Direct numerical simulation of interfacial instability in gas-liquid flows

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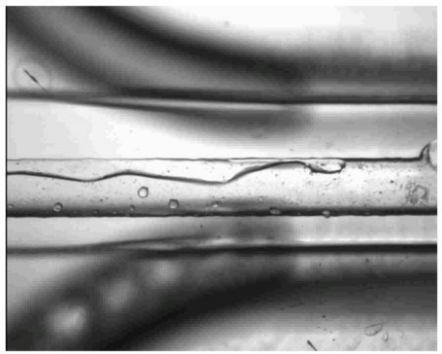
15th May 2017

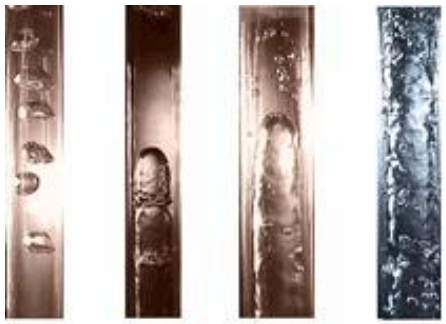
Context

Two-phase stratified flow is ubiquitous in nature.

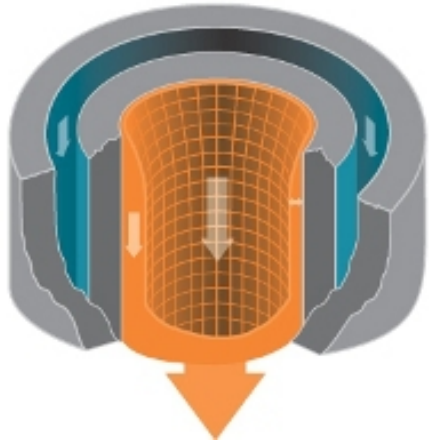
(a) Kelvin-Helmholtz instability



(b) Stratified flow in pipelines



(c) Slug flow

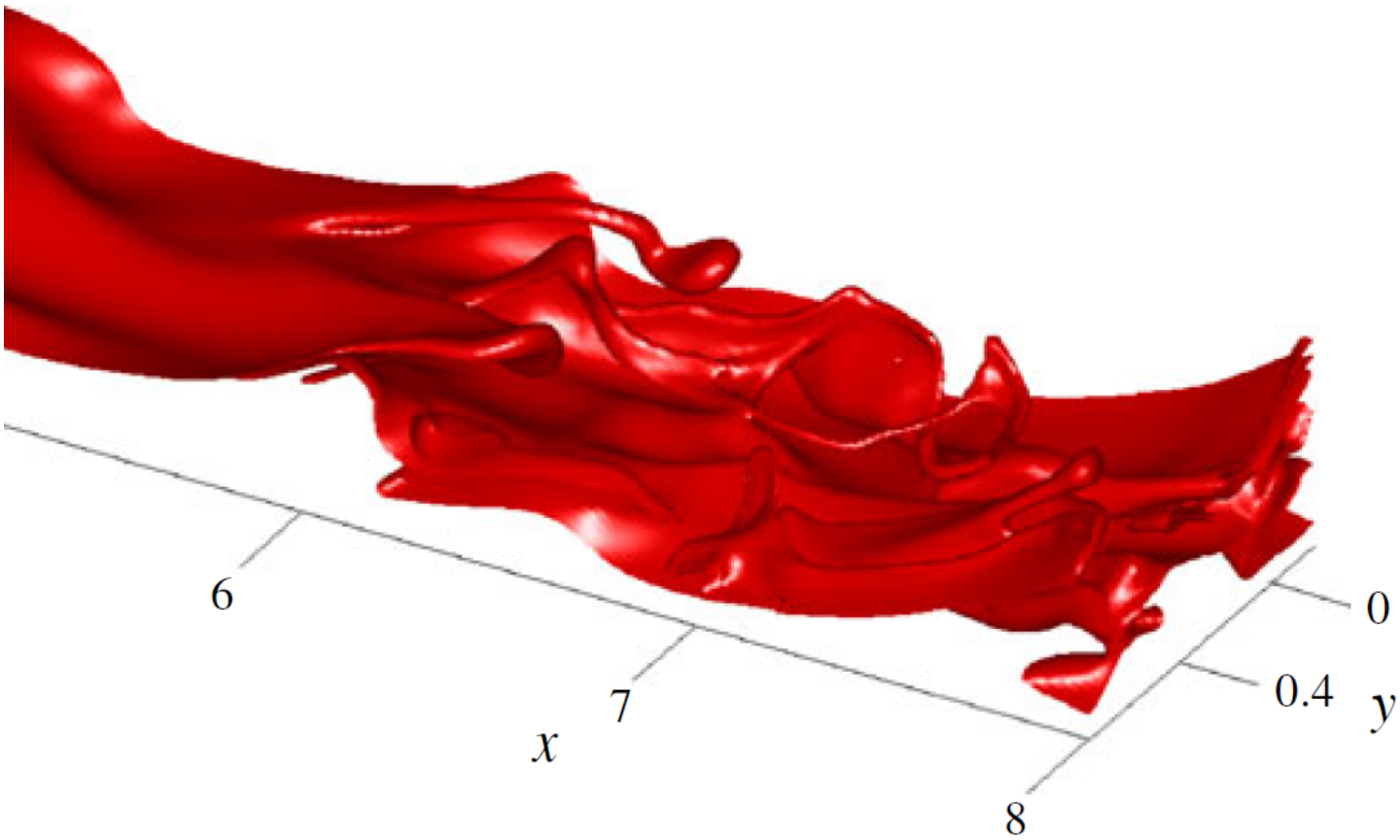


(d) Falling-film reactors

- Mathematically, and computationally, a tough problem – turbulence, extreme nonlinearity, topological change in interfaces, a range of instabilities that need to be captured.
- Even the laminar regime is tough - current focus of the research.

Context: The numerical challenge

- Flows involving many length- and time-scales
- Flows with sharp changes in interfacial topologies
- Transient three-dimensional simulations required over long periods of time, requiring **scalable** codes run at very high **resolutions**.



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- Not a silver bullet – levelset methods – tradeoff between capturing interfacial topology with great fidelity, and mass loss. But mass loss minimized at high resolution.

TPLS: Equations of motion

Numerical solution of two-phase Navier–Stokes equations with interface capturing:

$$\rho(\phi)\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=-\nabla p+\frac{1}{\mathrm{Re}}\nabla\cdot\left[\mu(\phi)\left(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^{T}\right)\right]+\boldsymbol{f}_{\mathrm{st}}(\phi)-\rho(\phi)\mathcal{G}\hat{\boldsymbol{z}},$$

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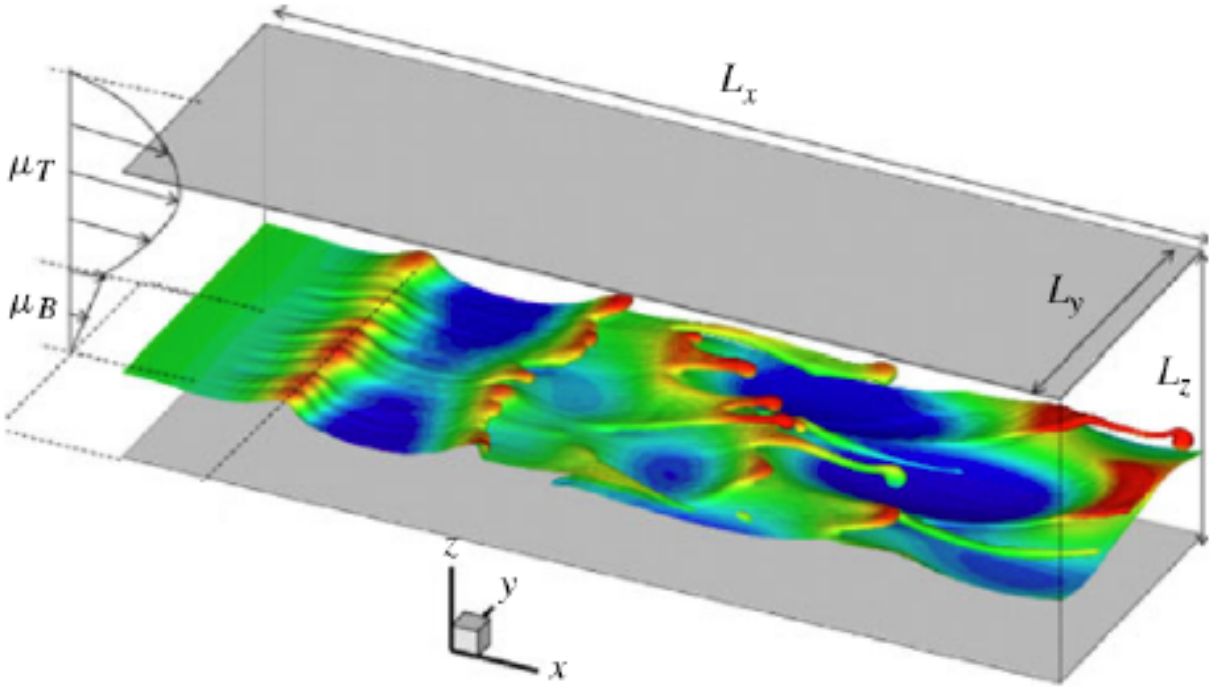
Dimensionless groups:

$$\mathrm{Re}=\frac{\rho_T V L}{\mu_T},\qquad \mathcal{G}=\frac{gL}{V^2},\qquad \mathrm{We}=\frac{\rho_T L V^2}{\gamma},$$

(I also use $\mathcal{S}=1/\mathrm{We}$, for historical reasons!)

TPLS: Problem geometry and configuration

- Simple channel geometry: periodic boundary conditions at $x = 0$, $x = L_x$; walls (no slip) at $z = 0$, $z = L_z$.
- Constant pressure drop drives flow in streamwise direction (forcing).
- Basic version involves hydrodynamics only. TPLS with physics under development, for applications including contact-line dynamics, and mass transfer.



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- The levelset function $\phi(x, y, z, t)$ is carried with the flow (3rd-order WENO) but is corrected at each timestep ('redistancing').

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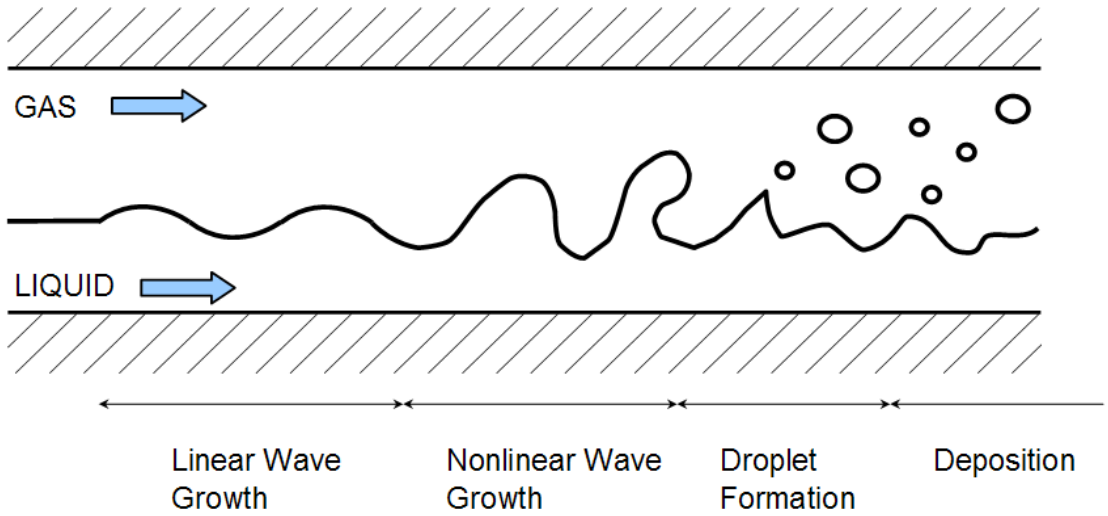
- Parallel efficiency with 2000 MPI processes is only 0.6 – there is a tradeoff between robustness/simplicity and performance. Underscores the need to replace hand-coded linear-algebra solvers with libraries.

Strict benchmarks for code's accuracy

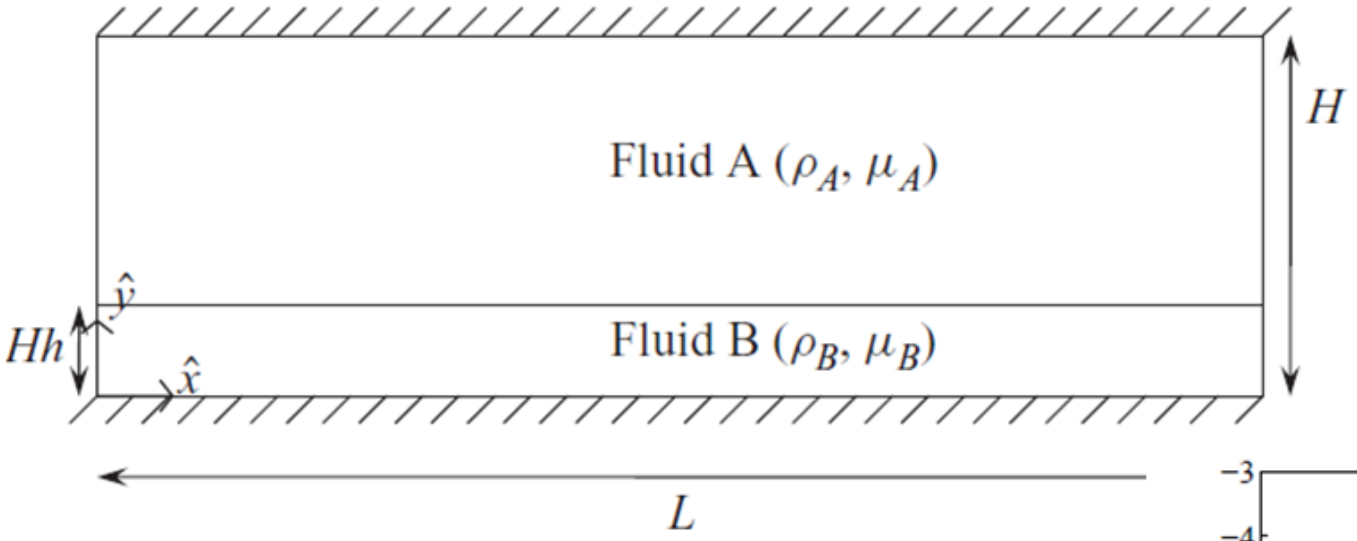
- Introduce a tiny sinusoidal perturbation at the interface.
- Produces pressure and velocity fluctuations that satisfy linear equations of motion.
- Linearized equations of motion solved via eigenvalue analysis (independent, quasi-analytical).
- Gives growth rate and wave speed of wave-like fluctuations.



Focus on finding agreement between OS analysis and wave growth in the code.

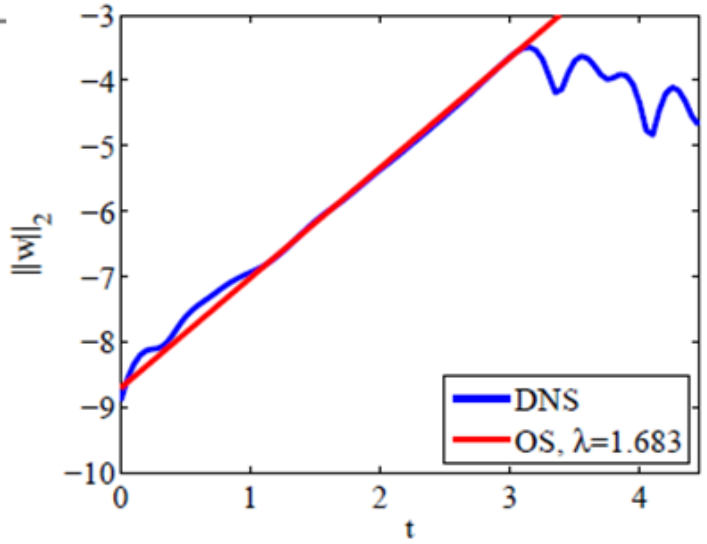
Orr–Sommerfeld analysis – Results

Stratified co-flow test case (h=0.3)

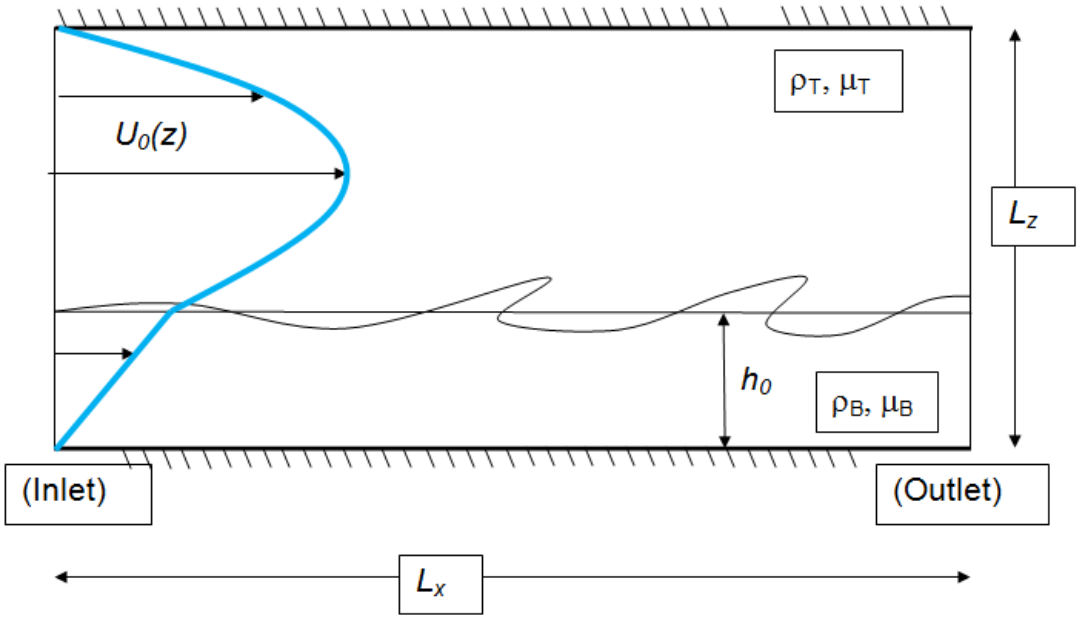


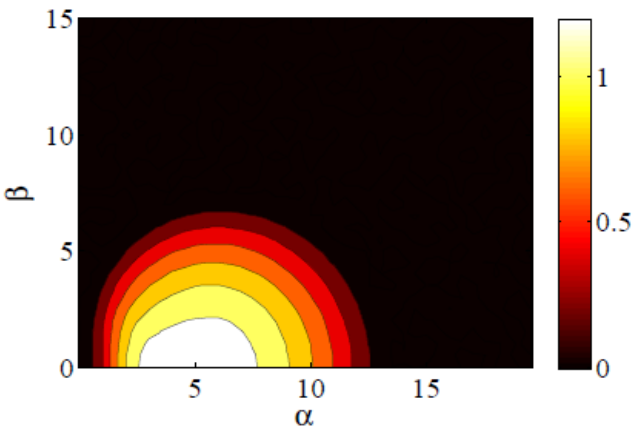
$$(Re, r, m, \mathcal{S}) = (100, 1, 30, 0.01)$$

- *L x W x H = (3 x 1 x 1)*
- *12 million grid points*
- *1024 processors, 12 h*



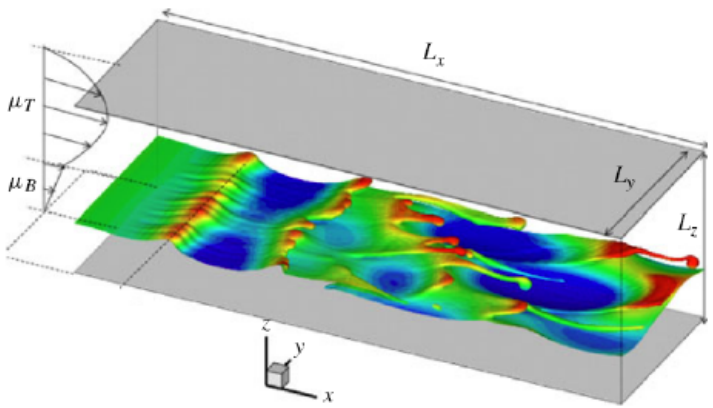
Application of TPLS: where do 3D waves in parallel flows come from?





$$r = 1, \mathcal{S} = 0.1$$

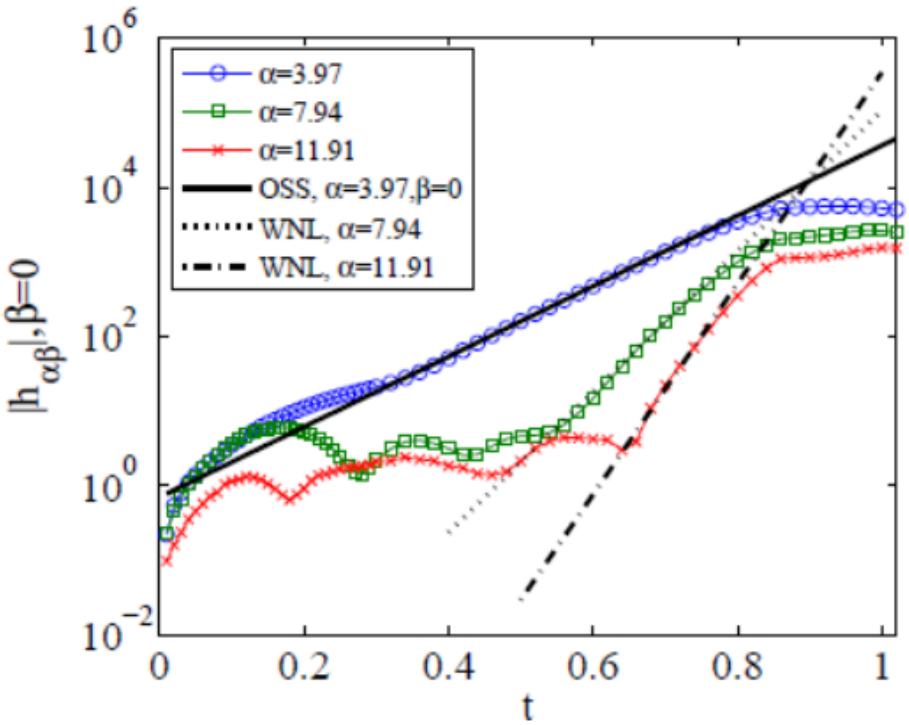
Linear instability of 2D parallel flow is dominated by 2D waves.
So how do 3D structures form?

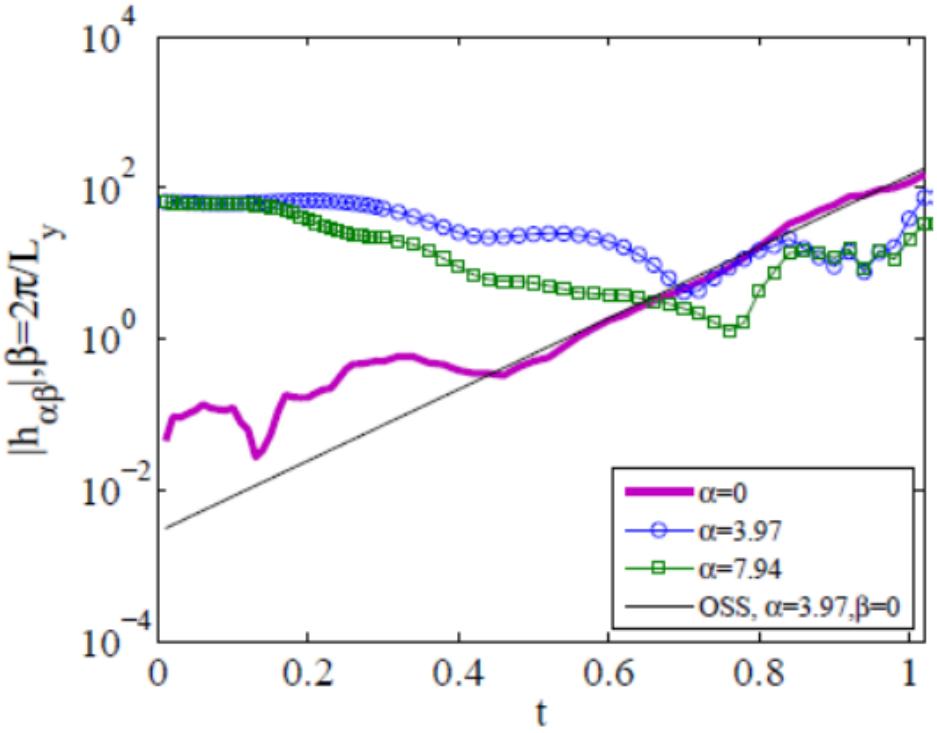


Brief review for liquid-liquid flows

We know the answer for liquid-liquid flows ($r = 1$) – it is weakly nonlinear analysis.

Streamwise waves – Large temporal growth, Spanwise waves – No temporal growth rate



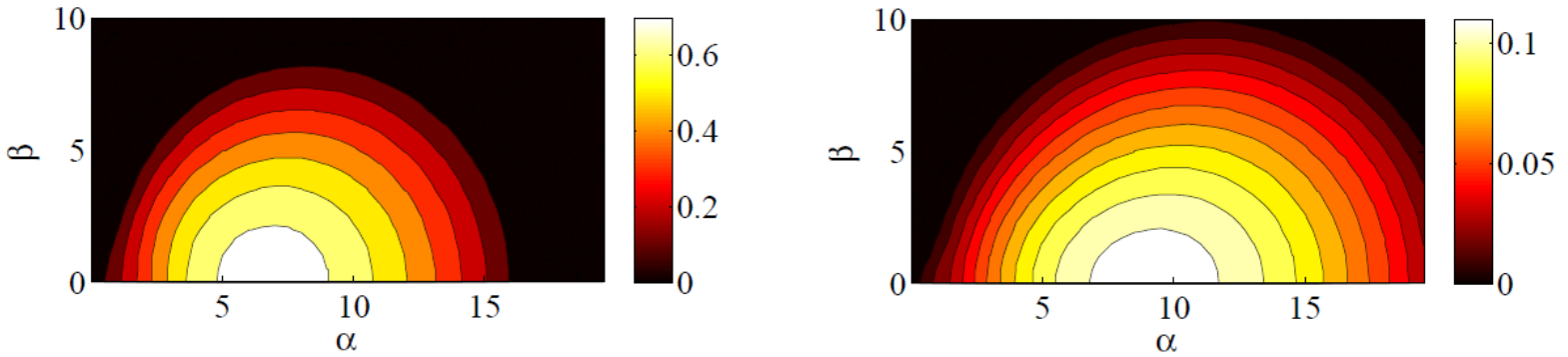


- Streamwise overtones are enslaved to the streamwise dominant mode
- Purely spanwise mode enslaved to the dominant streamwise mode

Periodic boundary conditions, $(Re, m, r, \mathcal{S}) = (300, 30, 1, 0.3)$.

New study required for gas-liquid flows

For gas-liquid flows, linear theory predicts a **direct route**.



(a) $r = 100, \mathcal{S} = 0.1$ (b) $r = 1000, \mathcal{S} = 0.1$

Eigenvalue analysis of the two-phase Orr–Sommerfeld–Squire equations for $Re = 100$, $m = 30$, $h_0 = 0.3$, and $\mathcal{S} = 0.1$, and $\mathcal{G} = 0.1$.

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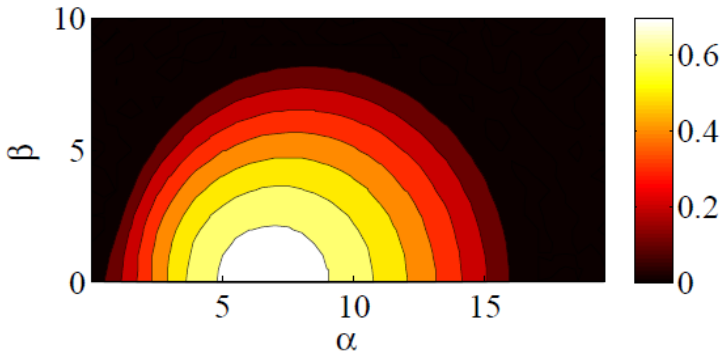

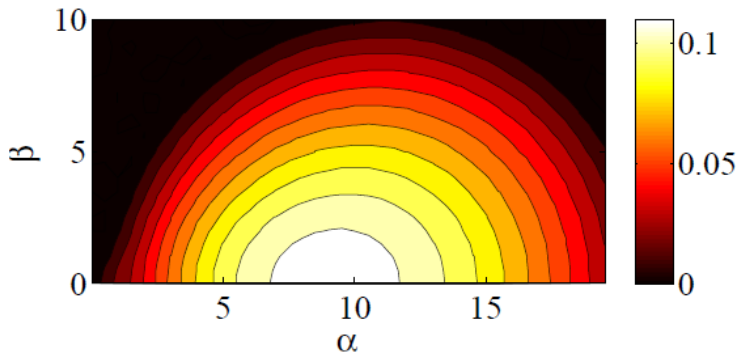

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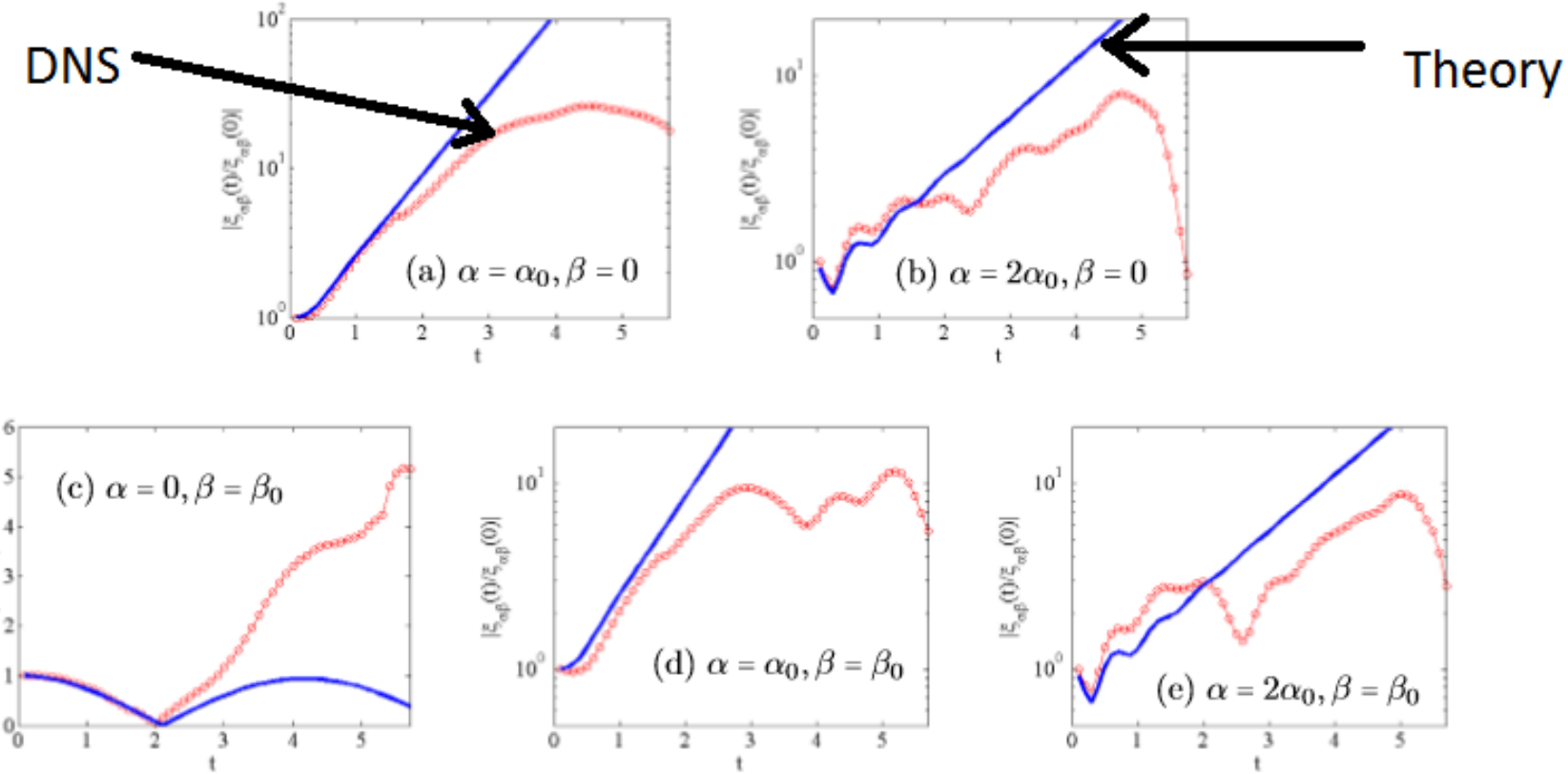
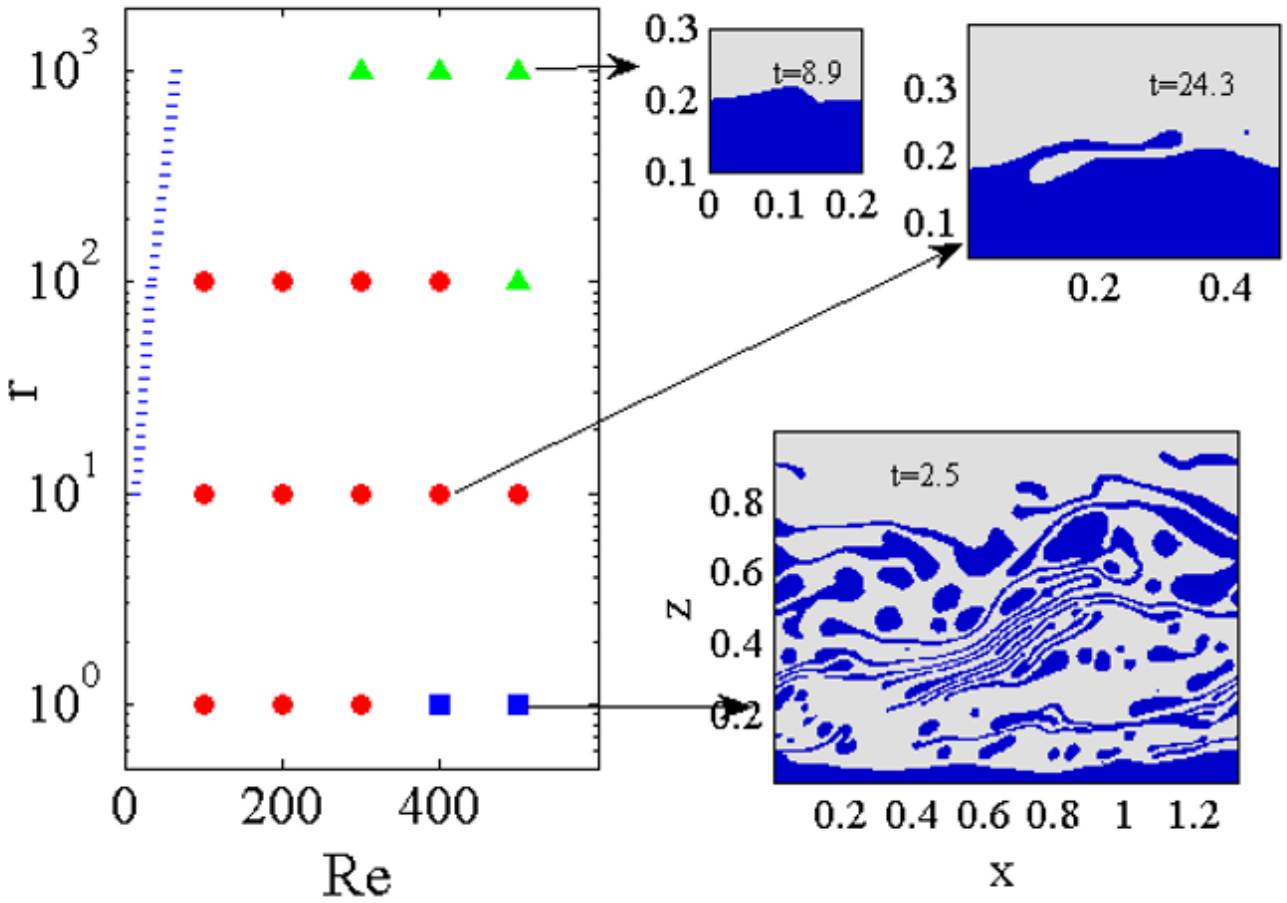
Overall trend: increasing r means that more modes become unstable (both streamwise and spanwise), but with a smaller growth rate.

Theoretical Prediction confirmed by DNS



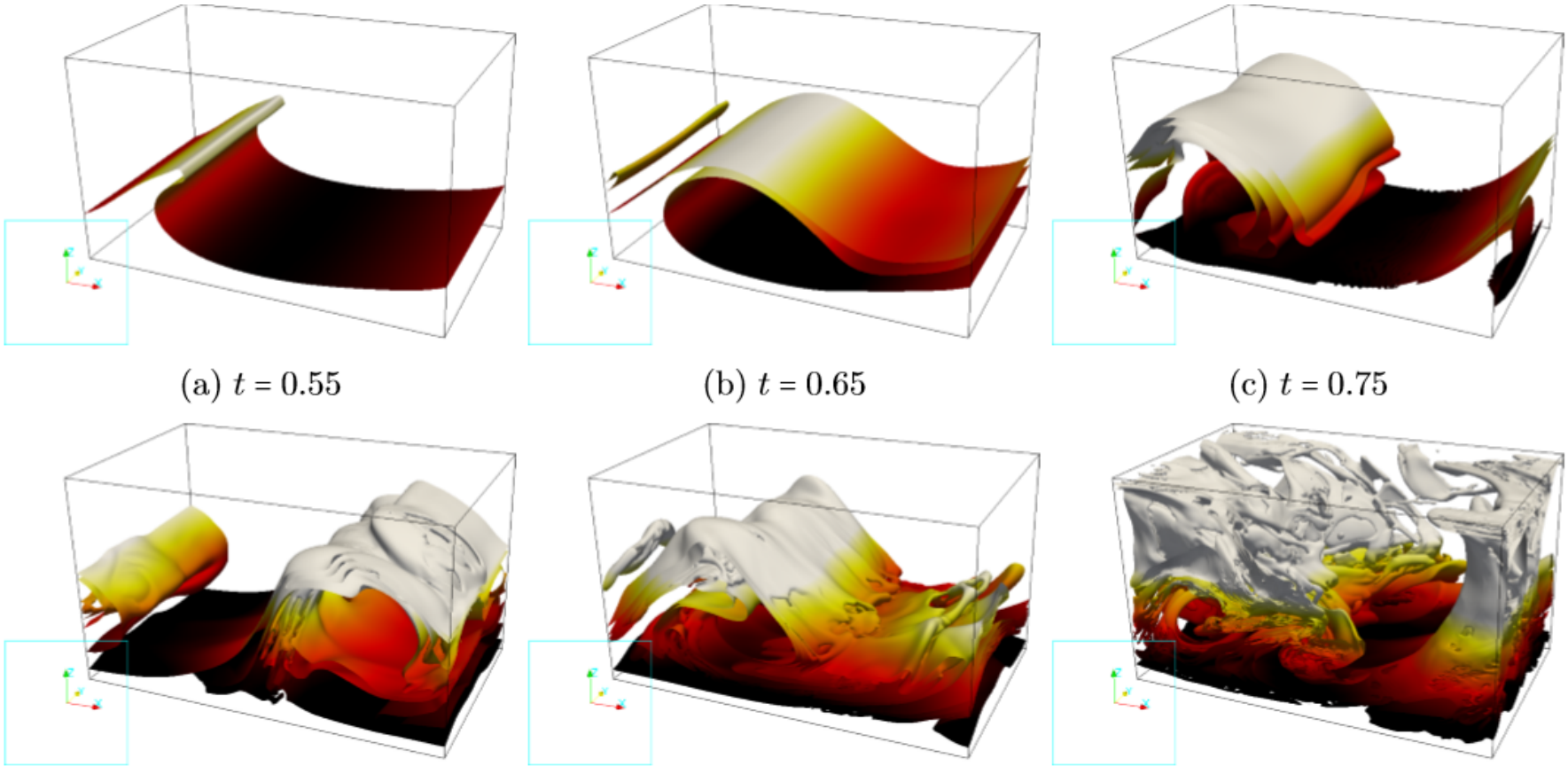
DNS results (lines with circles) for the case ($m = 50, h_0 = 0.2, \mathcal{G} = 0.1, \mathrm{We} = 10$), with $r = 1000$ and $\mathrm{Re} = 500$. Shown also is a comparison with linearized DNS (unadorned lines). Here, $\alpha_0 = 2\pi/L_x$ and $\beta_0 = 2\pi/L_y$ denote the fundamental wavenumber in the streamwise and spanwise directions respectively. In panel (c) the growth of the relevant amplitude is modest and a vertical linear (as opposed to logarithmic) scale is used. Also, the 'kink' at $t = 2$ in the same panel simply corresponds to a a zero of $\xi_{0\beta_0}(t)$, as this particular Fourier amplitude does not grow exponentially.

2D-DNS used to construct a flow-pattern map



Flow-pattern map for the two-dimensional simulations. The non-dimensionalization is based on the upper-layer properties, with $(m = 50, h_0 = 0.2, \mathcal{G} = 0.1, \mathrm{We} = 10)$. Squares – Dispersed liquid phase. Circles – ligaments. Triangle – saturated travelling wave. The insets show snapshots of the three different flow regimes.

Carefully-chosen 3D simulations show the results carry over

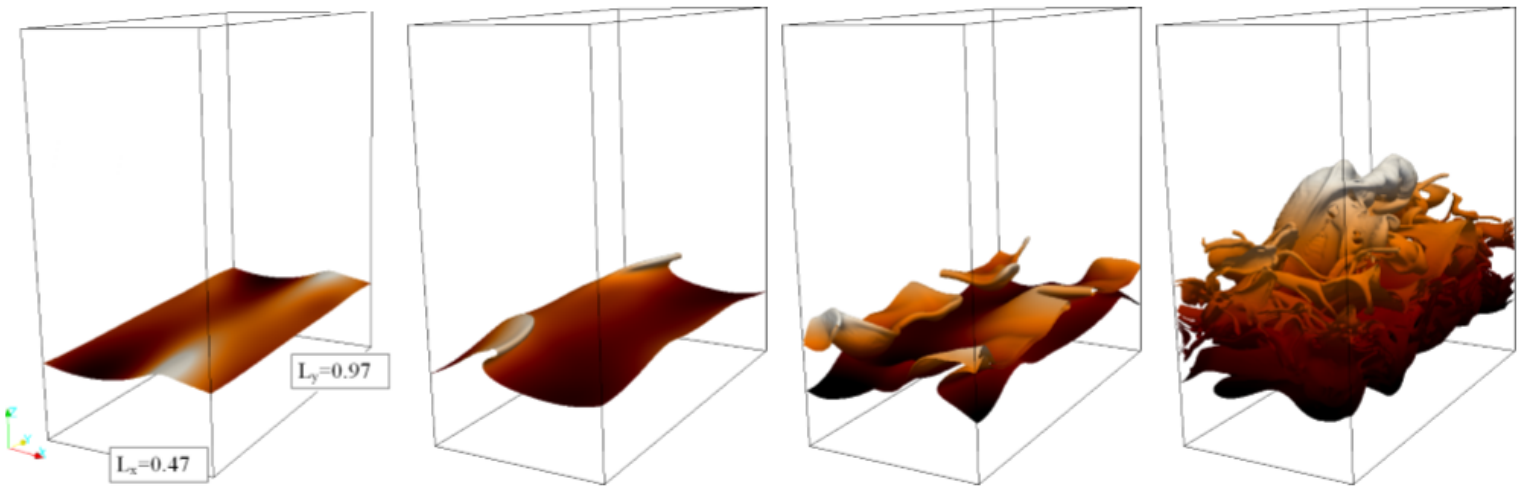


(a) $t = 0.55$ (b) $t = 0.65$ (c) $t = 0.75$

(d) $t = 0.8$ (e) $t = 0.85$ (f) $t = 0.95$

DNS results for the case $(m = 50, h_0 = 0.2, \mathcal{G} = 0.1, \mathrm{We} = 10)$, with $r = 1$ and $\mathrm{Re} = 500$.

The case $r = 10$



(a) $t = 0.4$ (b) $t = 0.5$ (c) $t = 0.6$ (d) $t = 0.7$

FIG. 30. DNS results for the case $(m = 50, h_0 = 0.2, \mathcal{G} = 0.1, \mathrm{We} = 10)$, with $r = 10$ and $\mathrm{Re} = 500$.

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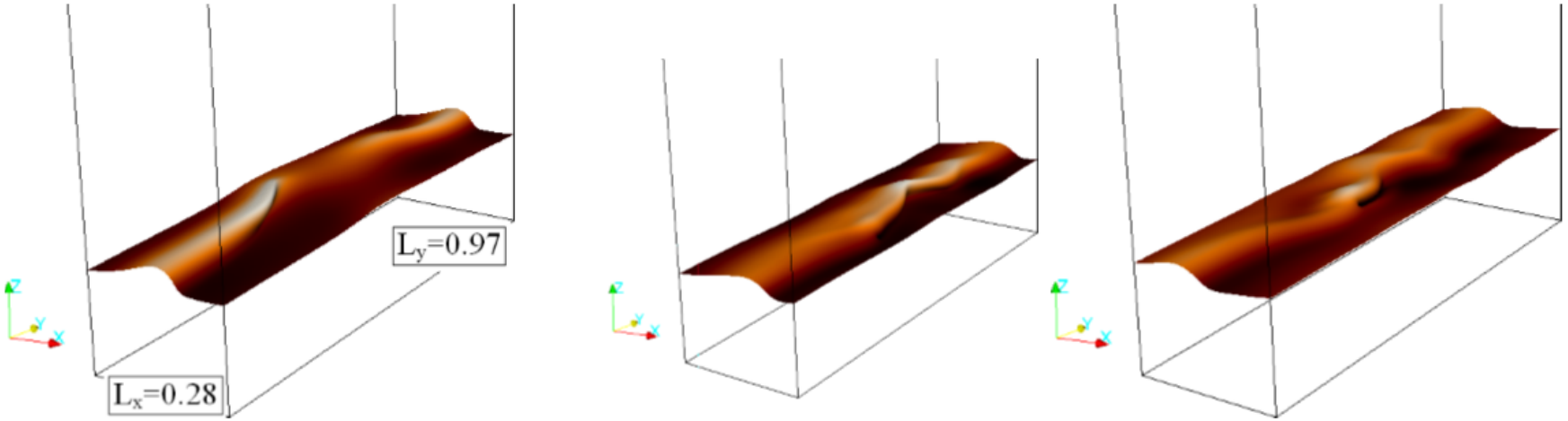

(a) $t = 1.1$ (b) $t = 1.4$ (c) $t = 1.6$

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- Using theory and DNS, our current work shows a different scenario at work in gas-liquid flows:
  - The interfacial waves in gas-liquid ows grow much more slowly than those in corresponding liquid-liquid flows.
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Therefore, three-dimensional waves form in gas-liquid ows via a **direct route**: by waiting long enough, streamwise and spanwise modes form as a result of small-amplitude perturbations.

- Beyond this early-stage wave growth, a zoo of dierent phenomena is possible, depending on the particular ow parameters involved.