# Mechanics and Special Relativity (MAPH10030)
# Assignment 3

Issue Date: 03 March 2010
Due Date: 24 March 2010

**In question 4, a numerical answer is required, with precision to three significant figures. Marks will be deducted for more or less precision. You may use $M_{\rm e} = 5.97 \times 10^{24}\,{\rm kg}$. In the other questions, a symbolic answer is fine.**

1. When an object is in circular orbit of radius $r$ about the earth (mass $M_{\rm e}$), the orbital period is
$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_{\rm e}}},$$
and the orbital velocity speed is
$$v = \sqrt{\frac{GM_{\rm e}}{r}}.$$
Show that when the object is moved into a circular orbit of slightly larger radius $r + \Delta r$, where $\Delta r \ll r$, its new period is $T + \Delta T$ and its new orbital velocity is $v - \Delta v$, where
$$\Delta T = \frac{3\pi \Delta r}{v}, \qquad \Delta v = \frac{\pi \Delta r}{T}$$
[4 points].

2. See Fig. 1. A projectile of mass $m$ is fired from the surface of the earth at an angle $\alpha$ from the vertical. The initial speed $v_0$ is equal to $\sqrt{GM_{\rm e}/R_{\rm e}}$. How high does the projectile rise? Neglect air resistance and the earth's rotation [4 points].

   Hint: Please do not try to solve for the orbit! Instead, use the conservation laws directly.

3. A satellite of mass $m$ is in circular orbit about the earth. The radius of the orbit is $r_0$ and the mass of the earth is $M_{\rm e}$.

   (a) Find the total mechanical energy of the satellite [2 points].

   (b) Now suppose that the satellite moves in the extreme upper atmosphere of the earth where it is retarded by a constant but small friction force $f$. The satellite will slowly spiral towards the earth. Since the friction force is weak, the change in radius will be very slow. Therefore, we assume that at any instant the satellite is effectively in a circular orbit of average radius $r$. Find the approximate change in radius per revolution of the satellite, $\Delta r$ [2 points].

(c) Find the approximate change in the kinetic energy of the satellite per revolution, $\Delta K$ [2 points].

4. A space vehicle is in circular orbit around the earth. The mass of the vehicle is $3,000\,\mathrm{kg}$ and the radius of the orbit is $2R_\mathrm{e} = 12,800\,\mathrm{km}$. It is desired to transfer the vehicle to a circular orbit or radius $4R_\mathrm{e}$.

   (a) What is the minimum energy expenditure required for the transfer? [3 points]

   (b) An efficient way to accomplish the transfer is to use a semi-elliptical orbit (known as a Hohmann transfer orbit), shown in the figure. What velocity changes are required at the points of intersection, points $A$ and $B$ in Fig. 2 [3 points].

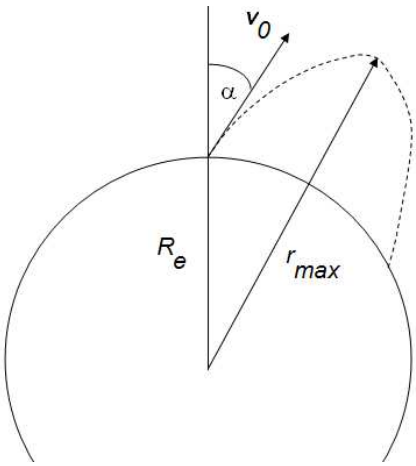


Figure 1: Problem 1

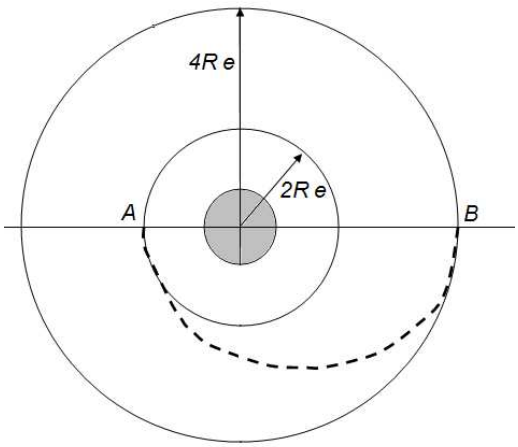


Figure 2: Problem 3