Mechanics and Special Relativity (MAPH10030)
Assignment 1

Issue Date: 01 February 2010
Due Date: 08 February 2010

1. Recall the definition of the dot product for vectors $\boldsymbol{a} = (a_1, a_2, a_3)$ and $\boldsymbol{b} = (b_1, b_2, b_3)$:

$$\boldsymbol{a} \cdot \boldsymbol{b} = \sum_{i=1}^{3} a_i b_i, \qquad |\boldsymbol{a}|^2 = \sum_{i=1}^{3} a_i^2.$$

   (a) Show that $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ [2 Points].

   (b) Show that if $|\boldsymbol{a} - \boldsymbol{b}| = |\boldsymbol{a} + \boldsymbol{b}|$, then $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b}$ [2 Point].

2. Refer to Eqs. (1) and (2). A girl throws a water balloon at an angle α above the horizontal with a speed $|\boldsymbol{v}_0|$. The horizontal component of the balloon's velocity $u = |\boldsymbol{v}_0| \cos\alpha$ is directed towards a car that is approaching the girl with a constant speed V. If the balloon is to hit the car at the same height at which it leaves her hand, what is the maximum distance the car can be from the girl when the balloon is thrown?

   The answer, H, involves V, v_0, α, and g [4 Points].

3. Consider a particle experiencing the force $F = kx$, a repulsive spring force.

   (a) Write down the equation of motion and the energy [1 Point].

   (b) Reduce the motion to an integral using the energy [1 Point].

   (c) Solve this integral using any method you know [2 Points].

4. A particle with mass m moves in one dimension with the celebrated *Lennard–Jones potential*

$$\mathcal{U}(x) = 4\epsilon \left[\left(\frac{x_0}{x} \right)^{12} - \left(\frac{x_0}{x} \right)^{6} \right].$$

   where ϵ and x_0 are positive constants.

   (a) Construct a timescale based on the energy ϵ, the lengthscale x_0, and the mass m. Hence, write down the non-dimensional equation of motion [2 points].

   (b) From this, identify the non-dimensional potential function, $\overline{\mathcal{U}}(s)$. Evaluate any maxima, minima, and zeros of the function. Then plot it [2 points].

(c) What is the non-dimensional period of small oscillations around the stable minimum? What is the corresponding dimensional value? [2 points]

(d) If the particle starts from rest at non-dimensional distance $s = x/x_0 = 1$, what is its ultimate fate? [2 points]

Recall the equations of motion for trajectory motion in a uniform gravitational field g:

$$x = x_0 + ut, \quad (1)$$
$$y = y_0 + vt - \tfrac{1}{2}gt^2. \quad (2)$$

where (x_0, y_0) is the initial location of the particle relative to a given inertial frame and (u, v) is the initial velocity. Neglect air resistance.