Mechanics and Special Relativity (MAPH10030)
Assignment 2

Issue Date: 16 February 2010
Due Date: 23 February 2010

1. Consider a particle that is constrained on top of a semicircle (See Fig. 1). Gravity points downwards. Suppose that the particle starts from rest. At what angle does the particle fall off the semicircle? [4 points]

   Hint: Please give the solution in two forms: in terms of the angle ϕ, and the angle θ. The answer in the ϕ-angle is given in the e-book mentioned in Lecture 1.

2. One force acting on a machine part is $\boldsymbol{F} = (-5.00\,\mathrm{N})\,\hat{\boldsymbol{x}} + (4.00\,\mathrm{N})\,\hat{\boldsymbol{y}}$. The vector from the origin to the point where the force is applied is $\boldsymbol{r} = (-0.450\,\mathrm{m})\,\hat{\boldsymbol{x}} + (0.150\,\mathrm{m})\,\hat{\boldsymbol{y}}$.

   - In a sketch show $\boldsymbol{r}$, $\boldsymbol{F}$, and the origin [1 point].
   - Use the right-hand rule to determine the direction of the torque. Then, compute the torque from the determinant definition. Make sure that the direction obtained in both calculations is the same. [3 points]

3. (a) Show that if the total linear momentum of a system of particles is zero, the angular momentum of the system is the same about all origins. [3 points]

   (b) Show that if the total force on a system of particles is zero, the torque on the system is the same about all origins. [3 points]

4. Recall the law of gravity for point particles m_1 and m_2: the force on particle 1 due to particle 2 is given by

$$\boldsymbol{F}_{12} = -\frac{Gm_1m_2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|^2}\left(\frac{\boldsymbol{x}_1 - \boldsymbol{x}_2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}\right). \tag{1}$$

   In class, we stated that the same law holds for spherical bodies at finite separations, and that the proof of this statement follows by integration. In this problem we obtain a hint at how this integration might be done by considering the gravitational force exerted by a continuous line of particles on a point particle of mass m.

   Consider the system shown in Fig. 2. A continuous line of particles extends from $x = -a$ to $x = a$, at $y = 0$. A point mass lies at $x = 0$, $y = L$.

   (a) Show that the force on the particle due to a point-like mass $dm\,(x)$ extending from x to $x + \mathrm{d}x$ is

$$\mathrm{d}\boldsymbol{F}_{1,x} = -\frac{Gm\,\mathrm{d}m\,(x)}{(x^2 + L^2)^{3/2}}\,(L\hat{\boldsymbol{y}} - x\hat{\boldsymbol{x}})\,.$$

[3 points]

(b) Assume a linear mass density $\mathrm{d}m = \rho\,\mathrm{d}x$ and thus obtain the total force $\boldsymbol{F}_1$ on the point mass m. You might have to use your favourite table of integrals to do this. [3 points]

(c) How would the force distribution change if $\mathrm{d}m = \rho_0\,[1 + \varepsilon\,(x/L)]\,\mathrm{d}x$? **Bonus question: up to four top-up points for fully worked-out answer**

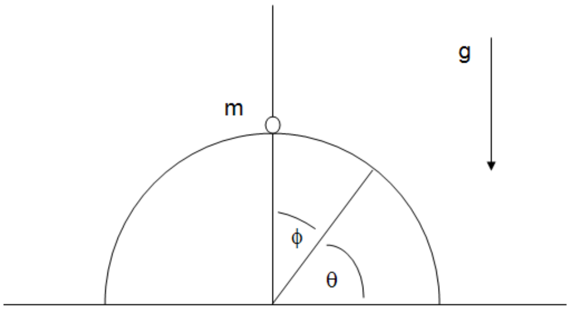

Figure 1: Problem 1

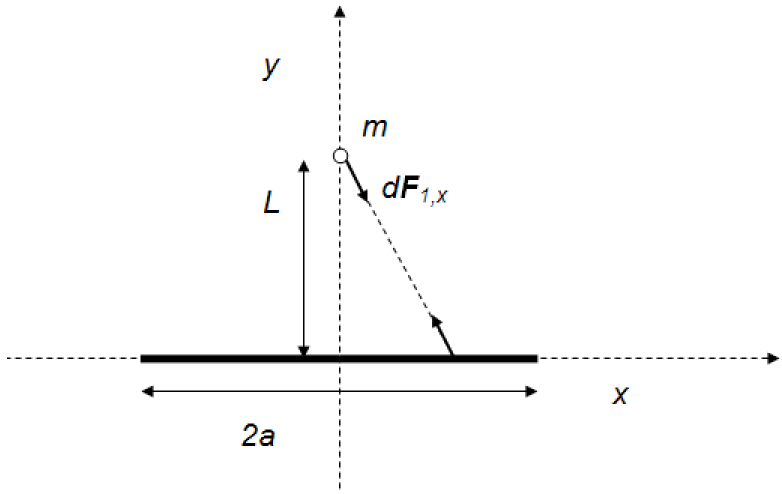

Figure 2: Problem 4