

Sum-Enchanted Evenings

The Fun and Joy of Mathematics



LECTURE 0

Peter Lynch

**School of Mathematics & Statistics
University College Dublin**

Evening Course, UCD, Autumn 2017



Outline

Introduction

Solar Eclipse

Symmetry and Group Theory

Beautiful Symmetry

The Utility of Mathematics

Euler's Gem

Shackleton's Rescue Voyage

Recreational Mathematics



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Meaning and Content of Mathematics

The word **Mathematics** comes from Greek *μαθημα* (*máthēma*), meaning “knowledge” or “study” or “learning”.

It is the study of topics such as

- ▶ Quantity (numbers)
- ▶ Structure (patterns)
- ▶ Space (geometry)
- ▶ Change (analysis).



Taster Lecture

The course **Sum-enchanted Evenings** will run over ten (10) lectures from 25 September to 4 December.

The aim of the course is to show you

- ▶ The tremendous **beauty** of mathematics;
- ▶ Its great **utility** in our daily lives;
- ▶ The **fun** we can have studying maths.



Taster Lecture

Last year, I taught a course with the title

AweSums: The Majesty of Maths

It was well received, but the pace was too fast for some of the participants.

So, I have decided to break it into two parts:

- ▶ **Sum-enchanted Evenings**
(Autumn 2017)
- ▶ **AweSums: Outstanding Problems of Maths.**
(Spring 2018)



Taster Lecture

In this **Taster Lecture** I will give a small sample of some of the topics covered in the course.

But to start: Today there is a **SYZGY**, long predicted by the power of mathematics.



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But to start: Today there is a **SYZGY**, long predicted by the power of mathematics.

A Syzygy:

A straight-line configuration of three celestial bodies.



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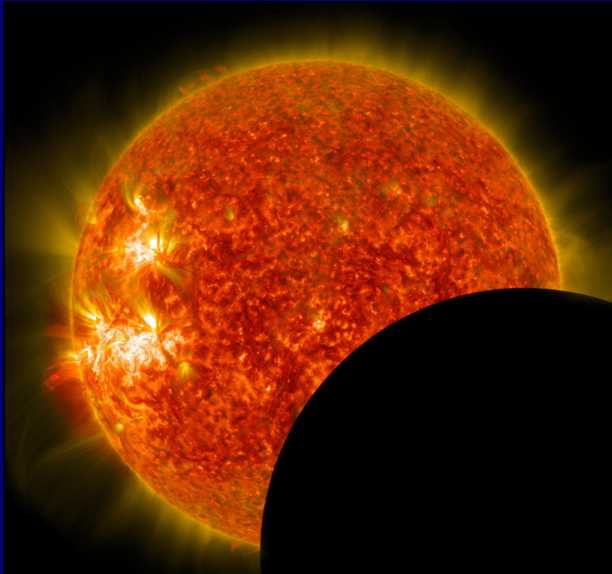
Euler's Gem

Shackleton's Rescue Voyage

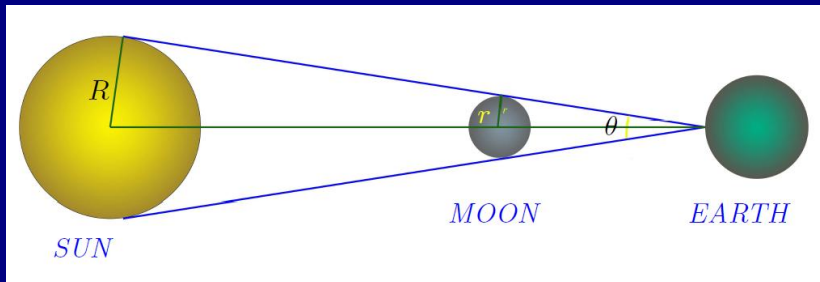
Recreational Mathematics



Saros 145/22: The Great American Eclipse



Earth, Moon and Sun in Line



It is remarkable that the Sun and Moon appear to be about the same size.

A solar eclipse occurs when the Moon passes between Earth and Sun, obscuring the Sun.



Saros 145/22

This afternoon, the Moon will eclipse the Sun, sweeping across the United States along a path from Oregon to South Carolina.

This eclipse is No. 22 in the **Saros series 145**.

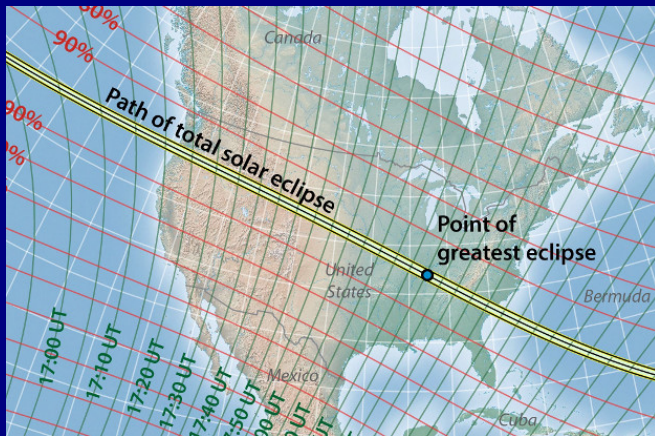
The eclipse near Paris in August 1999 was the 21st event in this series.



Total Solar Eclipse



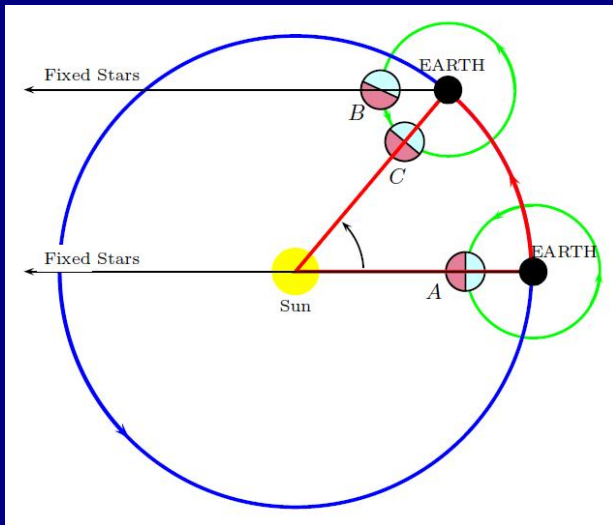
Path of Totality over USA



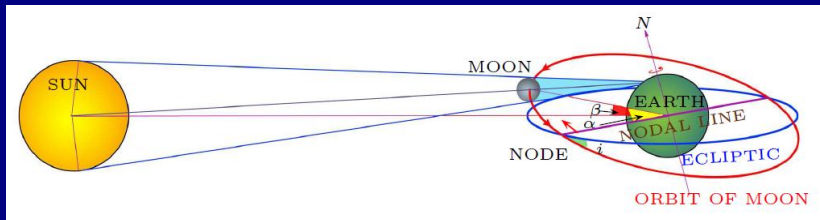
Starting: Oregon, 17:15 UT (18:15 IST)
Ending: South Carolina, 20:15 UT (21:15 IST)



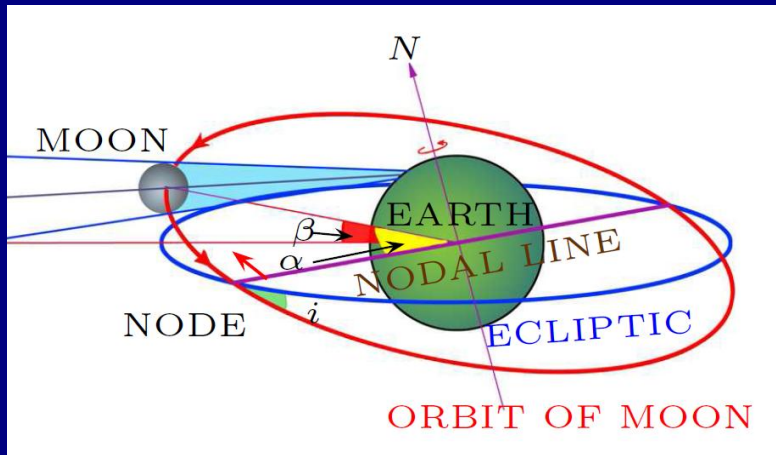
Lunar and Draconic Months



The Tilt of the Moon's Orbit



The Tilt of the Moon's Orbit



The Saros Series

In a **draconic month**, averaging 27.21 days, the Moon orbits the Earth and is back at its original position.

But the Earth has moved on, so we have not reached the next New Moon, and there is no eclipse.

The period to the next New Moon is called a **synodic month**, lasting 29.53 days.

When a large whole number of draconic months equals a large whole number of synodic months, we get another eclipse.

242 draconic months is close to 223 synodic months.



Saros: Coincidence of Periods

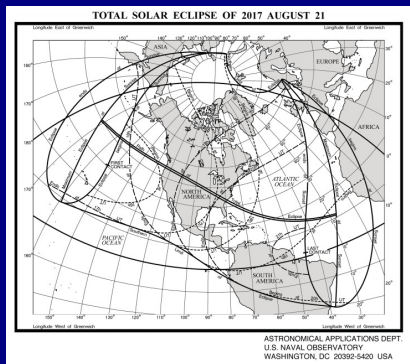
$$\begin{aligned} S &= 223 T_{\text{lun}} = 6585.32 \text{ days,} \\ &242 T_{\text{dr}} = 6585.36 \text{ days,} \\ &19 J_{\text{ecl}} = 6585.78 \text{ days,} \\ &239 T_{\text{an}} = 6585.54 \text{ days.} \end{aligned}$$

223 Synodic months and 242 nodical or draconic months are almost exactly equal, at $6585\frac{1}{3}$ days.

$6585\frac{1}{3}$ days \approx 18 years, 11 days and 8 hours.



Saros 145/22: Full Extent of the Eclipse



NASA site with maps of the eclipse:
<https://eclipse2017.nasa.gov/>



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Symmetry and Group Theory

Symmetry is an essentially **geometric** concept.

The mathematical theory of symmetry is **algebraic**.
The key concept is that of a **group**.

A group is a set of elements such that any two elements can be combined to produce another.

Instead of giving the mathematical **definition**,
I give an **example** to make things clear.



A Face with Symmetry: Halle Berry



Halle Berry

Berry Halle



An Asymmetric Face: You know Who!



The *Janus Group*: D_1

The group of symmetries of the human face,
& of all biological forms with **bilateral symmetry**.

I : The Identity transformation

R : Reflection about central line

Table : First Dihedral Group D_1 .

	I	R
I	I	R
R	R	I

This is how we combine, or **multiply** transformations.



Formal Definition of a Group

Let G be a set, for example

$$G = \{g_1, g_2, g_3, g_4\}$$

Suppose we can **combine** two elements of G to get another element of G .

For G to be a group, we must have:

1. Closure
2. Associativity
3. Identity
4. Inverse.



The Book Group (Klein 4-Group)

- I** : Identity transformation
- R** : Rotation about central point
- H** : Rotation about horizontal line
- V** : Rotation about vertical line.

Table : Second Dihedral Group D_2 .

	I	R	H	V
I	I	R	H	V
R	R	I	V	H
H	H	V	I	R
V	V	H	R	I



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Mathematics and Art

The link between maths and art goes back thousands of years.

- Greek Architecture
- Renaissance Painting
- Gothic Cathedrals
- Oriental Carpets
- Islamic Mosaics



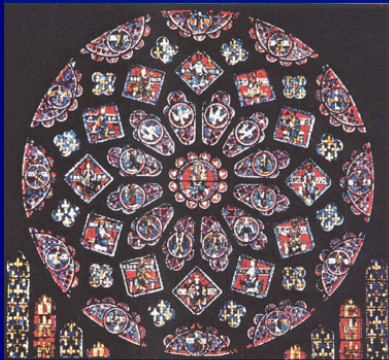
The Parthenon



Chartres cathedral



Rose window, Chartres



Raphael's School of Athens



Mosaics in the Alhambra



Persian Carpet



Alloy Wheels



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**I hope you agree that
maths is Beautiful**

But is it any use?



Useful: Maths is crucial for technology



Find us on:
facebook®



Useful

Maths is used in many aspects of our lives.

Searching for information: Google matrix [algebra].

Facebook & Twitter: Network analysis. Graph Theory.

Download music or photos: Data compression [MP3,JPEG].

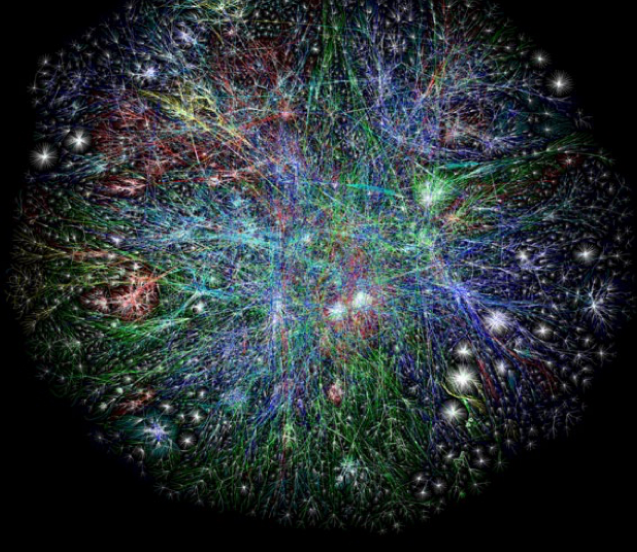
Commerce and Finance: Coding and Cryptography.

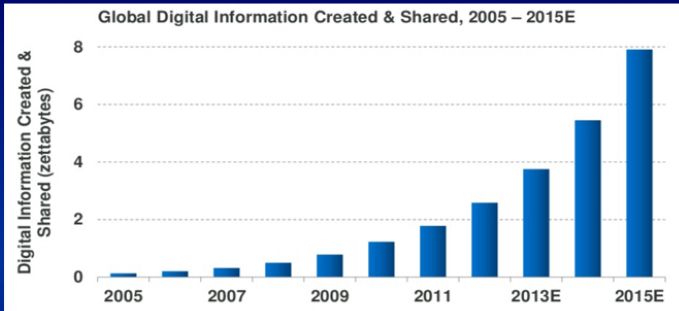
Biology and medicine. CAT Scans. Epidemiology.

Etc. etc. etc.



Useful: Maths is crucial for the Internet

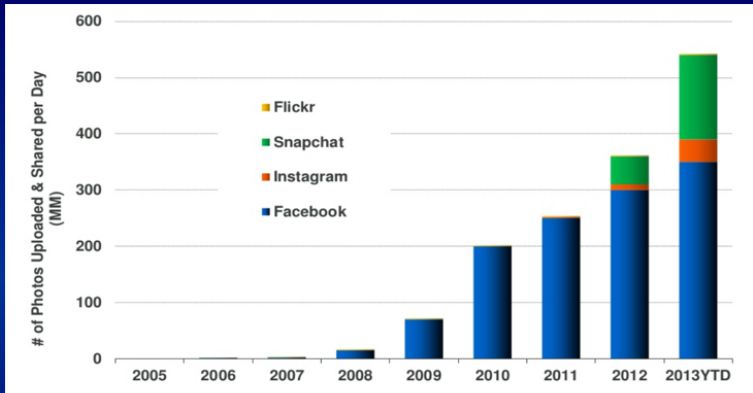




Digital Information is growing exponentially:
> 3 Zbytes shared in 2013.

1 Zettabyte is
 $10^{21} = 1,000,000,000,000,000,000,000$ bytes





500 million photos uploaded EVERY DAY.
That's half a billion !!!



Useful: Maths is crucial for Security



Maths Week 2013



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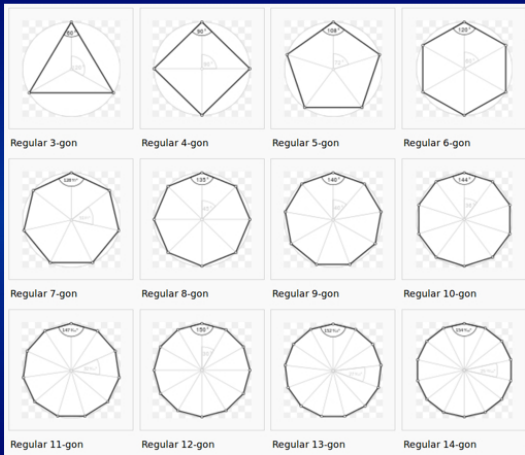


Euler's polyhedron formula.






Carving up the globe.



Regular Polygons



The Platonic Solids (polyhedra)

Tetrahedron (four faces)	Cube or hexahedron (six faces)	Octahedron (eight faces)	Dodecahedron (twelve faces)	Icosahedron (twenty faces)
				

These five regular polyhedra were discovered in ancient Greece, perhaps by **Pythagoras**.

Plato used them as models of the universe.

They are analysed in Book XIII of **Euclid's Elements**.



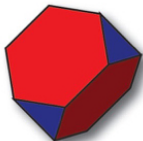


There are only five **Platonic** solids.

But **Archimedes** found, using different types of polygons, that he could construct 13 new solids.



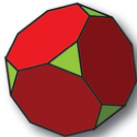
The Thirteen Archimedean Solids



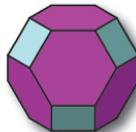
TRUNCATED TETRAHEDRON



CUBOCTAHEDRON



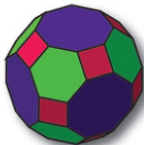
TRUNCATED CUBE



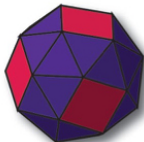
TRUNCATED OCTAHEDRON



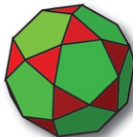
RHOMBICUBOCTAHEDRON



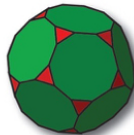
TRUNCATED CUBOCTAHEDRON



SNUB CUBE



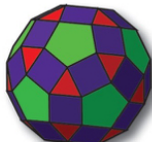
ICOSIDODECAHEDRON



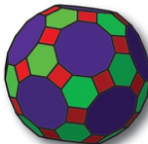
TRUNCATED DODECAHEDRON



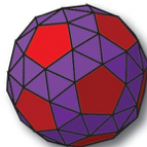
TRUNCATED ICOSAHEDRON



RHOMBICOSIDODECAHEDRON



TRUNCATED ICOSIDODECAHEDRON



SNUB DODECAHEDRON



Euler's Polyhedron Formula

The great Swiss mathematician, **Leonard Euler**, noticed that, for all (convex) polyhedra,

$$V - E + F = 2$$

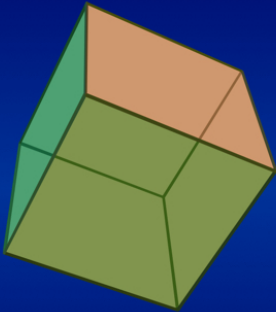
where

- **V** = Number of vertices
- **E** = Number of edges
- **F** = Number of faces

Mnemonic: Very Easy Formula



For example, a Cube



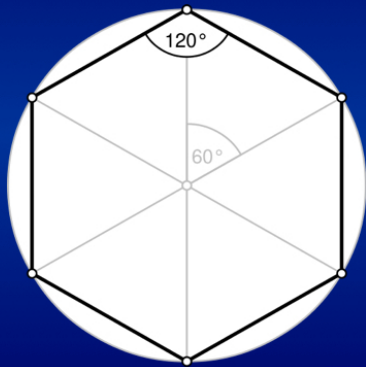
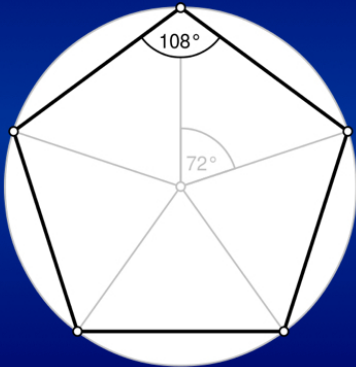
Number of vertices: $V = 8$
Number of edges: $E = 12$
Number of faces: $F = 6$

$$(V - E + F) = (8 - 12 + 6) = 2$$

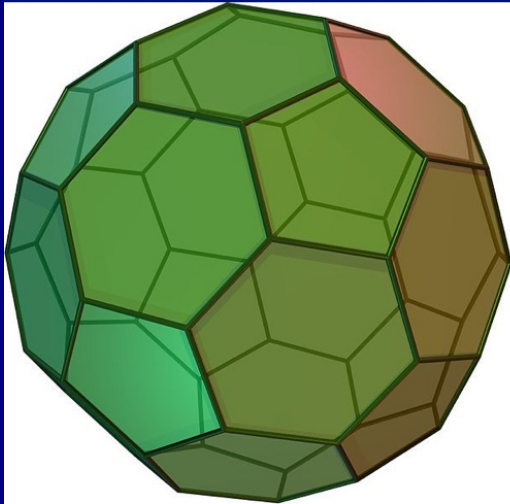
Mnemonic: Very Easy Formula



Pentagons and Hexagons



The Truncated Icosahedron



**An Archimedean solid
with
pentagonal and
hexagonal faces.**



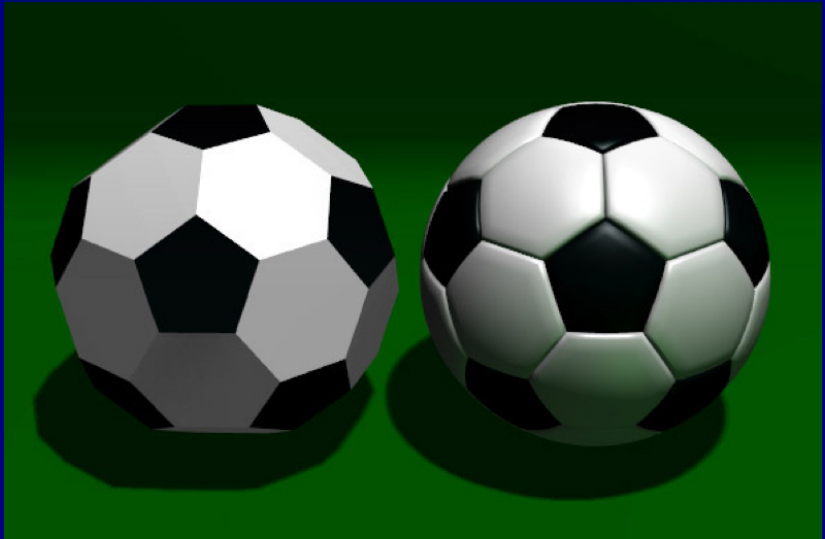
The Truncated Icosahedron



Where have
you seen this
before?



The Truncated Icosahedron





The "**Buckyball**", introduced at the 1970 World Cup Finals in Mexico.

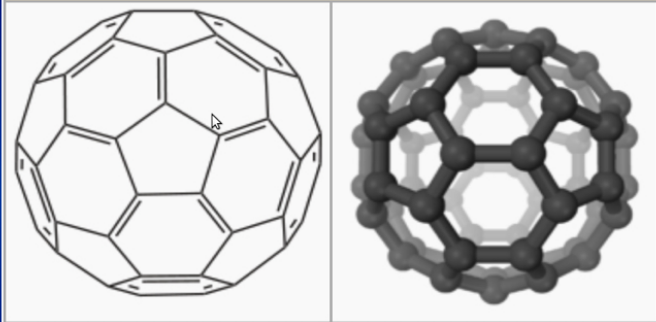
It has 32 panels: 20 hexagons and 12 pentagons.



**A Geodesic Dome designed by the American architect
Richard Buckminster "Bucky" Fuller.**



Buckminsterfullerene



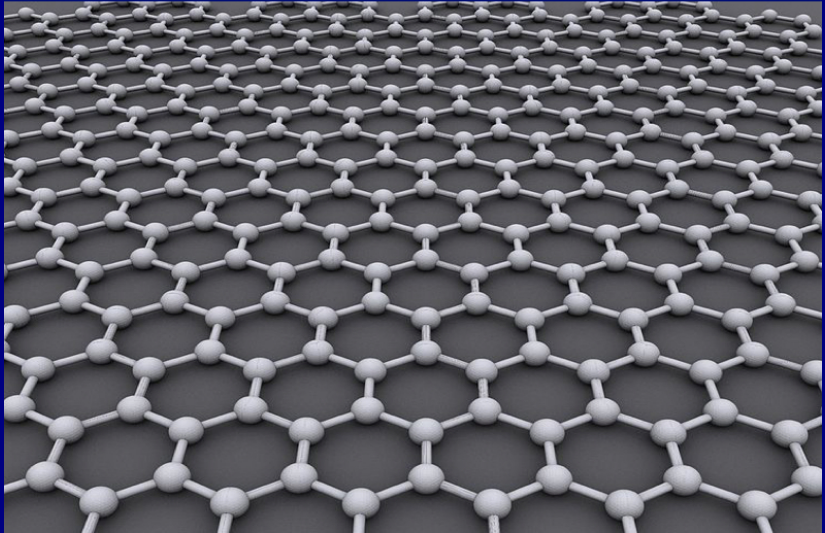
Buckminsterfullerene is a molecule with formula C_{60}

It was first synthesized in 1985.

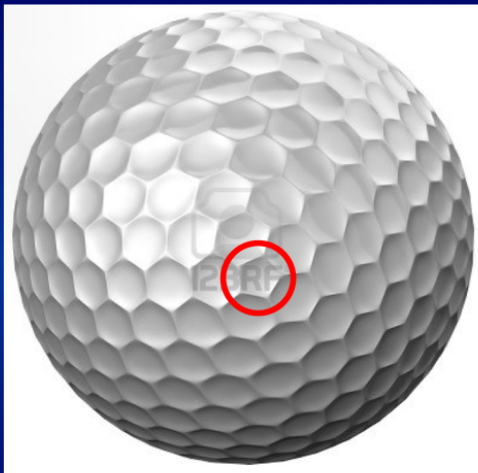


Graphene

A hexagonal pattern of carbon one atom thick



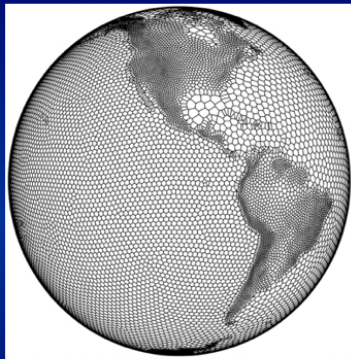




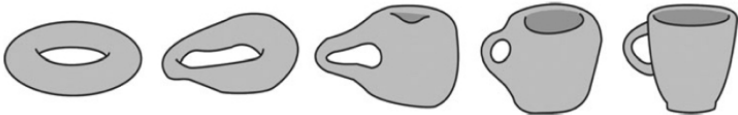
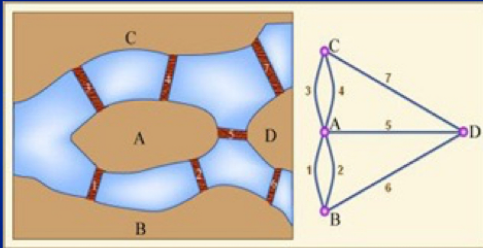
Euler's Polyhedron Formula

$$V - E + F = 2$$

still holds.

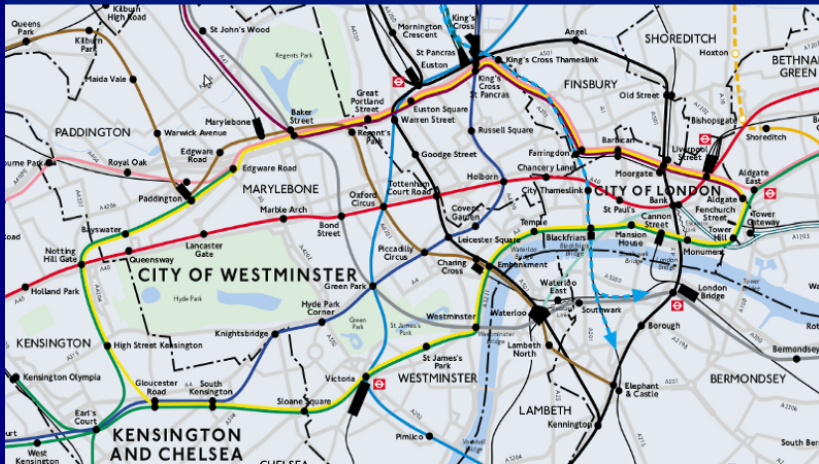


Topology is often called Rubber Sheet Geometry



Topology and the London Underground

Topographical Map



Topology and the London Underground

Topological Map



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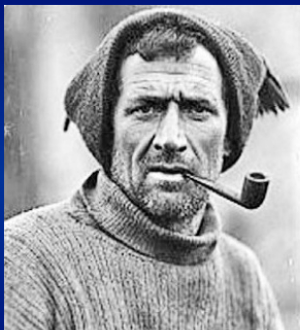
Who is this?



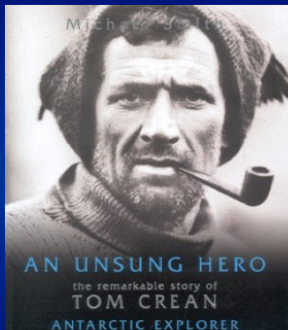
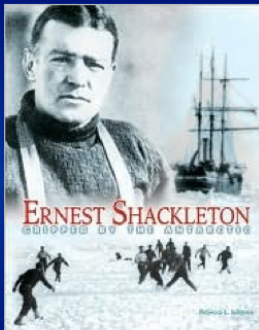
Who is this?



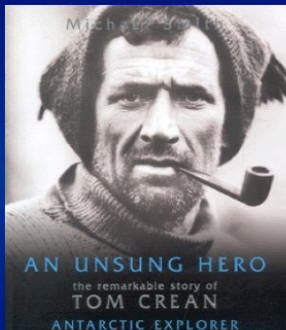
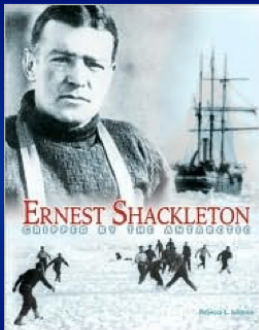
Who is this?



Ernest Shackleton Tom Crean



Ernest Shackleton Tom Crean



Two great Antarctic explorers, both born in Ireland



Shackleton's Imperial Trans-Antarctic Expedition (1914)



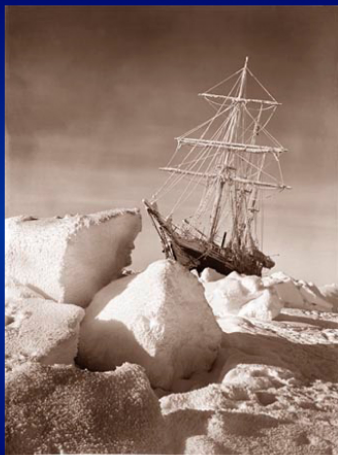
Shackleton's Imperial Trans-Antarctic Expedition (1914)



Shackleton's Imperial Trans-Antarctic Expedition (1914)



Endurance is Icebound



Shackleton's Imperial Trans-Antarctic Expedition (1914)



Shackleton's Imperial Trans-Antarctic Expedition (1914)





Six men sailed 800 miles across the Southern Ocean to South Georgia.



Six men sailed 800 miles across the Southern Ocean to South Georgia.

How did they find their way?

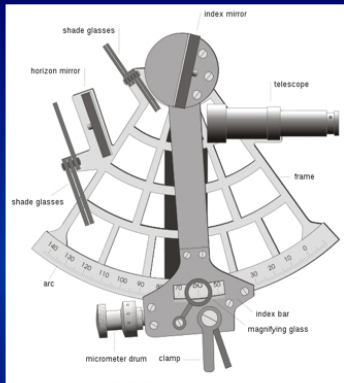


**Six men sailed 800 miles across the
Southern Ocean to South Georgia.**

How did they find their way?

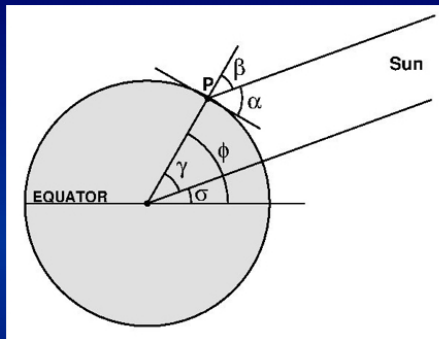
MATHEMATICS !!!





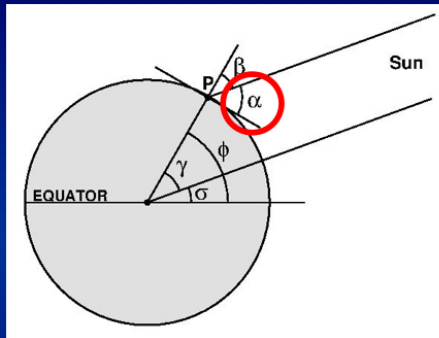
A sextant, used to determine latitude.





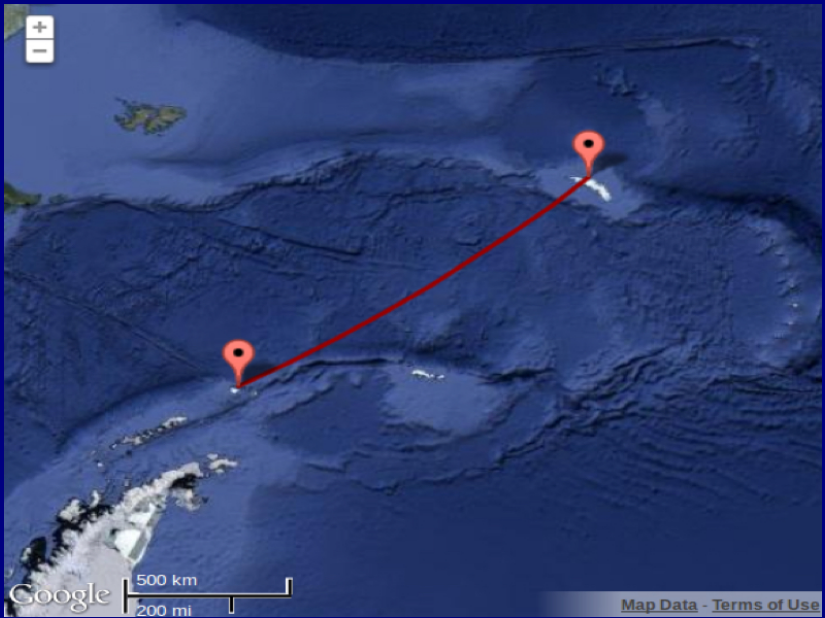
Angles used to calculate the latitude.





Angles used to calculate the latitude.





[Map Data - Terms of Use](#)



**The boat journey to South Georgia
was a spectacular feat of navigation.**

It resulted in the saving of 28 lives.

**This was possible thanks to
elementary geometry.**



**The boat journey to South Georgia
was a spectacular feat of navigation.**

It resulted in the saving of 28 lives.

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That's Maths!



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Recreational Mathematics

Recreational mathematics puts the focus on **insight, imagination and beauty.**

Recreational Maths includes the study of

- ▶ The culture of mathematics,
- ▶ Its relevance to art, music and literature,
- ▶ Its role in technology,
- ▶ Mathematical games and puzzles,
- ▶ The lives of the great mathematicians.



Many Resources Available

Great variety of books on popular mathematics.

Wealth of literature suitable for a general audience

Magazines available free online.

One of the best is the e-zine **Plus:**

<https://plus.maths.org/>.

All past content is available and is a valuable resource for school students and teachers.



Content of Last Year's Course

Lecture	Content
1	Outline of Course. Emergence of Numbers.
2	Georg Cantor. Set Theory.
3	Pythagoras. Irrational Numbers.
4	Hilbert. Gauss. The Real Number Line
5	Powers. Logarithms. Prime Numbers.
6	Functions. Archimedes. Natural Logs.
7	Exponential Growth. Euler. Sequences & Series.
8	Trigonometry. Taylor Series.
9	Basel Problem. Complex Numbers. Euler's Formula.
10	Prime Number Theorem. Riemann Hypothesis.

**This years course will be different.
If you want to know how, **come along!****



Thank you

