Sum-Enchanted Evenings

The Fun and Joy of Mathematics

•

LECTURE 2

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Evening Course, UCD, Autumn 2017

Outline

Outline

Meaning and Content of Mathematics

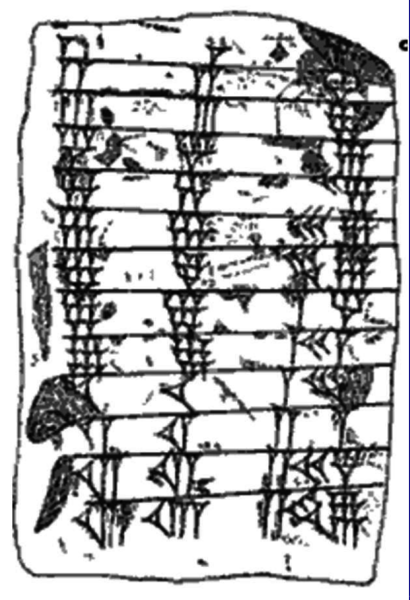
The word Mathematics comes from Greek $\mu\alpha\theta\eta\mu\alpha$ (máthéma), meaning “knowledge” or “study” or “learning”.

It is the study of topics such as

- Quantity (numbers)
- Structure (patterns)
- Space (geometry)
- Change (analysis).

Outline

The Nippur Tablet



The Nippur Tablet

What purpose could the Nippur Tablet have had?

What use could there be for a list of squares?

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What use could there be for a list of squares?

Perhaps it was used for multiplication!

We show how this is done in the next slide.

Multiplication by Squaring

Let a and b be any two numbers.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Subtracting, we get

$$(a+b)^2 - (a-b)^2 = 4ab$$

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Thus, we can find the product using squares:

$$ab = \frac{1}{4}[(a+b)^2 - (a-b)^2]$$

Multiplication by Squaring

Again,

$$ab = \frac{1}{4}[(a+b)^2 - (a-b)^2]$$

Let us take a particular example: 37×13**.**

$$a = 37 \quad b = 13 \quad a+b = 50 \quad a-b = 24\,.$$

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Let us take a particular example: 37×13**.**

$$a = 37 \quad b = 13 \quad a+b = 50 \quad a-b = 24\,.$$

$$\begin{aligned}
\tfrac{1}{4}[(a+b)^2 - (a-b)^2] &= \tfrac{1}{4}[50^2 - 24^2] \\
&= \tfrac{1}{4}[2500 - 576] \\
&= \tfrac{1}{4}[1924] \\
&= 481 \\
&= 37 \times 13\,.
\end{aligned}$$

Perhaps this was the function of the Nippur tablet.

Outline

Georg Cantor

Inventor of Set Theory

Born in St. Petersburg, Russia in 1845.

Moved to Germany in 1856 at the age of 11.

His main career was at the University of Halle.

Georg Cantor (1845–1918)

- Invented Set Theory.
- One-to-one Correspondence.
- Infinite and Well-ordered Sets.
- Cardinal and Ordinal Numbers.
- Proved: $\#(\mathbb{Q}) = \#(\mathbb{N})$.
- Proved: $\#(\mathbb{R}) > \#(\mathbb{N})$.
- Infinite Hierarchy of Infinities.

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Outline Galileo's arguments on infinity.

Set Theory: Controversy

Cantor was strongly criticized by

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Set Theory is a "grave disease" (HP).
Set Theory is "nonsense; laughable; wrong!" (LW).

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Cantor is a "corrupter of youth" (LK).
Set Theory is a "grave disease" (HP).
Set Theory is "nonsense; laughable; wrong!" (LW).

Adverse criticism like this may well have contributed to Cantor's mental breakdown.

Set Theory: A Difficult Birth

Set Theory brought into prominence several paradoxical results.

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To illustrate the difficulty of accepting new ideas, let's consider the problem of a river flowing uphill.

Describe the blog post "Paddling Uphill".

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It was so innovative that many mathematicians could not appreciate its fundamental value and importance.

Gösta Mittag-Leffler was reluctant to publish it in his *Acta Mathematica.* He said the work was "100 years ahead of its time".

David Hilbert said:
"We shall not be expelled from the paradise that Cantor has created for us."

A Passionate Mathematician

In 1874, Cantor married Vally Guttmann.

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According to Wikipedia:

“During his honeymoon in the Harz mountains, Cantor spent much time in mathematical discussions with Richard Dedekind.”

[Cantor had met the renowned mathematician Dedekind two years earlier while he was on holiday in Switzerland.]

Distraction: The Simpsons

Several writers of the Simpsons scripts have advanced mathematical training.

They "sneak" jokes into the programmes.

Books on a Shelf

Ten books are arranged on a shelf.
They include an Almanac (A) and a Bible (B).

Suppose A must be to the left of B
(not necssarily beside it).

How many possible arrangements are there?

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BIG IDEA: SYMMETRY.

Every SOLUTION correponds to a NONSOLUTION.

Just switch positions of A and B!

Books on a Shelf

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BIG IDEA: SYMMETRY.

Every SOLUTION correponds to a NONSOLUTION.

Just switch positions of A and B!

The total number of arrangements is 10!.
For half of these, A is to the left of B.

So, answer is $\frac{1}{2}(10 \times 9 \times \cdots \times 1) = \frac{1}{2} \times 10!$ Q.E.D.

Outline

Set Theory I

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Sets are the basic building-blocks of mathematics.

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The objects in a set are called the elements.

Examples:

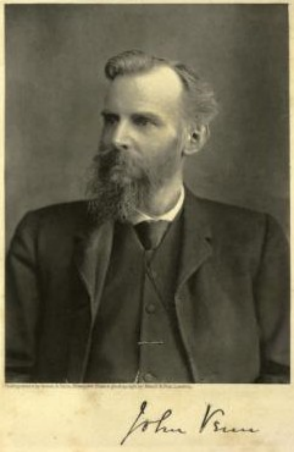
- All the prime numbers, $\mathbb{P}$
- All even numbers: $\mathbb{E} = \{2, 4, 6, 8 \ldots\}$
- All the people in Ireland: See Census returns.
- The colours of the rainbow: {Red, . . . , Violet}.
- Light waves with wavelength $\lambda \in [390 - 700\mathrm{nm}]$

Do You Remember Venn?

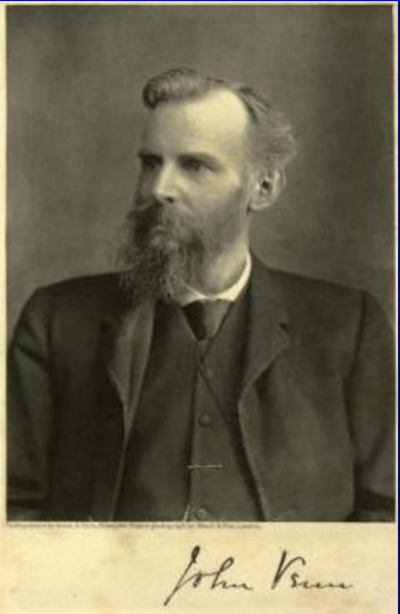
John Venn was a logician and philosopher, born in Hull, Yorkshire in 1834.

He studied at Cambridge University, graduating in 1857 as sixth Wrangler.

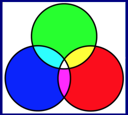
Venn introduced his diagrams in *Symbolic Logic*, a book published in 1881.




JOHN VENN
FELLOW 1857-1923
PRESIDENT 1903-23


John Venn

Venn Diagrams



Venn diagrams are very valuable for showing elementary properties of sets.

They comprise a number of overlapping circles.

The interior of a circle represents a collection of numbers or objects or perhaps a more abstract set.

The Universe of Discourse

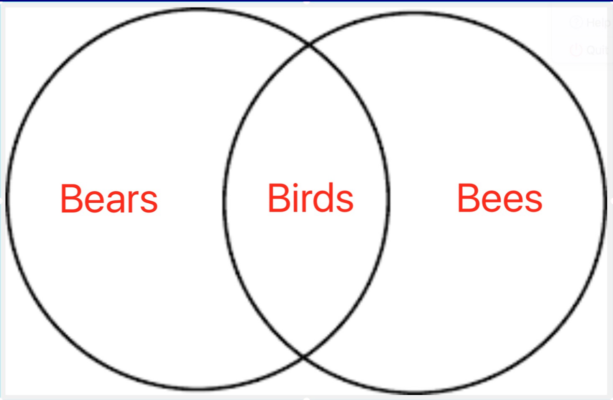
We often draw a rectangle to represent the **universe**, the set of all objects under current consideration.

For example, suppose we consider all species of animals as the universe.

A rectangle represents this universe.

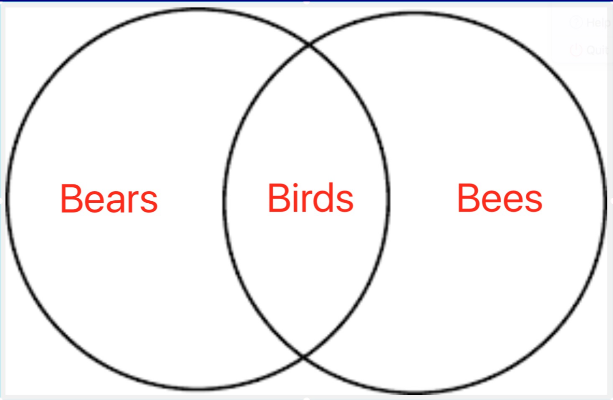
Two circles indicate subsets of animals of two different types.

The Birds and the Bees



Two-legged Animals **Flying Animals**

The Birds and the Bees



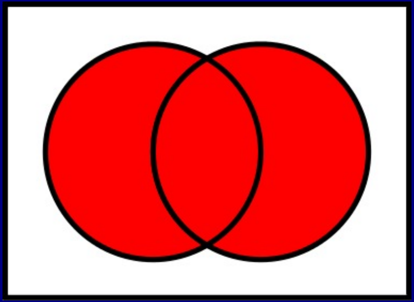
Two-legged Animals Flying Animals

Where do we fit in this diagram?

The Union of Two Sets

The aggregate of two sets is called their union.

Let one set contain all two-legged animals and the other contain all flying animals.



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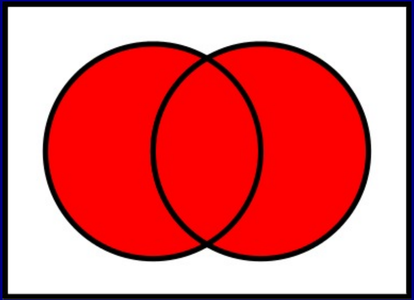
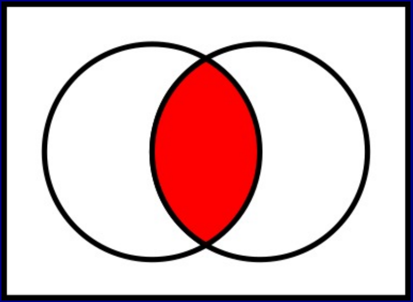


Bears, birds and bees (and we) are in the union.

The Intersection of Two Sets

The elements in both sets make up the intersection.

Let one set contain all two-legged animals
and the other contain all flying animals.



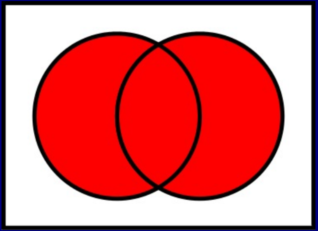
Birds are in the intersection. Bears and bees are not.

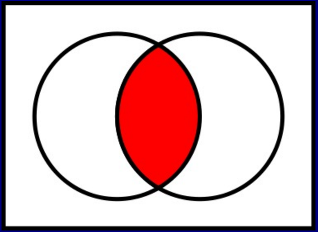
The Notation for Union and Intersection

Let A and B be two sets

The **union** of the sets is

$$A \cup B$$





The **intersection** is

$$A \cap B$$

The Technical (Logical) Definitions

Let A and B be two sets.

The **union** of the sets $A \cup B$ is defined by

$$[x \in A \cup B] \iff [(x \in A) \vee (x \in B)]$$

The **intersection** of the sets $A \cap B$ is defined by

$$[x \in A \cap B] \iff [(x \in A) \wedge (x \in B)]$$

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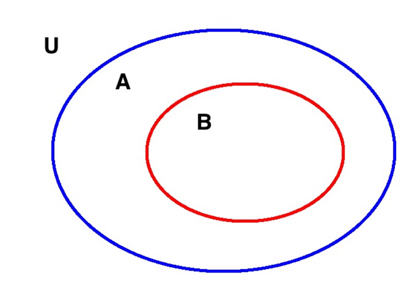
There is an intimate connection between Set Theory and Symbolic Logic.

Subset of a Set



For two sets A and B we write

$$B \subset A \qquad \text{or} \qquad B \subseteq A$$

to denote that B is a subset of A.

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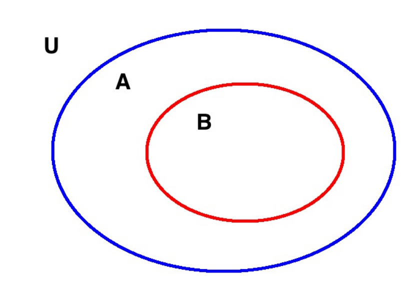

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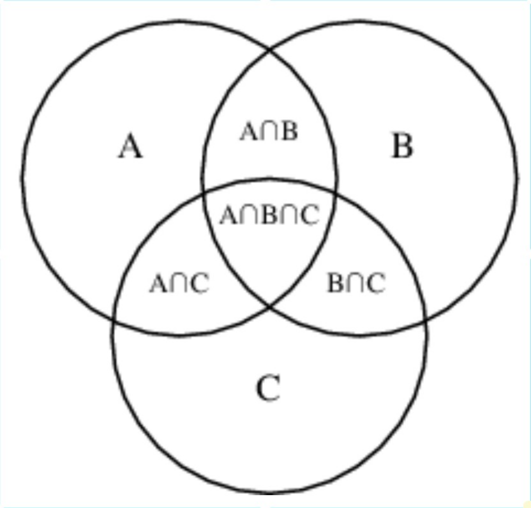
For more on set theory, see website of Claire Wladis
http://www.cwladis.com/math100/Lecture4Sets.htm

Intersections between 3 Sets



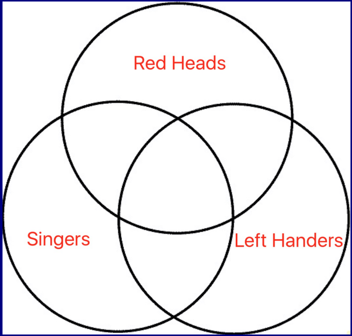
Example: Intersection of 3 Sets

In the diagram the elements of the universe are all the people from Connacht.

Three subsets are shown:

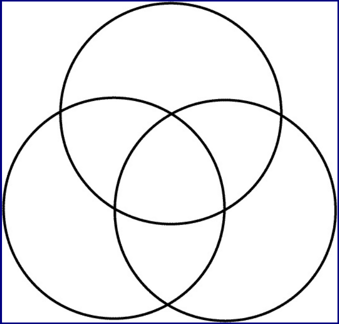
- Red-heads
- Singers
- Left-handers.

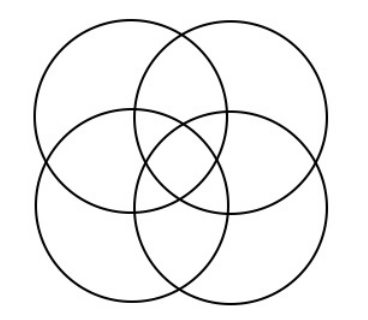
All are from Connacht.



These sets overlap and, indeed, there are some copper-topped, crooning cithogues in Connacht.

Three and Four Sets





8 Domains **14 Domains**

How Many Possibilities?

With just one set A, there are 2 possibilities:

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$$(x \in A) \wedge (x \in B) \qquad \text{or} \qquad (x \in A) \wedge (x \notin B)$$
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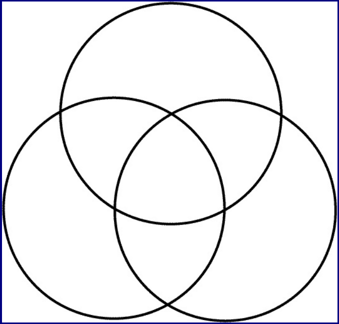
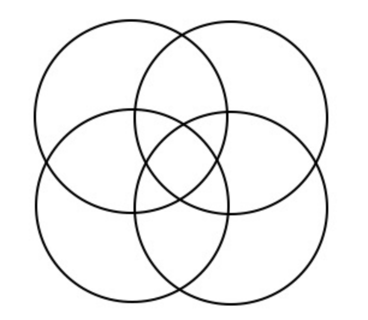
With three sets there are 8 possible conditions.

With four sets there are 16 possible conditions.

Three and Four Sets





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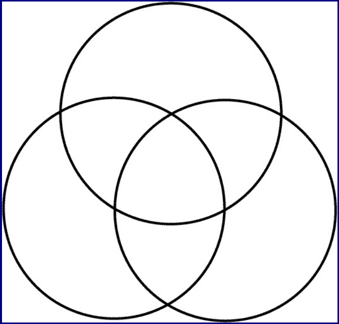
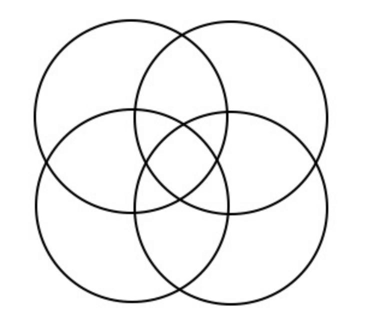
Three and Four Sets





8 Domains 14 Domains

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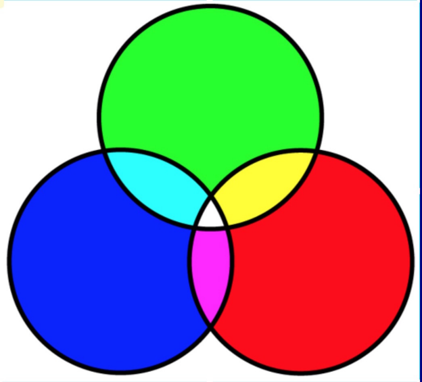
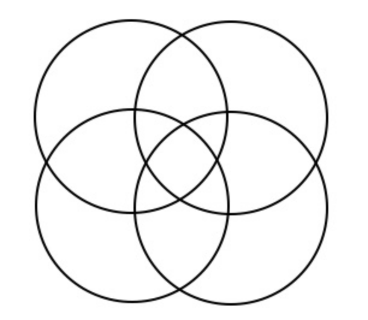
The Intersection of 3 Sets

The three overlapping circles have attained an iconic status, seen in a huge range of contexts.

It is possible to devise Venn diagrams with four sets, but the simplicity of the diagram is lost.



Exercise: Four Set Venn Diagram



Can you modify the 4-set diagram to cover all cases.
(You will not be able to do it with circles only)

Hint: Venn Diagrams for 5 and 7 Sets

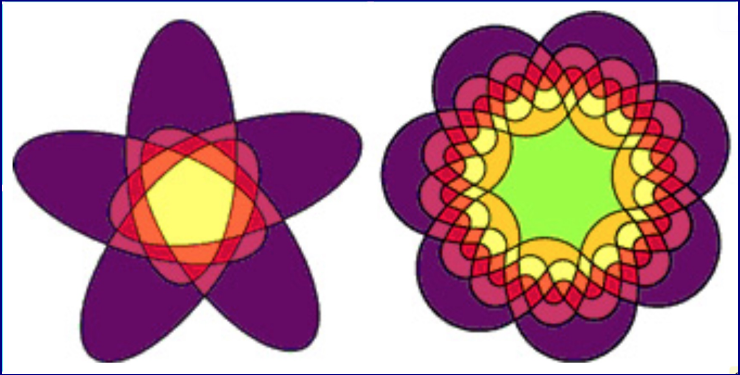
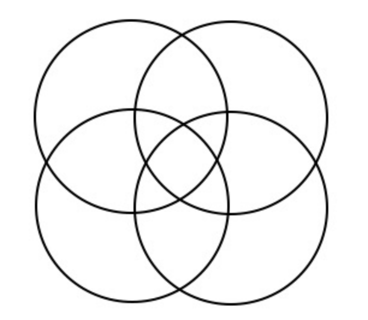


Image from Wolfram MathWorld: Venn Diagram

Solution: Next Week (if you are lucky)



We will find a surprising connection with a Cube

Outline

The Greek Alphabet, Part 1

Ελληνικό αλφάβητο

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Ελληνικό αλφάβητο

Some Motivation

- Greek letters are used extensively in maths.
- Greek alphabet is the basis of the Roman one.
- Also the basis of the Cyrillic and others.

The Greek Alphabet, Part 1

Ελληνικό αλφάβητο

Some Motivation

- Greek letters are used extensively in maths.
- Greek alphabet is the basis of the Roman one.
- Also the basis of the Cyrillic and others.
- A great advantage for touring in Greece.
- You already know several of the letters.
- It is simple to learn in small sections.

Letter	Name	Sound	
		Ancient[5]	Modern[6]
Α α	alpha, άλφα	[a] [aː]	[a]
Β β	beta, βήτα	[b]	[v]
Γ γ	gamma, γάμμα	[g], [ŋ][7]	[ɣ] ~ [ʝ], [ŋ][8] ~ [ɲ][9]
Δ δ	delta, δέλτα	[d]	[ð]
Ε ε	epsilon, έψιλον	[e]	[e]
Ζ ζ	zeta, ζήτα	[zd][A]	[z]
Η η	eta, ήτα	[ɛː]	[i]
Θ θ	theta, θήτα	[tʰ]	[θ]
Ι ι	iota, ιώτα	[i] [iː]	[i], [ʝ],[10] [ɲ][11]
Κ κ	kappa, κάππα	[k]	[k] ~ [c]
Λ λ	lambda, λάμδα	[l]	[l]
Μ μ	mu, μυ	[m]	[m]

Letter	Name	Sound	
		Ancient[5]	Modern[6]
Ν ν	nu, νυ	[n]	[n]
Ξ ξ	xi, ξι	[ks]	[ks]
Ο ο	omicron, όμικρον	[o]	[o]
Π π	pi, πι	[p]	[p]
Ρ ρ	rho, ρώ	[r]	[r]
Σ σ/ς[13]	sigma, σίγμα	[s]	[s]
Τ τ	tau, ταυ	[t]	[t]
Υ υ	upsilon, ύψιλον	[y] [yː]	[i]
Φ φ	phi, φι	[pʰ]	[f]
Χ χ	chi, χι	[kʰ]	[x] ~ [ç]
Ψ ψ	psi, ψι	[ps]	[ps]
Ω ω	omega, ωμέγα	[ɔː]	[o]

Figure : The Greek Alphabet (from Wikipedia)

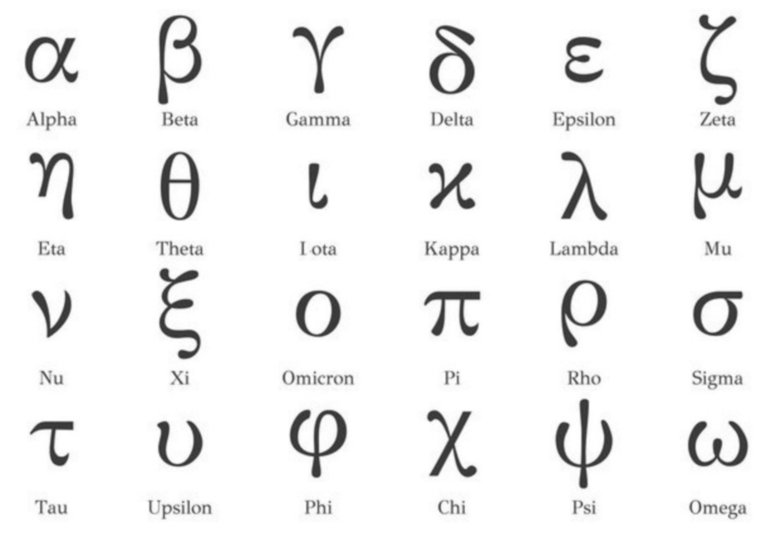

Figure : 24 beautiful letters

The First Six Letters

We will take the alphabet in groups of six letters.

$$\alpha \qquad \beta \qquad \gamma \qquad \delta \qquad \epsilon \qquad \zeta$$

$$A \qquad B \qquad \Gamma \qquad \Delta \qquad E \qquad Z$$

Let us focus first on the small letters and come back to the big ones later.

You know α and β from the word Alphabet: α β

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You have heard of gamma-rays, or γ-rays

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Both δ and ϵ are widely used in maths. For example, the definition of continuity of function $f(x)$ at $x = a$ is

$$\forall\epsilon > 0 \; \exists\delta > 0 : |x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$$

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Now we already know the first six letters!

End of Greek 101

Outline

Topology: a Major Branch of Mathematics

Topology is all about continuity and connectivity, but the meaning of that will appear later.

We will look at a few aspects of Topology.

- The Bridges of Königsberg
- Doughnuts and Coffee-cups
- Knots and Links
- Nodes and Edges: Graphs
- The Möbius Band

In this lecture, we study The Bridges of Königsberg.

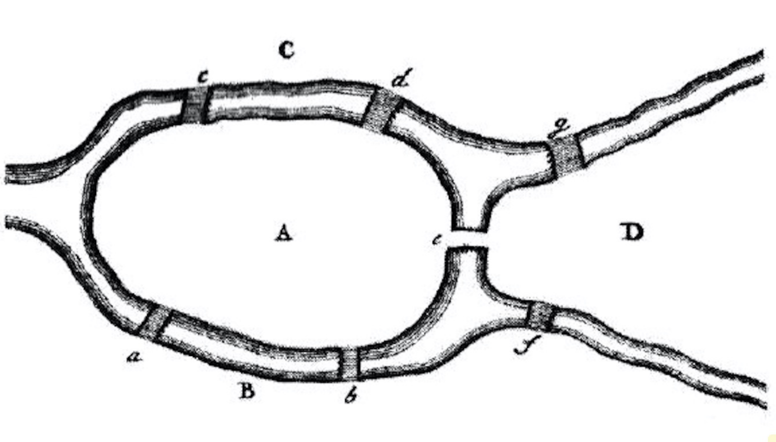
The Bridges of Königsberg

One of the earliest topological puzzles was studied by the renowned Swiss mathematician Leonard Euler.

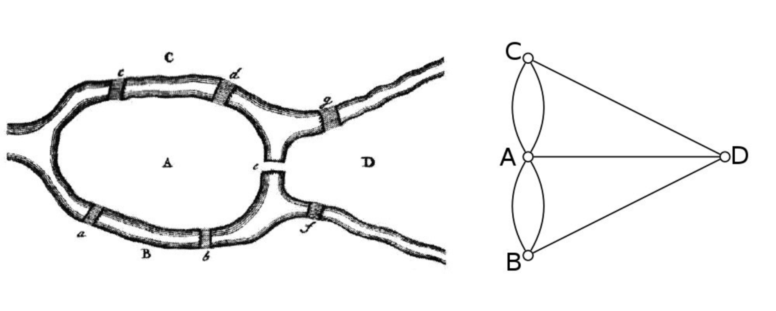
It is called ’The Seven Bridges of Königsberg’.

The goal is to find a route through that city, crossing each of seven bridges exactly once.

The Bridges of Königsberg



The Bridges of Königsberg



Euler reduced the problem to its essentials, removing all extraneous details.

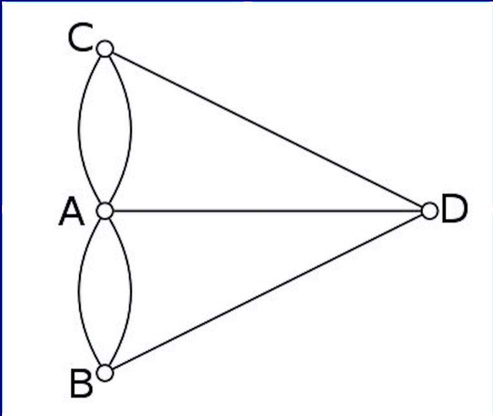
He replaced the map above by the graph on the right.

A simple argument showed that no journey that crosses each bridge exactly once is possible.

Except at the termini of the route, each path arriving at a vertex must have a corresponding path leaving it.

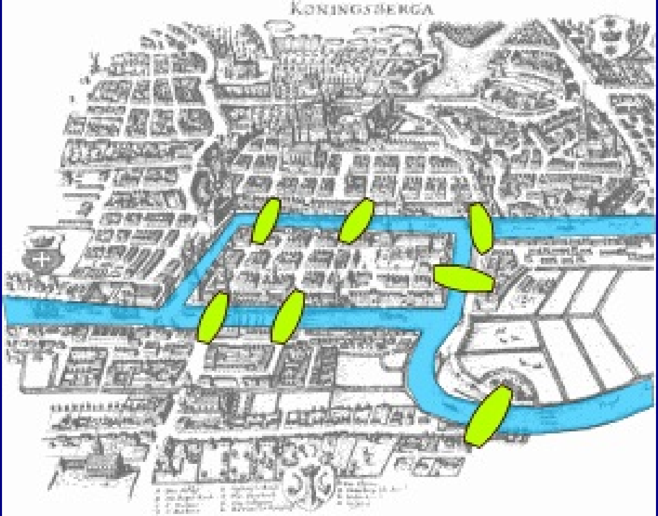
Only two vertices with an odd number of edges are possible for a solution to exist.

The Bridges of Königsberg

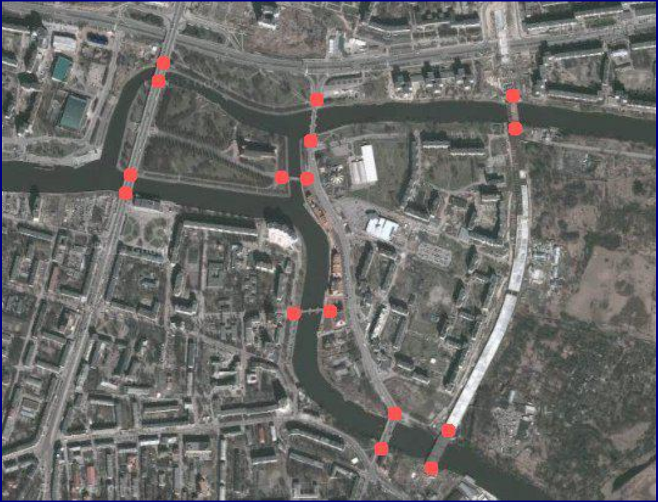


Exercize: Draw the diagram with *A*, *B*, *C* and *D* arranged clockwise at the corners of a square.

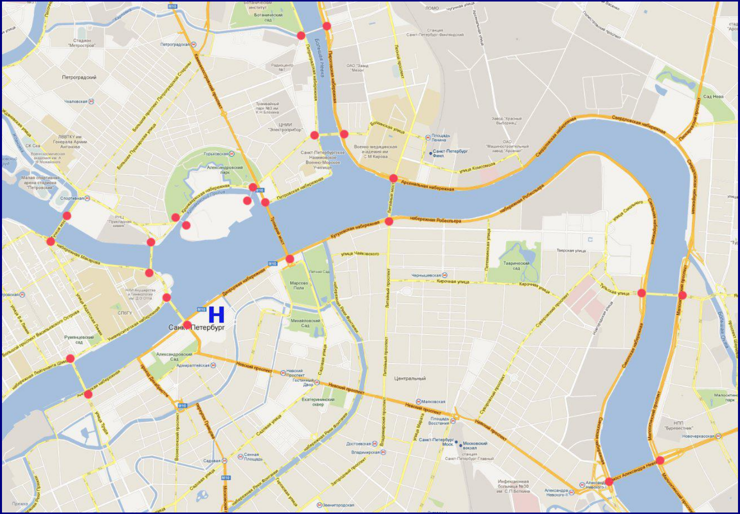
The Bridges of Königsberg



Königsberg Today



The Bridges of St Petersburg



The Bridges of St Petersburg

Euler spend much of his life in St Petersburg,
a city with many rivers, canals and bridges.

Did he think about another problem like the
Königsberg Bridges problem while there?

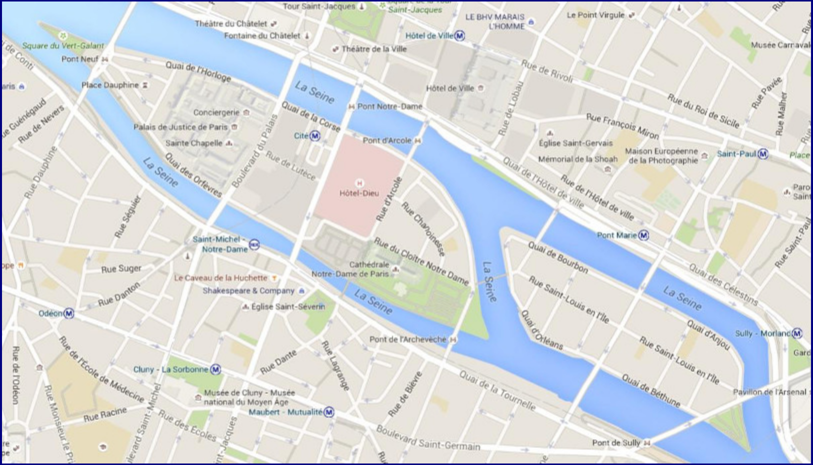
The map of central St Petersburg has twelve bridges.

An Euler cycle is a route that crosses all bridges
exactly once and returns to the starting point?

Is there an Euler cycle starting at the Hermitage
(marked "H" on the map)?

The Bridges of Paris

Cue romantic music



The Bridges of Paris

In central Paris, two small islands, Île de la Cité and Île Saint-Louis, are linked to the Left and Right Banks of the Seine and to each other.

The number of bridges for each land-mass are:

- Left Bank: 7 bridges
- Right Bank: 7 bridges
- Île de la Cité: 10 bridges
- Île Saint-Louis: 6 bridges

The total is 30. How many bridges are there?

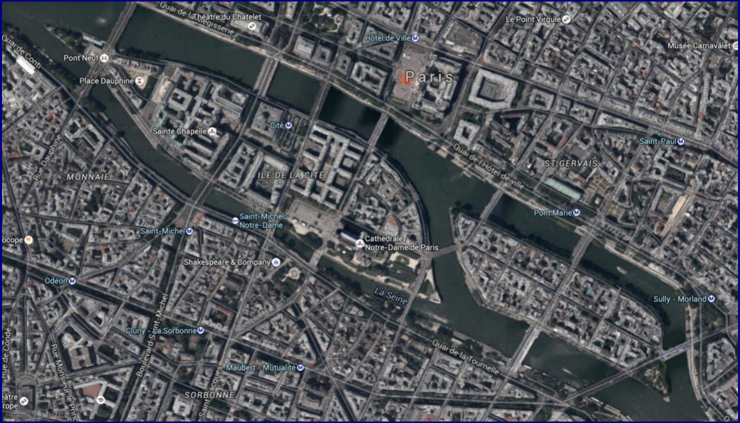
The Bridges of Paris

The Bridges of Paris

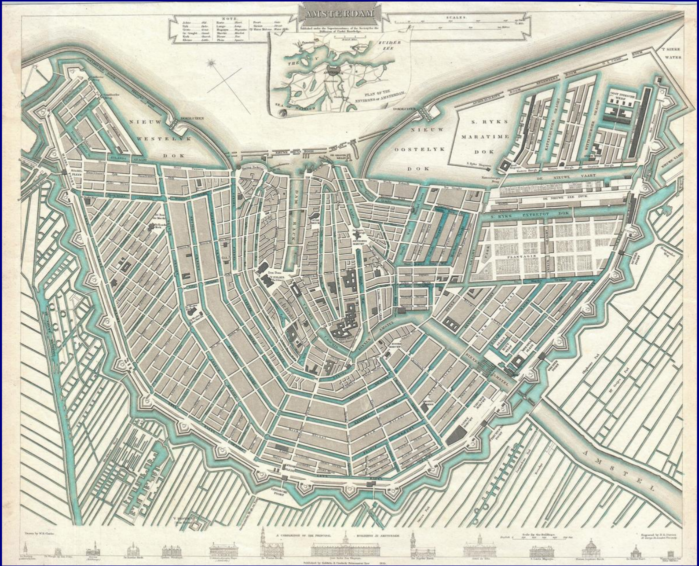
1. Starting from Saint-Michel on the Left Bank, walk continuously so as to cross each bridge once.
2. Start at Saint-Michel but find a closed route that ends back at the starting point.
3. Start at Notre-Dame Cathedral, on Île de la Cité, and cross each bridge exactly once.
4. Find a closed route that crosses each bridge once and arrives back at Notre-Dame.

Try these puzzles yourself. Use logic, not brute force!

The Bridges of Paris



The Bridges of Amsterdam



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Seven Bridges of Königsberg

From Wikipedia, the free encyclopedia

Coordinates: 54°42′12″N 20°30′56″E

This article is about an abstract problem. For the historical group of bridges in the city once known as Königsberg, and those of them that still exist, see § Present state of the bridges.

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The **Seven Bridges of Königsberg** is a historically notable problem in mathematics. Its negative resolution by Leonhard Euler in 1736 laid the foundations of graph theory and prefigured the idea of topology.[1]

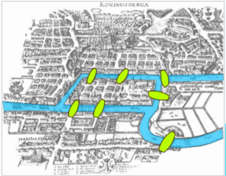
The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other, or to the two mainland portions of the city, by seven bridges. The problem was to devise a walk through the city that would cross each of those bridges once and only once.

By way of specifying the logical task unambiguously, solutions involving either

1. reaching an island or mainland bank other than via one of the bridges, or
2. accessing any bridge without crossing to its other end

are explicitly unacceptable.

Euler proved that the problem has no solution. The difficulty he faced was the development of a suitable technique of analysis, and of subsequent tests that established this assertion with mathematical rigor.



Map of Königsberg in Euler's time showing the actual layout of the seven bridges, highlighting the river Pregel and the bridges

Outline

The Unary System

The simplest numeral system is the unary system.

Each natural number is represented by
a corresponding number of symbols.

If the symbol is “ | ” , the number seven
would be represented by | | | | | | | .

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Tally marks represent one such system,
which is still in common use.

The unary system is only useful for small numbers.

The unary notation can be abbreviated,
with new symbols for certain values.

Sign-Value Notation

The **five-bar gate** system groups 5 strokes together.

Normally, distinct symbols are used for powers of 10.

If “ | ” stands for one, “ Λ ” for ten and “ ϒ ” for 100, then the number **123** becomes ϒ ΛΛ | | |

Sign-Value Notation

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If “ | ” stands for one, “ Λ ” for ten and “ ϒ ” for 100,
then the number 123 becomes ϒ ΛΛ | | |

There is no need for a symbol for zero.

This is called sign-value notation.

Ancient Egyptian numerals were of this type.

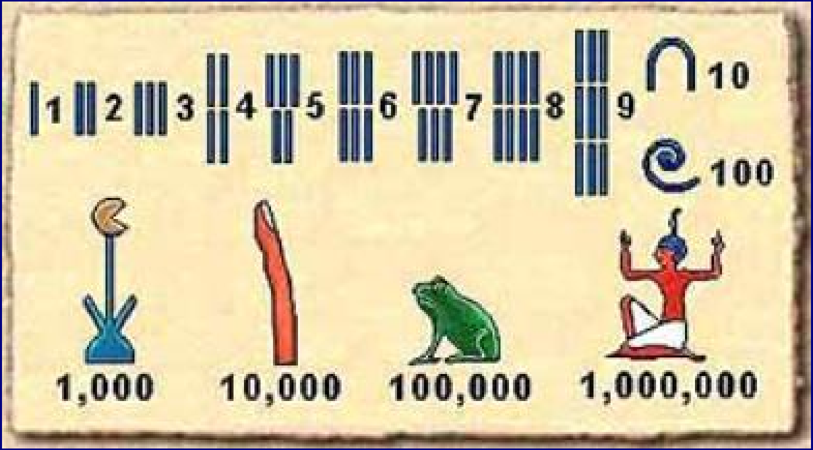
Roman numerals were a modification of this idea.

Egypyian Numerals

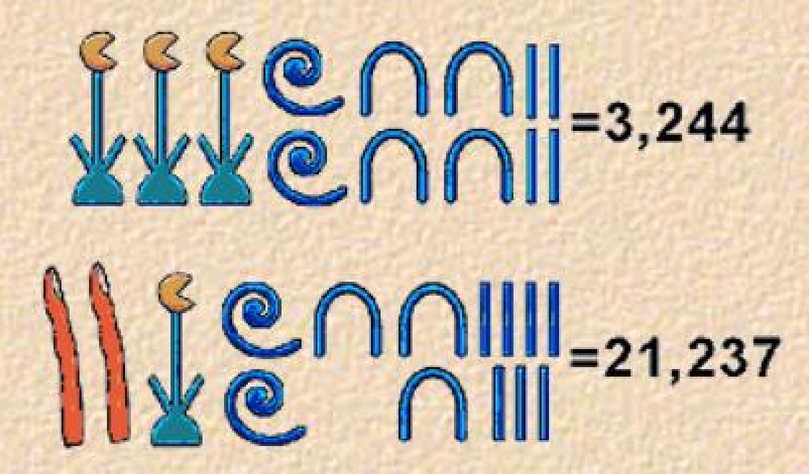
Value	1	10	100	1,000	10,000	100,000	1 million, or many
Hieroglyph						 or 	
Description	Single stroke	Heel bone	Coil of rope	Water lily (also called Lotus)	Bent Finger	Tadpole or Frog	Man with both hands raised, perhaps Heh.[3]

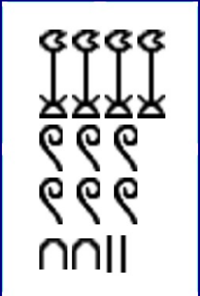
Figure : From Wikipedia page `https://en.wikipedia.org/wiki/Egyptian_numerals`

Egypyian Numerals



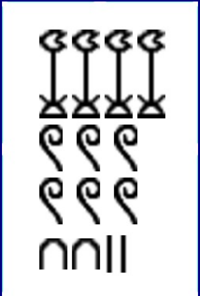
Egypyian Numerals





The arrangement of symbols is not important.

What number is this?



The arrangement of symbols is not important.

What number is this?

This pattern represents the number 4622.

Hebrew Numerals

Hebrew Number Values

א	Aleph -	1	ל	Lamed -	30
ב	Beth -	2	מ	Mem -	40
ג	Gimel -	3	נ	Nun -	50
ד	Daleth -	4	ס	Samekh -	60
ה	Heh -	5	ע	Ayin -	70
ו	Vav -	6	פ	Peh -	80
ז	Zain -	7	צ	Tzaddi -	90
ח	Cheth -	8	ק	Qoph -	100
ט	Teth -	9	ר	Resh -	200
י	Yod -	10	ש	Shin -	300
כ	Kaph -	20	ת	Tau -	400

The 22 letters of the Hebrew alphabet were used also as numerals.

Each letter corresponded to a numerical value.

Greek Numerals

	Units	Tens	Hundreds
1	α alpha	ι iota	ρ rho
2	β beta	κ kappa	σ sigma
3	γ gamma	λ lambda	τ tau
4	δ delta	μ mu	υ upsilon
5	ε epsilon	ν nu	φ phi
6	ϝ digamma	ξ xi	χ chi
7	ζ zeta	ο omicron	ψ psi
8	η eta	π pi	ω omega
9	θ theta	ϙ koppa	ϡ sampi

The 24 letters of the Greek alphabet had corresponding numerical values.

They were supplemented by three additional letters, which are now archaic.

$$243 = \sigma\mu\gamma$$

Greek Numerals

Arabic number	1	2	3	4	5	6	7	8	9
Greek number	α	β	γ	δ	ε	Ϝ	ζ	η	θ
Greek name	alpha	beta	gamma	delta	epsilon	digamma	zeta	eta	theta
Sound	a	b	g	d	short e		z	long e	th
Arabic number	10	20	30	40	50	60	70	80	90
Greek number	ι	κ	λ	μ	ν	ξ	ο	π	ϟ
Greek name	iota	kappa	lambda	mu	nu	xi	omicron	pi	koppa
Sound	i	k/c	l	m	n	x	short o	p	
Arabic number	100	200	300	400	500	600	700	800	900
Greek number	ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
Greek name	rho	sigma	tau	upsilon	phi	chi	psi	omega	sampi
Sound	r	s	t	u	f/ph	ch	ps	long o	

Thank you