Agglomeration and transport of drilling-generated particles in directional oil wells – analytical study

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Context of work

Problem posed by IRIS at ESGI 102 in UCD (July 2014):

- Drilling an inclined well creates cuttings, which have to be removed;
- Drilling fluid is pumped down drillstring through drillbit and up to annulus;
- Drilling cuttings transported to surface by circulating drilling fluid;
- Poor control of cuttings may cause critical situations and loss of well.

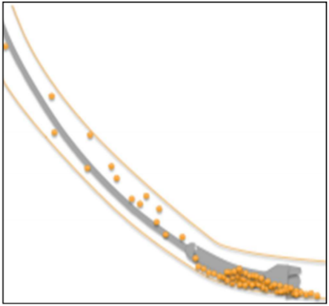

Figure: Drillstring, drillbit and annulus filled with drilling fluid and cuttings

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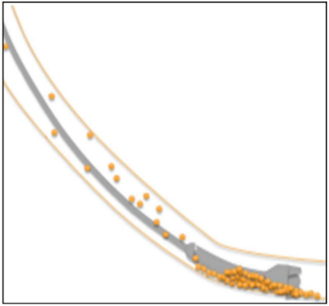
Can this process be modelled such that control of this complex multiphase flow can be guaranteed?



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Two complementary modelling approaches were taken:

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Future work – Connect the two approaches with a view to building up the analytical approach, testing its validity and hence improving its applicability.

Mathematical Modelling

Fundamentals:

- Cuttings are small - can be treated in continuum theory
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- Formulate equations of motion for average velocity of such parcels
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- Stratification is not assumed – transition from dispersed phase to stratified phase is **predicted** by the model.

Definition Sketch

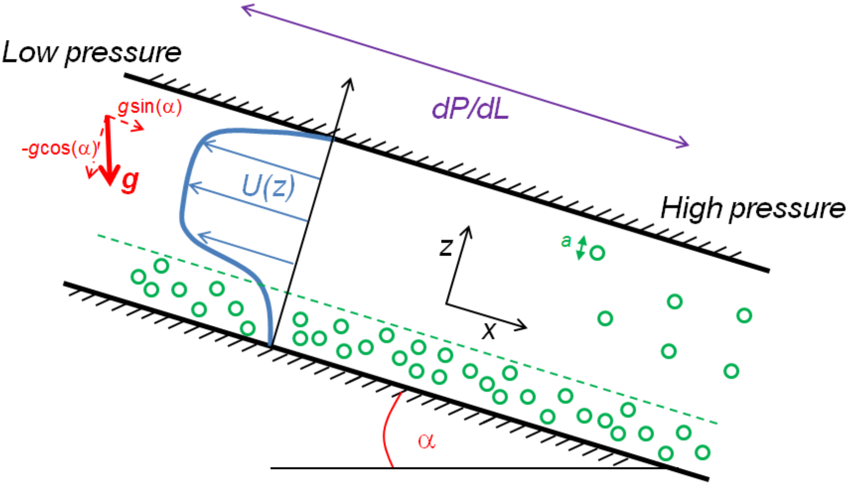

Figure: Definition sketch for mathematical model

Diffusive flux model

- Equation of motion for the mixture (particles+suspending fluid):

$$\rho(\phi)\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=\nabla\cdot\boldsymbol{T}+\rho(\phi)\boldsymbol{g}, \qquad \nabla\cdot\boldsymbol{u}=0,$$

  where $\boldsymbol{u}(\boldsymbol{x},t)$ is the Eulerian velocity for a parcel comprising a mixture of particles and fluid.

- Density:

$$\rho(\phi)=\rho_{\mathrm{b}}\phi+\rho_{\mathrm{f}}(1-\phi),$$

  where ϕ is the particle volume-fraction.

- Hydrodynamics coupled to the ϕ-field via diffusive flux equation

$$\frac{\partial\phi}{\partial t}+\boldsymbol{u}\cdot\nabla\phi=\nabla\cdot\boldsymbol{J}_{\phi}.$$

- **Constitutive modelling** for the stress tensor $\boldsymbol{T}$ and the flux $\boldsymbol{J}_{\phi}$.

Modelling the diffusive flux

- Shear-induced migration – particles in a shear flow collide and move to regions where the collisions are fewer (lower shear), giving a shear-induced contribution (where $\dot{\gamma}$ is the shear rate):

$$\boldsymbol{J}_{\mathrm{c}} = -D_c \phi a^2 \nabla(\phi \dot{\gamma}),$$

- Viscous migration – particles move from regions of high viscosity into regions of lower viscosity, giving a contribution

$$\boldsymbol{J}_{\mathrm{v}} = -D_v \phi a^2 \dot{\gamma} \left(\frac{\nabla \mu}{\mu} \right)$$

- Gravitational settling (Stokes' Law):

$$\boldsymbol{J}_{\mathrm{g}} = -\frac{2a^2(\rho_{\mathrm{b}} - \rho_{\mathrm{f}}) f(\phi)}{9\mu_{\mathrm{f}}} \boldsymbol{g}$$

where $f(\phi)$ is the hindrance function.

Modelling the stress tensor

For parallel flow $U(z)$ we use

$$T_{xz} = -p + \mu(\phi)\frac{\mathrm{d}U}{\mathrm{d}z},$$

where

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \mathrm{Const.}$$

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Throughout, we use the Krieger–Dougherty form for the viscosity of a suspension:

$$\mu(\phi) = \left(1 - \frac{\phi}{\phi_\mathrm{m}}\right)^{-\xi}, \qquad \xi > 0, \qquad \phi_\mathrm{m} > 0.$$

Assumption: suspending fluid is laminar in the absence of particles (first approximation).

Basic Model

Closed form of basic model for parallel unidirectional flow:

- Constitutive relation for stress:

$$\frac{\mathrm{d}U}{\mathrm{d}z} = \sigma/\mu,$$

- Force balance:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = 1 - \mathcal{G}\left[r\phi + (1-\phi)\right]\sin\alpha.$$

- Diffusive flux model:

$$D_{\mathrm{c}}\phi\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\sigma}{\mu}\phi\right) + D_{\mathrm{v}}\phi\frac{\sigma}{\mu^2}\frac{\mathrm{d}\mu}{\mathrm{d}z} + \frac{2(r-1)\phi(1-\phi)}{9\mu(\phi)}\mathcal{G}\cos\alpha = 0$$

These are closed equations with boundary conditions

$$U(0) = U(1) = 0, \qquad \Phi = \int_0^1 \phi(z)\,\mathrm{d}z.$$

Basic Model – nondimensionalization

- Basic model has been nondimensionalized with respect to channel height H, characteristic velocity
$$V = \frac{H^2}{\mu_{\mathrm{f}}} \left| \frac{\mathrm{d}p}{\mathrm{d}x} \right|,$$
along with the fluid properties (viscosity, density).

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- Dimensionless parameters:

$$\mathcal{G} = \frac{\mathrm{Re}}{\mathrm{Fr}^2} = \left(\frac{V H \rho_\mathrm{f}}{\mu_\mathrm{f}} \right) \left(\frac{gH}{V^2} \right),$$

together with inclination α, D_c, D_v (nondimensional) and $\epsilon = a/H$ (particle radius).

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together with inclination α, D_c, D_v (nondimensional) and $\epsilon = a/H$ (particle radius).

- Apart from the KD parameters, a key feature of the model is the **absence of any further empirical correlations**

Basic Model – issues

- So far all we have done is to add gravity to a basic Diffusive-Flux model and to tilt it.
- This would be fine if it worked, but it doesn't:
  - Explicit dependence of model equations on particle radius has dropped out.
  - Model has an unphysical singularity at the critical point $\mathrm{d}U/\mathrm{d}z = 0$.

  Both of these points are related and can be tackled in the same way.
- Reason is that model collision rate is set proportional to $\dot{\gamma}\phi = \sigma\phi/\mu$, which vanishes at critical point.
- This is unphysical – collision rate should be proportional to σ **averaged** over the extent of a particle.
- We have performed this averaging for small particles and found the averaged stress to be equal to

$$\widehat{\sigma} = \sqrt{\sigma^2 + \epsilon^2 \left(\frac{\mathrm{d}\sigma}{\mathrm{d}z}\right)^2},$$

with $\widehat{\sigma} \neq 0$ when $\sigma = 0$.

Improved model

- With σ replaced by $\widehat{\sigma}$ at the appropriate places, we obtain an improved model of three ODEs without a cusp at the critical point – **main innovation**.
- Equations too complicated to solve in closed form but we can solve numerically via shooting.
- A shooting method is constructed in MATLAB. In practice, the implemented boundary conditions are $U(0) = U(1) = 0$ and $\phi(0) = \phi_1$, and ϕ_1 is adjusted until the desired bulk cuttings volume fraction $\Phi = \int_0^1 \phi(z)\,\mathrm{d}z$ is obtained.

Preliminary results – weak gravity effect

- We fix $\alpha = \pi/12$ $(= 15^{\text{o}})$, $r = 2$, $\epsilon = 0.01$ and study the effects of varying Φ and $\mathcal{G}$.
- Recall, $\mathcal{G} = Re/Fr^2 = Re(gH/V^2)$, so that $\mathcal{G}$ small means the gravity effect is weak compared to the pressure gradient.
- So we first fix $\Phi = 0.35$ and study weak, intermediate, and strong gravity effects.

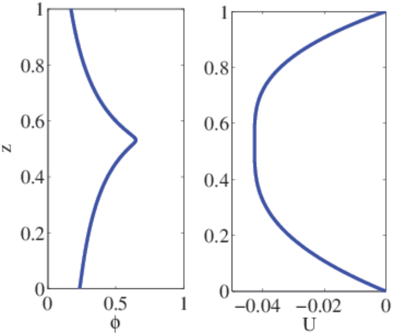


Figure: $\mathcal{G} = 0.1$ $\Phi = 0.35$

Preliminary results – intermediate gravity effect I

- With $\mathcal{G} = 2$ a dense lower bed forms together with an upper layer that is clear of particles:
- However, $U(z) < 0$ strictly throughout the domain, meaning that particles are still transported upward and are therefore still removed from the system under this intermediate gravity regime.

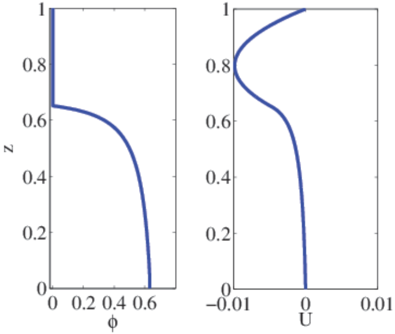


Figure: $\mathcal{G} = 2, \ \Phi = 0.35$

Preliminary results – intermediate gravity effect II

- Increasing $\mathcal{G}$ further to $\mathcal{G} = 2.5$, the lower bed becomes stationary and a clear layer of drilling fluid is transported upward (example has $\Phi = 0.4$).

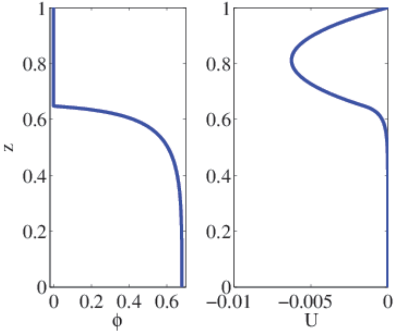


Figure: $\mathcal{G} = 2, \ \Phi = 0.4$

Preliminary results – large gravity effect

- For $\mathcal{G} = 10$ we have complete flow reversal, and the particles and the fluid both are transported in the positive x-direction ('back down') in the channel.

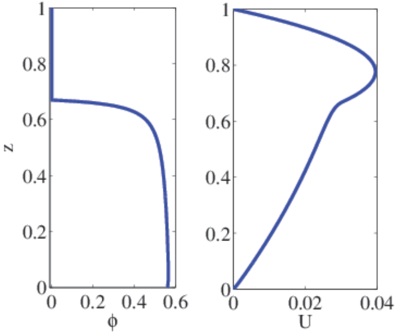
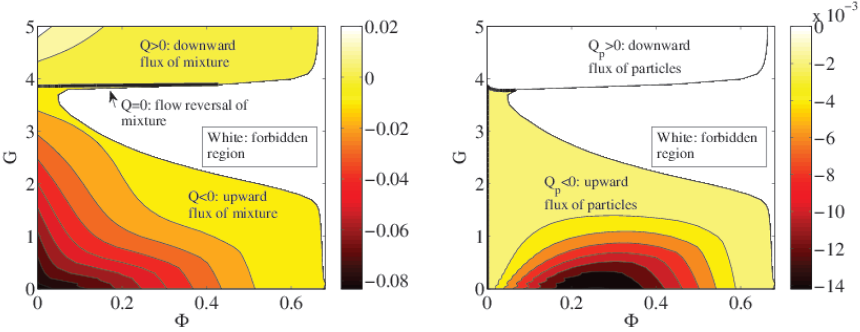


Figure: $\mathcal{G} = 10, \;\; \Phi = 0.35$

Systematic Parameter Study

We have tried to do a more systematic parameter study by looking at the mixture and particle fowrates as a function of $(\Phi, \mathcal{G})$, where

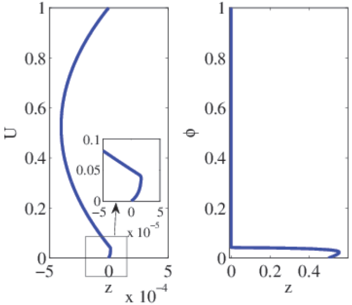
$$Q_{\text{mixture}} = \int_0^1 U(z)\,\mathrm{d}z, \qquad Q_{\text{particles}} = \int_0^1 \phi(z)U(z)\,\mathrm{d}z.$$



(a) Q_{mixture} (b) $Q_{\text{particles}}$

Forbidden Regions and countercurrent flow

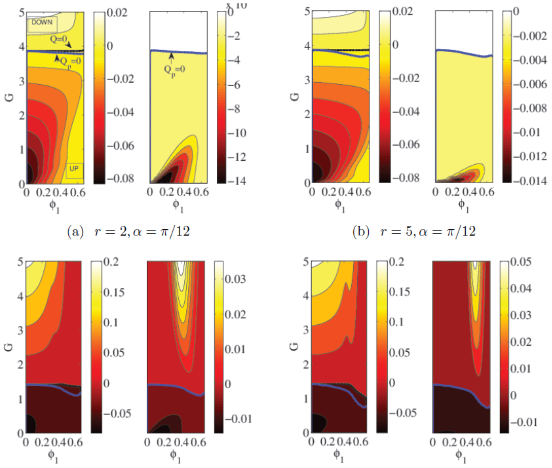
- **Forbidden regions** correspond to a very high density of particles in a slow-moving or stationary bed, which is unsustainable as an equilbrium solution and corresponds to 'clogging'.
- Second possibility: $Q = 0$ and Q_p level curves do not overlap. Closed region bounded by these curves corresponds to **countercurrent flow**.



Countercurrent flow regime corresponding to the parameter values $(\mathcal{G}, \phi_1) = (3.85, 0.5)$. Additional parameter values $r = 2$, $\alpha = \pi/12$. The corresponding value of the bulk volume fraction is $\Phi = 0.021$.

Enhancement of countercurrent flow by parameter changes

Enlarge the region in parameter space corresponding to CC flow and increase the magnitude of the CC flow.



(a) $r = 2, \alpha = \pi/12$ (b) $r = 5, \alpha = \pi/12$

(c) $r = 2, \alpha = \pi/4$ (d) $r = 2, \alpha = \pi/4$

Criterion for flow reversal

- Conclusion so far – starting from $\mathcal{G} = 0$ and then increasing $\mathcal{G}$, flow goes countercurrent and then fully reversed (bad).
- Lower bound for onset of flow reversal can be estimated. Anything beyond this lower bound is 'risky' and can lead to flow reversal and cuttings flowing in the wrong direction.
- Back to model equations - review force balance:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = 1 - \mathcal{G}\left[r\phi + (1-\phi)\right]\sin\alpha.$$

- Lower bound for critical value $\mathcal{G}_{\mathrm{c}}$ for onset of flow reversal when $(\mathrm{d}\sigma/\mathrm{d}z)_0 = 0$, hence

$$\mathcal{G}_{\mathrm{c}} \geq \frac{1}{\sin\alpha}\frac{1}{r\phi_1 + (1-\phi_1)}.$$

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- Also, we need to systematically evaluate the performance of the regularized Diffusive Flux model (e.g. comparison with suspension balance model).
- **Crucially, we need to connect the predictive results of the model (e.g. flow-pattern maps) to DNS** and from there, to connect the model to realistic flow scenarios. These are exciting avenues for future work.

Acknowledgements

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