Mechanics and Special Relativity (ACM10030)
Assignment 4

Issue Date: 13 April 2010
Due Date: 20 April 2010

In these questions, you may use the following conversion factor relating the electron-volt to Joules: $1\,\mathrm{eV} = 1.60217646 \times 10^{-19}\,\mathrm{Joules}$, where the Joule is the SI unit of energy, $\mathrm{J} = \mathrm{kg\,m^2/s^2}$. Furthermore, you may use the fact that the rest energy of an electron is $m_\mathrm{e}c^2 = 0.511\,\mathrm{MeV}$.

1. **Satellite motion, Galilean physics** A satellite of mass m is in circular orbit about the earth. The radius of the orbit is r_0 and the mass of the earth is M_e.

   (a) Find the total mechanical energy of the satellite.

   (b) Now suppose that the satellite moves in the extreme upper atmosphere of the earth where it is retarded by a constant but small friction force f. The satellite will slowly spiral towards the earth. Since the friction force is weak, the change in radius will be very slow. Therefore, we assume that at any instant the satellite is effectively in circular orbit of average radius r. Find the approximate change in radius per revolution of the satellite, Δr.

   (c) Find the approximate change in the kinetic energy of the satellite per revolution, ΔK.

2. **The Lorenz transformations**

   (a) An observer in frame S' is moving to the right at speed $V = 0.600c$ away from a stationary observer in frame S. The observer in S' measures the speed v' of a particle moving to the right away from her. What speed v does the observer in S measure the particle if $v' = 0.900c$?

   (b) A pursuit spacecraft from the planet Tatooine is attempting to catch up with a Trade Federation cruiser. As measured by an observer on Tatooine, the cruiser is travelling away from the planet with a speed $0.600c$. The pursuit ship is travelling at a speed $0.800c$ relative to Tatooine, in the same direction as the cruiser. What is the speed of the cruiser relative to the pursuit ship?

   (c) Two particles are created in a high-energy accelerator and move off in opposite directions: one to the left, and one to the right. The speed of one particle as measured in the lab is $0.650c$ and the speed of each particle relative to the other is $0.950c$. What is the speed of the second particle, as measured in the lab?

3. **Energy**

   (a) What is the speed of a particle if its kinetic energy is 1.0% larger than $mv^2/2$? Hint: Use the Binomial Theorem.

   (b) The kinetic energy of a certain electron is 0.520-MeV. To create x-rays (high-energy photons), the electron travels down a tube and hits a target. When it arrives at the target, what is its kinetic energy in eV? What is its total energy? What is its speed? What is the speed of the electron, computed (incorrectly) from Newtonian mechanics?

4. **Scattering experiments**

   (a) A photon with energy E is emitted by an atom with mass m, which recoils in the opposite direction. Assuming that the motion of the atom can be treated non-relativistically, compute the recoil speed of the atom. From this result, show that the recoil speed is much smaller than c whenever E is much smaller than the rest energy mc^2 of the atom.

   (b) Two pions π^+ and π^- collide and produce a neutral kaon. If the event is a head-on collision in which the pions have velocities v_0 and $-v_0/2$ in the laboratory frame, what is the mass of the kaon in terms of the velocity v_0 and the pion mass m_π? Find a numerical result (with m_π still undetermined) if $v_0 = 0.95c$.