= (b\sin\alpha\theta) \end{aligned}$$

These are the parametric equations for the twisted bands, with $\theta \in [0, 2\pi]$ and $b \in [-\ell, \ell]$.

For the Möbius band, $\alpha = \frac{1}{2}$.

Outline

Distraction: A Curious Year, AD 1089

What is so special about the year 1089?

- Palmyra destroyed by an earthquake.
- First Cistercian monastery, Cîteaux Abbey, founded in southern France.
- The Synod of Melfi issues decrees against simony and clerical marriage.

Such vital information is obtained from Wikipedia.

Distraction: A Curious Number

Think of a three-digit number, for example 275.

Calculate the difference between this number and the number formed by reversing digits:

$$572 - 275 = 297$$

Distraction: A Curious Number

Think of a three-digit number, for example 275**.**

Calculate the difference between this number and the number formed by reversing digits:

$$572 - 275 = 297$$

Now repeat the process, this time adding numbers:

$$297 + 792 = 1089$$

Distraction: A Curious Number

Think of a three-digit number, for example 275**.**

Calculate the difference between this number and the number formed by reversing digits:

$$572 - 275 = 297$$

Now repeat the process, this time adding numbers:

$$297 + 792 = 1089$$

What is so special about the number 1089?

Distraction: A Curious Number

This "trick" nearly always works.

But it can fail in some cases.

Can you find the conditions for success?

See the Wikipedia page "1089 (number)".

Thank you