

On the generation of nonlinear 3D interfacial waves in gas-liquid flows

Lennon Ó Náraigh ¹, Peter Spelt ²

¹ School of Mathematics and Statistics and CASL, University College Dublin

² LMFA, Ecole Centrale de Lyon

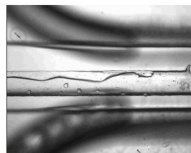
8th September 2015

Context

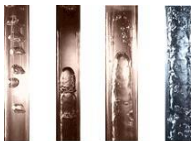
Two-phase stratified flow is ubiquitous in nature and industry.



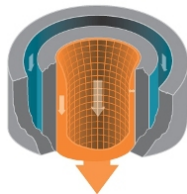
(a) Kelvin-Helmholtz instability



(b) Stratified flow in pipelines



(c) Slug flow

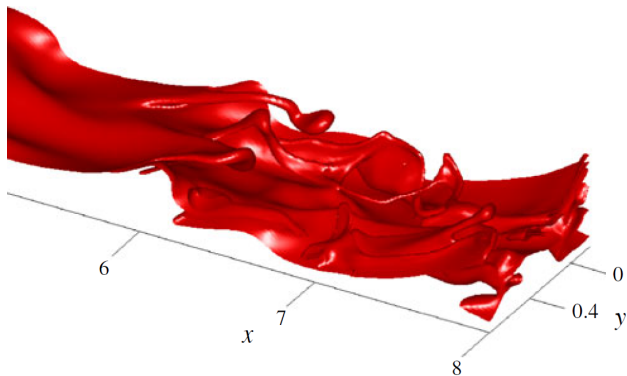


(d) Falling-film reactors

- Mathematically, and computationally, a tough problem – turbulence, extreme nonlinearity, topological change in interfaces, a range of instabilities that need to be captured.
- Even the laminar regime is tough - current focus of the research.

The numerical challenge

- Flows involving many length- and time-scales
- Flows with sharp changes in interfacial topologies
- Transient three-dimensional simulations required over long periods of time, requiring **scalable** codes run at very high **resolutions**.



TPLS

Numerical solution of two-phase Navier–Stokes equations with interface capturing:

$$\rho(\phi) \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \frac{1}{Re} \nabla \cdot \left[\mu(\phi) \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \mathbf{f}_{st}(\phi) + \rho(\phi) \mathbf{g},$$

where $\nabla \cdot \mathbf{u} = 0$ and ϕ is the interface-capturing field:

TPLS

Numerical solution of two-phase Navier–Stokes equations with interface capturing:

$$\rho(\phi) \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \frac{1}{Re} \nabla \cdot \left[\mu(\phi) \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \mathbf{f}_{st}(\phi) + \rho(\phi) \mathbf{g},$$

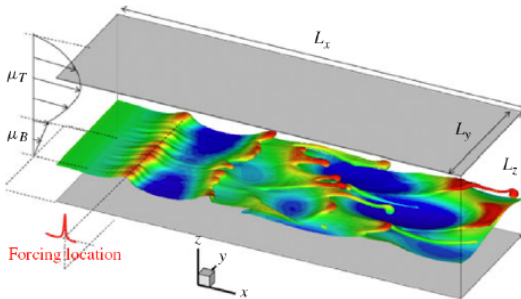
where $\nabla \cdot \mathbf{u} = 0$ and ϕ is the interface-capturing field:

Levelset method:

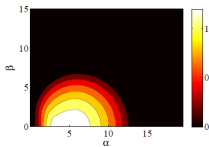
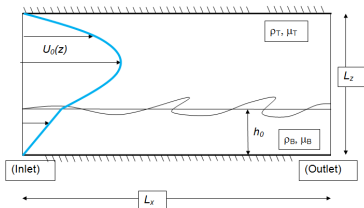
$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0, \quad \mathbf{f}_{st} = \delta_\epsilon(\phi) \frac{1}{We} \hat{\mathbf{n}} \nabla \cdot \hat{\mathbf{n}}, \quad \hat{\mathbf{n}} = \frac{\nabla \phi}{|\nabla \phi|}.$$

Problem geometry and configuration

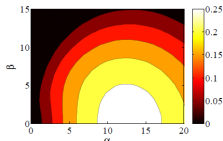
- Simple channel geometry: periodic OR inlet/outlet conditions at $x = 0$, $x = L_x$; walls (no slip) at $z = 0$, $z = L_z$.
- Basic version involves hydrodynamics only. TPLS with physics available, e.g. evaporating droplets, contact-line dynamics, mass transfer.



Where do 3D waves in parallel flows come from?

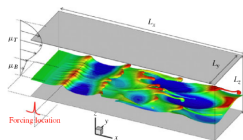


$$r = 1, S = 0.1$$



$$r = 1000, S = 0.1$$

Linear instability of 2D parallel flow is dominated by 2D waves.
So how do 3D structures form?



We want to keep an open mind and examine all possibilities.

- Direct route for supercritical cases – wherein linear stability analysis predicts 2D and 3D waves are present in more-or-less equal strengths. **This is found to play an important role across a variety of density ratios and parameter values.**

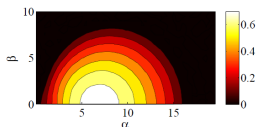
We want to keep an open mind and examine all possibilities.

- Direct route for supercritical cases – wherein linear stability analysis predicts 2D and 3D waves are present in more-or-less equal strengths. **This is found to play an important role across a variety of density ratios and parameter values.**
- Subcritical transition to 3D state – **weakly nonlinear mechanisms excite 3D waves even though linear theory says they shouldn't be there.**

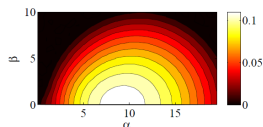
We want to keep an open mind and examine all possibilities.

- Direct route for supercritical cases – wherein linear stability analysis predicts 2D and 3D waves are present in more-or-less equal strengths. **This is found to play an important role across a variety of density ratios and parameter values.**
- Subcritical transition to 3D state – **weakly nonlinear mechanisms excite 3D waves even though linear theory says they shouldn't be there.**
- Also, investigate possibility of **secondary instability.**

The direct route – linear stability analysis



(a) $r = 100, \mathcal{S} = 0.1$



(b) $r = 1000, \mathcal{S} = 0.1$

Eigenvalue analysis of the two-phase Orr-Sommerfeld-Squire equations for $Re = 100$, $m = 30$, $h_0 = 0.3$, and $\mathcal{S} = 0.1$, and $\mathcal{G} = 0.1$.

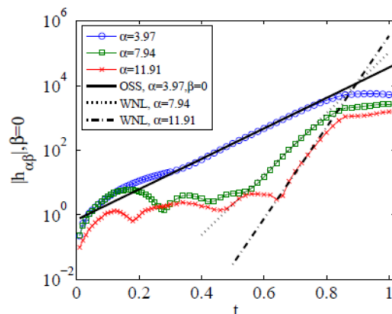
The direct route is most important at high density ratios

$$r = \frac{\rho_B}{\rho_T},$$

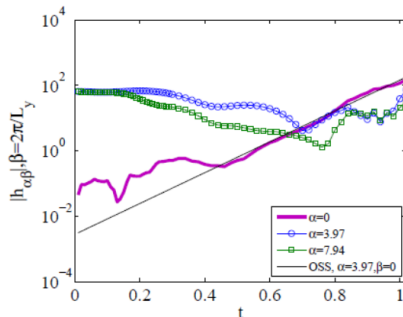
with $r > 1$ for a gas-liquid flow.

Weakly nonlinear route below ‘criticality’

Streamwise waves – Large temporal growth, Spanwise waves – No temporal growth rate



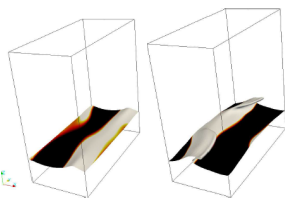
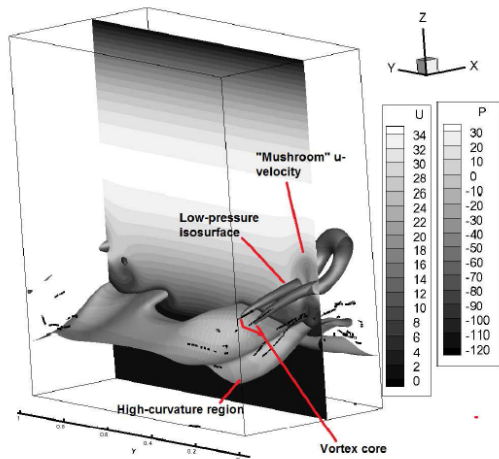
- Streamwise overtones are enslaved to the streamwise dominant mode



- Purely spanwise mode enslaved to the dominant streamwise mode

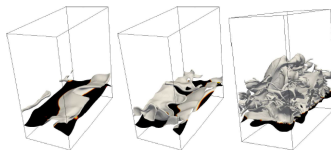
Most relevant for liquid-liquid flows. Periodic boundary conditions, $(Re, m, r, S) = (300, 30, 1, 0.3)$.

More exotic possibilities for gas-liquid flows



(a) $t = 0.4$

(b) $t = 0.5$



(c) $t = 0.56$

(d) $t = 0.6$

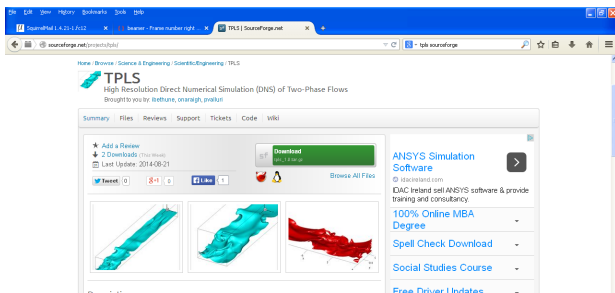
(e) $t = 0.7$

Evidence of **Gortler vortices** forming as a secondary instability.

Conclusions

- A variety of routes to 3D waves in strongly 2D systems - direct route, weakly nonlinear route, and secondary instability.
- Direct route is a strong source of 3D waves
- Weakly nonlinear route – most important for liquid-liquid flows
- Other interesting routes – via secondary instability – for gas-liquid flows.

TPLS is open-source – new collaborations / applications always welcome!



The screenshot shows a web browser window displaying the SourceForge project page for TPLS. The browser's address bar shows the URL `sourceforge.net/projects/tpls`. The page header includes navigation links: Home, Browse, Science & Engineering, Scientific Engineering, TPLS. The main content area features the TPLS logo and the title "High Resolution Direct Numerical Simulation (DNS) of Two-Phase Flows", attributed to `istefune, oneargh, prakun`. Below the title are tabs for Summary, Files, Reviews, Support, Tickets, Code, and Wiki. A "Download" button is visible, along with social media sharing options for Twitter, Facebook, and LinkedIn. Three 3D simulation visualizations are shown: a long, narrow channel with a blue wave, a 3D surface plot of a wave, and a red 3D surface plot. On the right side of the page, there is a sidebar with advertisements for "ANSYS Simulation Software" and "100% Online MBA Degree".