Mechanics and Special Relativity (MAPH10030)
Assignment 4

Issue Date: Tuesday 13 April 2010
Due Date: Wednesday 21 April 2010
Collection Date: Friday 23 April 2010

In these questions, you may use the following conversion factor relating the electron-volt to Joules: $1\,\text{eV} = 1.60217646 \times 10^{-19}\,\text{Joules}$, where the Joule is the SI unit of energy, $\text{J} = \text{kg}\,\text{m}^2/\text{s}^2$. Furthermore, you may use the fact that the rest energy of an electron is $m_e c^2 = 0.511\,\text{MeV}$.

1. **The Lorentz transformations**

   (a) An observer in frame S' is moving to the right at speed $V = 0.600c$ away from a stationary observer in frame S. The observer in S' measures the speed v' of a particle moving to the right away from her. What speed v does the observer in S measure the particle if $v' = 0.900c$? [2 points]

   (b) A pursuit spacecraft from the planet Tatooine is attempting to catch up with a Trade Federation cruiser. As measured by an observer on Tatooine, the cruiser is travelling away from the planet with a speed $0.600c$. The pursuit ship is travelling at a speed $0.800c$ relative to Tatooine, in the same direction as the cruiser. What is the speed of the cruiser relative to the pursuit ship? [2 points]

   (c) Two particles are created in a high-energy acclelerator and move off in opposite directions: one to the left, and one to the right. The speed of one particle as measured in the lab is $0.650c$ and the speed of each particle realtive to the other is $0.950c$. What is the speed of the second particle, as measured in the lab? [2 points]

2. **Energy**

   (a) What is the speed of a particle if its kinetic energy is 2.0% larger than $mv^2/2$? [2 points] Hint: Use the Binomial Theorem.

   (b) The kinetic energy of a certain electron is 0.420-MeV. To create x-rays (high-energy photons), the electron travels down a tube and hits a target. When it arrives at the target, what is its kinetic energy in eV? What is its total energy? What is its speed? What is the speed of the electron, computed (incorrectly) from Newtonian mechanics? [2 points]

3. **Scattering experiments**

(a) A photon with energy E is emitted by an atom with mass m, which recoils in the opposite direction. Assuming that the motion of the atom can be treated nonrelativistically, compute the recoil speed of the atom. From this result, show that the recoil speed is much smaller than c whenever E is much smaller than the rest energy mc^2 of the atom [2 points].

(b) In Compton scattering, what is the maximum possible wavelength shift? [2 points]

(c) Two pions π^+ and π^- collide and produce a neutral kaon. If the event is a head-on collision in which the pions have velocities v_0 and $-v_0/2$ in the laboratory frame, what is the mass of the kaon in terms of the velocity v_0 and the pion mass m_π? Find a numerical result (with m_π still undetermined) if $v_0 = 0.95c$. [6 points]

4. **Bonus question** [Top up for a maximum of 5 points] Two events observed in a frame of reference S have positions and times given by (x_1, t_1) and (x_2, t_2), respectively.

(a) Frame S' moves along the x-axis just fast enough that the two events occur at the same position in S'. Show that in S', the time interval $\Delta t'$ between the two events is given by

$$\Delta t' = \sqrt{(\Delta t)^2 - (\Delta x/c)^2}, \qquad \Delta x = x_2 - x_1, \Delta t = t_2 - t_1.$$

Hence, show that if $\Delta x > c\Delta t$, there is NO frame S' in which the two events occur at the same point [2 points].

(b) Show that if $\Delta x > c\Delta t$, there is a different frame of reference S' in which the two events occur SIMULTANEOUSLY. Find the distance between the two events in S'; express your answer in terms of Δx, Δt, and c. [3 points]