AweSums:

The Majesty of Mathematics

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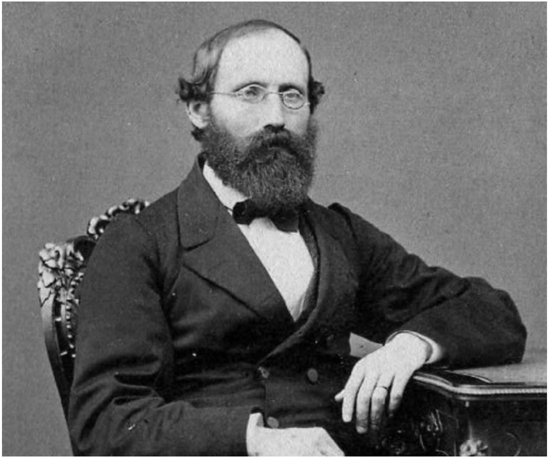
Evening Course, UCD, Autumn 2016

Outline

Outline

AweSums: The Majesty of Maths



Bernhard Riemann (1826-66)

Outline

Names for the Course

- Maths for Everyone
- The Fun of Maths
- Recreational Maths
- Our Mathematical World
- The History and Development of Maths
- Mathematics: Beautiful, Useful & Fun

AweSums: The Majesty of Maths

Notes and Slides

- **All the slides will be available online:**
  http://mathsci.ucd.ie/∼plynch/AweSums
- **No Notes are to be provided.**
  Why Not? See next slide.
- **Additional Reading Recommendations.**
- **Optional Exercises and Problems.**
- **No Assignments!**
- **No Assessments!**
- **No Examinations!**

Why No Notes?

- **Maths is NOT a Spectator Sport**
- **Active engagement is essential to understanding.**
- ***You should take your own notes:***
  - **This involves repetition of what you hear.**
  - **This involves repetition of what you see.**
  - **What you write passes through your mind!**
  - **This process is a great help to memory.**

Lectures

- Classes run from 7pm to 9pm.
- 120 minutes intensive lecturing too long
  (both for you and for me).
- Educational Theory:
  - Attention & retention both decrease with time.
- Class will be broken into short sections.

If you cannot attend a class:

- There is no need to offer reasons.
- Please do not bother to email me.
- The presentation slides will be available.

Typical Structure of a Class

1. Problem: Background and Theory (30 min)
2. Distraction (10 min)
3. Some History of the problem/theorem (30 min)
4. Another Distraction (10 min)
5. Exercises, Puzzles, History (20 min)
6. Questions & Discussion (20 min)

Total duration: about 120 minutes.

I will (normally) be available after classes to answer questions or offer clarifications.

Some Distractions

- Visual Awareness: Maths Everywhere
- Puzzles: E.g. Watermelon Puzzle
- Sieve of Eratosthenes
- The Greek Alphabet
- Lateral Thinking in Maths
- *Lecture sans paroles*
- How Cubic and Quartic Equations were cracked
- Four-colour Theorem
- Online Encyclopedia of Integer Sequences

Please ask me if you have a favorite topic!

It's Your Course

I expect a group with a wide range of knowledge and "mathematical maturity".

Everybody should benefit from the course.

If something is unclear, shout out!

If something is missing, let me know.

Feedback on the course is very welcome.

It's Your Course

Classes begin at 7 pm. and run till 9 pm.

Pi Restaruant (downstairs) closes at 7:30 (?).

There seem to be several options:

- Break at 7:20 for 20 minutes.
- Break at 7:50 for 15 or 20 minutes.
- Don't break at all !!!

I have no strong views, except that Option (1) is a bit soon after the beginning, and leaves a long session (c. 100 min) after the break.

Let's have a show of hands.

Popular Mathematics Books

1. **John H Conway and Richard K Guy, 1996:**
   The Book of Numbers. **Copernicus, New York.**
2. $\heartsuit \Rightarrow$ **John Darbyshire, 2004:**
   Prime Obsession. **Plume Publishing.**
3. $\heartsuit \Rightarrow$ **William Dunham, 1991:**
   Journey through Genius. **Penguin Books.**
4. **Marcus Du Sautoy, 2004:**
   The Music of the Primes. **Harper Perennial.**
5. $\heartsuit \Rightarrow$ **Richard Elwes, 2010:**
   Mathematics 1001. **Firefly Books.**
6. **Peter Lynch, 2016:**
   That's Maths. **Gill Books**
   (To be published in October 2016).

Outline

Distraction 1: Remember π

To 15-figure accuracy, π is equal to

$$3.14159265358979$$

How can we remember this without much effort?

Just remember this:

> ***How I want a drink,***
> ***Alcoholic of course,***
> ***After the heavy lectures***
> ***involving quantum mechanics.***

Explain this *piem* on the blackboard

Distraction 1: Remember π

How I want a drink,
Lemonsoda of course,
After the heavy lectures
involving quantum mechanics.

How I want a drink,
Sugarfree of course,
After the heavy lectures
involving quantum mechanics.

Repeat: To Remember π

To 15-figure accuracy, π is equal to

$$3.14159265358979$$

How can we remember this without much effort?

Just remember this:

How I want a drink,
Alcoholic of course,
After the heavy lectures
involving quantum mechanics.

Distraction 1: Remember $1/\pi$

The reciprocal of π is approximately 0.318310
Can I remember the reciprocal?

How I remember the reciprocal!
3 1 8 3 10

Now you know π and $1/\pi$ to an accuracy greater than you are ever likely to need!

Outline

The Greek Alphabet, Part 1

Ελληνικό αλφάβητο

Some Motivation

- Greek letters are used extensively in maths.
- Greek alphabet is the basis of the Roman one.
- Also the basis of the Cyrillic and others.
- A great advantage for touring in Greece.
- You already know several of the letters.
- It is simple to learn in small sections.

Letter	Name	Sound	
		Ancient[5]	Modern[6]
Α α	alpha, άλφα	[a] [aː]	[a]
Β β	beta, βήτα	[b]	[v]
Γ γ	gamma, γάμμα	[g], [ŋ][7]	[ɣ] ~ [ʝ], [ŋ][8] ~ [ɲ][9]
Δ δ	delta, δέλτα	[d]	[ð]
Ε ε	epsilon, έψιλον	[e]	[e]
Ζ ζ	zeta, ζήτα	[zd][A]	[z]
Η η	eta, ήτα	[ɛː]	[i]
Θ θ	theta, θήτα	[tʰ]	[θ]
Ι ι	iota, ιώτα	[i] [iː]	[i], [ʝ],[10] [ɲ][11]
Κ κ	kappa, κάππα	[k]	[k] ~ [c]
Λ λ	lambda, λάμδα	[l]	[l]
Μ μ	mu, μυ	[m]	[m]

Letter	Name	Sound	
		Ancient[5]	Modern[6]
Ν ν	nu, νυ	[n]	[n]
Ξ ξ	xi, ξι	[ks]	[ks]
Ο ο	omicron, όμικρον	[o]	[o]
Π π	pi, πι	[p]	[p]
Ρ ρ	rho, ρώ	[r]	[r]
Σ σ/ς[13]	sigma, σίγμα	[s]	[s]
Τ τ	tau, ταυ	[t]	[t]
Υ υ	upsilon, ύψιλον	[y] [yː]	[i]
Φ φ	phi, φι	[pʰ]	[f]
Χ χ	chi, χι	[kʰ]	[x] ~ [ç]
Ψ ψ	psi, ψι	[ps]	[ps]
Ω ω	omega, ωμέγα	[ɔː]	[o]

Figure : The Greek Alphabet (from Wikipedia)

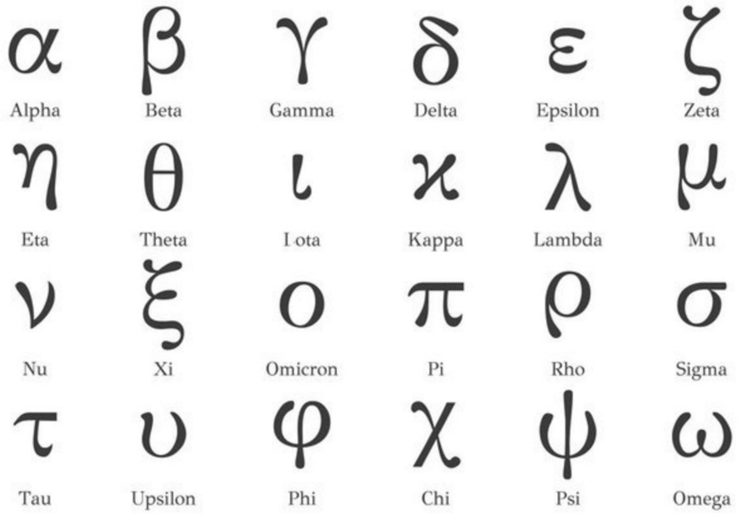


Figure : 24 beautiful letters

The First Six Letters

We will take the alphabet in groups of six letters.

α	β	γ	δ	ϵ	ζ
A	B	Γ	Δ	E	Z

Let us focus first on the *small letters*
and come back to the big ones later.

You know α and β from the word *Alphabet:* α β

You have heard of *gamma-rays*, or γ-rays

Both δ and ϵ are widely used in maths. For example, the definition of *continuity* of function $f(x)$ at $x = a$ is

$$\forall \epsilon > 0 \; \exists \delta > 0 : |x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$$

A central theme of this course, Riemann's Hypothesis is concerned with zeros of the *Riemann zeta-function:*

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$$

Now we already know the first six letters!

End of Greek 101

Outline

The Ancient Origins of Mathematics

Basic social living was possible without numbers

. . . but . . .

elementary comparisons and measures are needed to ensure fairness and avoid conflicts.

The need for mathematical thinking arose in problems like fair division of food.

Problem: How do you divide a woolly mammoth?

Division of Food

To divide a collection of apples, the idea of a *one-to-one correspondence* arose.

There was no direct need for *numbers* yet: the apples did not need to be counted, just broken into batches.

The problem of dividing up a slaughtered animal is more tricky: The forequarters and hindquarters of a woolly mammoth are not the same!

I Cut and You Choose

For two people or two families, the familiar technique "I cut and you choose" could keep everyone happy.

This is the method used by children to divide a cake. It works even for an inhomogeneous cake, say half chocolate and half lemon sponge.

To divide fairly between all members of a family is *much more difficult* (as many of you know!).

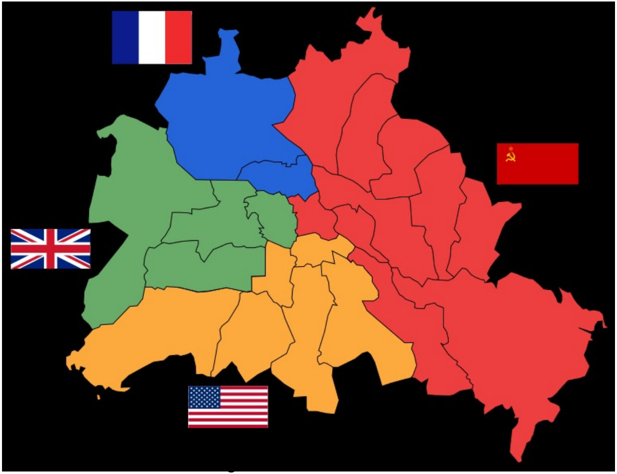
Exercise: Try to devise a generalization of the "cut-and-choose" method that works for three people . . . and one that works for four people.

This is a dfficult problem

It seems like an abstract problem, but it has practical implications:

Consider the partition of Berlin

Partition of Berlin (Potsdam Agreement, 1945)



Partition of Germany (Potsdam Agreement, 1945)

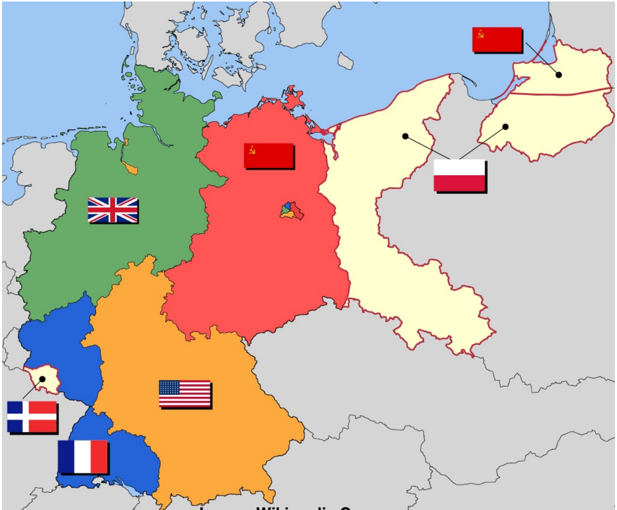
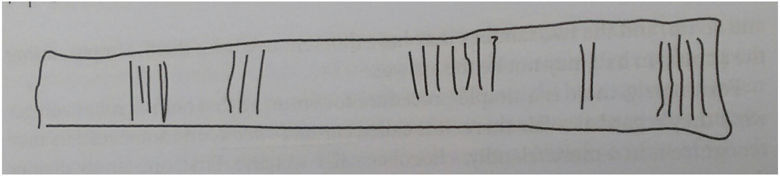


Image: Wikimedia Commons

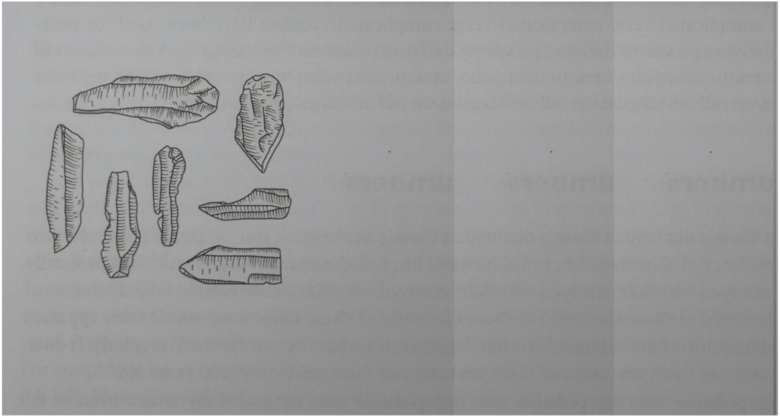
Tally Sticks

Keeping an account of sheep and such animals was done using a tally stick. The number of notches corresponded to the number of sheep.

Again, for small flocks, no concept of *actual numbers* was essential.



Keeping Stock without Counting



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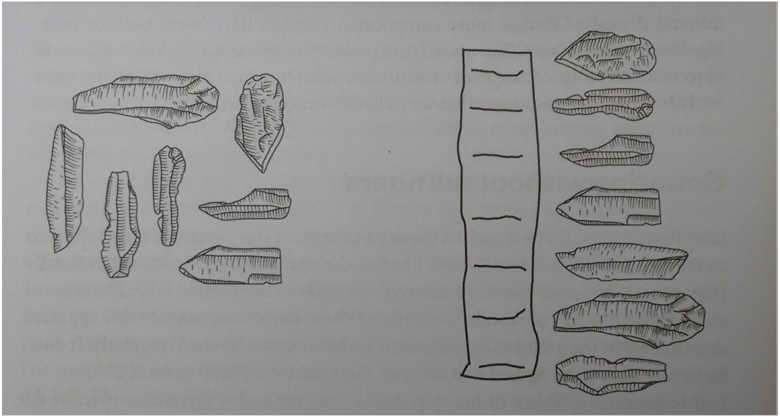


The origin of the number line ???

Numbers

At some stage, the general notion of a number arose. Even in considering the fingers of a hand, numbers up to five would arise.

Gradually the idea of five as a concept would emerge. Placing two hands together immediately gives us the idea of a one-to-one correspondence:

Both hands have five fingers.

Through repetition and familiarity, the concept of five would become natural. Any set of objects that are in one-to-one correspondence with the fingers of the hand must have five elements.

Numerals

Gradually all the small natural numbers,
at least up to about 10, came into use.

Sometimes, the connection between say two sheep and two bushels of corn was obscured.

Irish has distinct words for two apples and two people

Eventually, numerals, or symbols for the numbers, emerged.

Much numerical material is found in writings from Mesopotamia and from Ancient Egypt.

Bartering & Money

One group might have surplus *fish*
while another group have excess *fruit*.
Both gain by agreeing to an exchange.

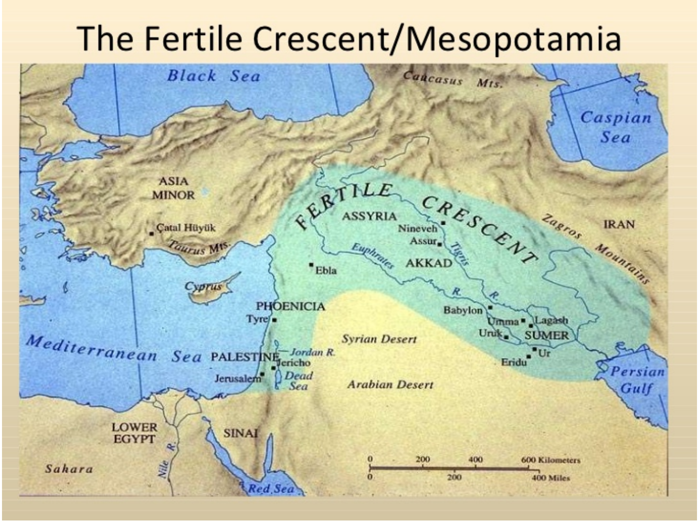
A common measure was needed. This eventually led to the idea of money.

In several cultures, objects like cowrie shells were used as a medium of exchange.

In some cases, the currency had some inherent value or at least scarcity. In others, it had not.

Exercise: Discuss the opinion of Aristotle in his *Ethics:* "With money we can measure everything."

The Fertile Crescent



Mesopotamia

Loosely called the Babylonian civilisation.

A vast number of cuneiform tablets survive.

The Babylonian numerical system used 60 as its base. Why?

It is uncertain why, but reasonable to speculate that, since there there are about 360 days in a year 60 was chosen to facilitate astronomical calculations.

The Sexagesimal System

The great advantage is that 60 has many divisors:
1, 2, 3, 4, 5, 6, 8, 10, 12,15, 20, 30.

This obviously facilitates all the division problems.

In Babylon, they wrote 70 = [1 | 10] and 254 = [4 | 14]

We can add these: 324 = [5 | 24].

Thus, basic arithmetic is possible with this system.

Time Measurement

Development of accurate *calendars* required mathematical development.

The relationship between days and months and years is not so simple.

Time and season could be measured by the length of shadow cast by a fixed pole.

Eventually this led to the *great obelisks* being erected in Egypt.

Exercise: Find out how high the Spire is. Using public web-cams, could it be used as a time-piece?

Time Measurement

There is a hangover from the sexagesimal system in our ‘modern’ units:

We have 60 seconds in a minute and 60 minutes in an hour.

We have 360 degrees in a circle so our latitude and longitude are influenced by Babylonian mathematics.

Can you think of any other examples?

Outline

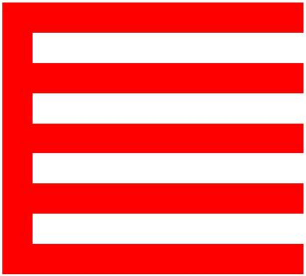
Source of Some Puzzles

Mathematical Lateral Thinking Puzzles
by
Paul Slone & Des MacHale

Slicing a Cake with One Cut

Can you bake a cake that you can slice into 6 equal pieces with only one knife-cut?

Hint: The cake can be any shape you like



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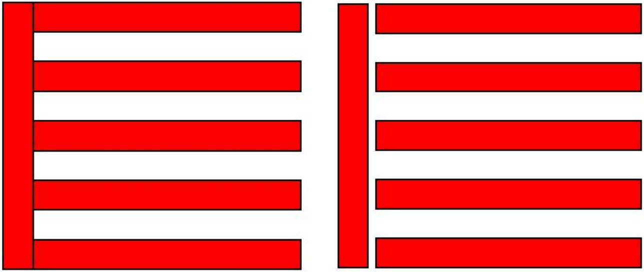
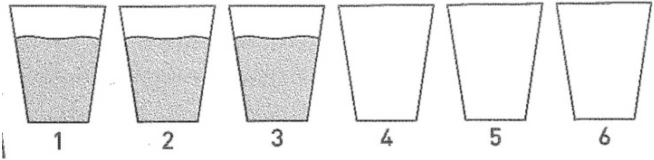


Rearrange Six Glasses



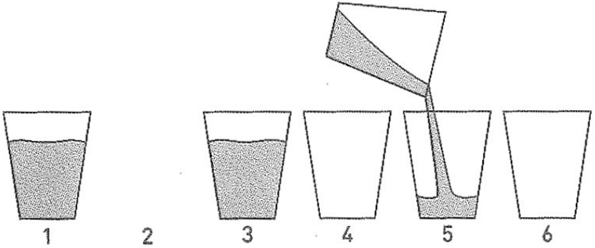
There are six glasses in a row.

Glasses 1, 2 and 3 are full.
Glasses 4, 5 and 6 are empty.

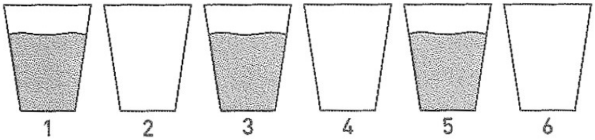
How can you arrange for the full and empty glasses to alternate, *moving only one glass?*

Rearrange Six Glasses

First, pour water from Glass 2 into glass 5:



Then, place Glass 2 in its original position:



Thank you