On the generation of nonlinear 3D interfacial waves in gas-liquid flows

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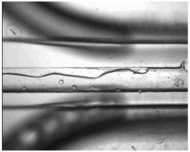
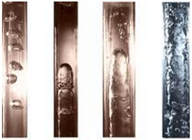
Context

Two-phase stratified flow is ubiquitous in nature and industry.

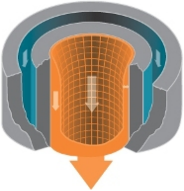
(a) Kelvin-Helmholtz instability



(b) Stratified flow in pipelines



(c) Slug flow

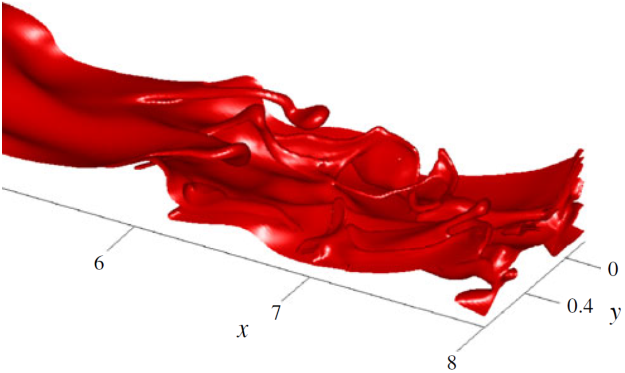


(d) Falling-film reactors

- Mathematically, and computationally, a tough problem – turbulence, extreme nonlinearity, topological change in interfaces, a range of instabilities that need to be captured.
- Even the laminar regime is tough - current focus of the research.

The numerical challenge

- Flows involving many length- and time-scales
- Flows with sharp changes in interfacial topologies
- Transient three-dimensional simulations required over long periods of time, requiring **scalable** codes run at very high **resolutions**.



TPLS

Numerical solution of two-phase Navier–Stokes equations with interface capturing:

$$\rho(\phi)\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \frac{1}{Re}\nabla\cdot\left[\mu(\phi)\left(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T\right)\right] + \boldsymbol{f}_{\mathrm{st}}(\phi) + \rho(\phi)\boldsymbol{g},$$

where $\nabla\cdot\boldsymbol{u} = 0$ and ϕ is the interface-capturing field:

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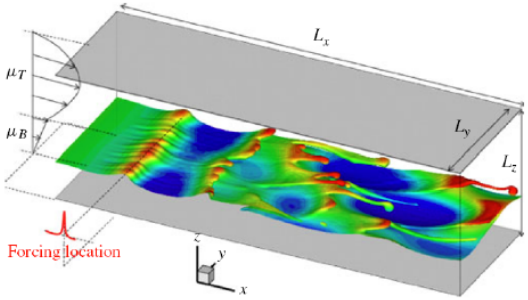
Levelset method:

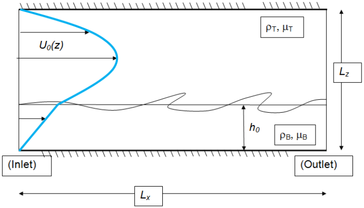
$$\frac{\partial\phi}{\partial t}+\boldsymbol{u}\cdot\nabla\phi=0,\qquad \boldsymbol{f}_{\mathrm{st}}=\delta_{\epsilon}(\phi)\frac{1}{We}\hat{\boldsymbol{n}}\nabla\cdot\hat{\boldsymbol{n}}.\qquad \hat{\boldsymbol{n}}=\frac{\nabla\phi}{|\nabla\phi|}.$$

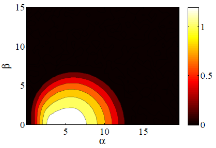
Problem geometry and configuration

- Simple channel geometry: periodic OR inlet/outlet conditions at $x = 0$, $x = L_x$; walls (no slip) at $z = 0$, $z = L_z$.
- Basic version involves hydrodynamics only. TPLS with physics available, e.g. evaporating droplets, contact-line dynamics, mass transfer.

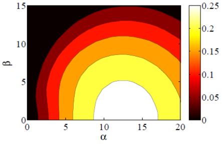


Where do 3D waves in parallel flows come from?



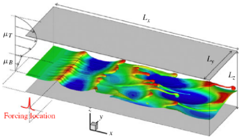


$r = 1, \mathcal{S} = 0.1$



$r = 1000, \mathcal{S} = 0.1$

Linear instability of 2D parallel flow is dominated by 2D waves.
So how do 3D structures form?



We want to keep an open mind and examine all possibilities.

- Direct route for supercritical cases – wherein linear stability analysis predicts 2D and 3D waves are present in more-or-less equal strengths. **This is found to play an important role across a variety of density ratios and parameter values.**

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- Direct route for supercritical cases – wherein linear stability analysis predicts 2D and 3D waves are present in more-or-less equal strengths. **This is found to play an important role across a variety of density ratios and parameter values.**
- Subcritical transition to 3D state – **weakly nonlinear mechanisms excite 3D waves even though linear theory says they shouldn't be there.**

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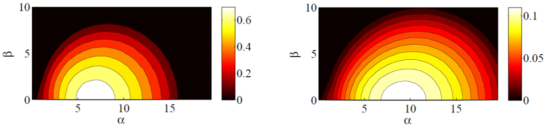
- Subcritical transition to 3D state – **weakly nonlinear mechanisms excite 3D waves even though linear theory says they shouldn't be there.**
- Also, investigate possibility of **secondary instability.**

The direct route – linear stability analysis



(a) $r = 100, \mathcal{S} = 0.1$ (b) $r = 1000, \mathcal{S} = 0.1$

Eigenvalue analysis of the two-phase Orr–Sommerfeld–Squire equations for $Re = 100$, $m = 30$, $h_0 = 0.3$, and $\mathcal{S} = 0.1$, and $\mathcal{G} = 0.1$.

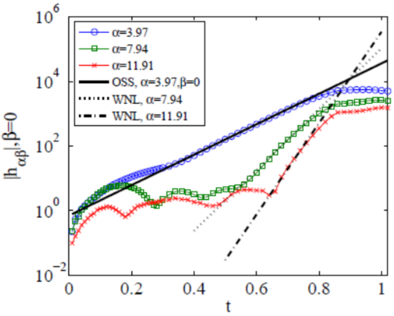
The direct route is most important at high density ratios

$$r = \frac{\rho_B}{\rho_T},$$

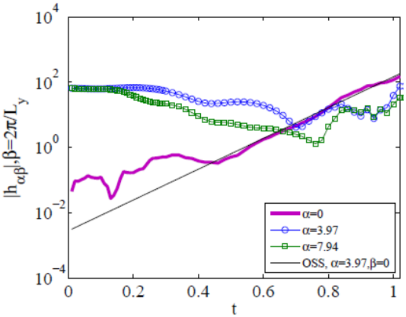
with $r > 1$ for a gas-liquid flow.

Weakly nonlinear route below 'criticality'

Streamwise waves – Large temporal growth, Spanwise waves – No temporal growth rate



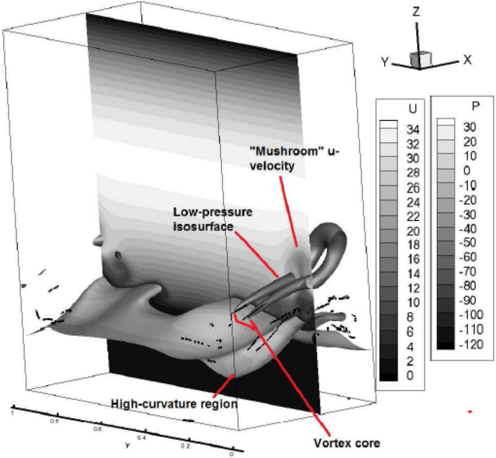
- Streamwise overtones are enslaved to the streamwise dominant mode

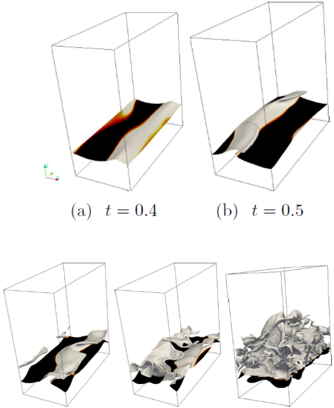


- Purely spanwise mode enslaved to the dominant streamwise mode

Most relevant for liquid-liquid flows. Periodic boundary conditions, $(Re, m, r, \mathcal{S}) = (300, 30, 1, 0.3)$.

More exotic possibilities for gas-liquid flows




(a) $t = 0.4$ (b) $t = 0.5$ (c) $t = 0.56$ (d) $t = 0.6$ (e) $t = 0.7$

Evidence of **Gortler vortices** forming as a secondary instability.

Conclusions

- A variety of routes to 3D waves in strongly 2D systems - direct route, weakly nonlinear route, and secondary instability.
- Direct route is a strong source of 3D waves
- Weakly nonlinear route – most important for liquid-liquid flows
- Other interesting routes – via secondary instability – for gas-liquid flows.

TPLS is open-source – new collaborations / applications always welcome!