

Irish Research Council
Government of Ireland 'New Foundations' Scheme 2013
Application Form

Informative text has been included throughout this application form: please delete this informative text when completing the Form.

1. SUMMARY PROPOSAL DETAILS:	
Name of applicant (including title):	Dr Lennon Ó Náraigh
Former award held:	<i>ULYSSES award, IRCSET, May 2012-November 2013</i>
Title of proposed proposal:	
Abbreviation of proposal title	<i>PDE-4</i>
Proposal abstract (100 words):	Several problems in Physics and Engineering involve the solution of fourth-order nonlinear partial differential equations, with applications in phase separation, polymer fluid mechanics, and multiphase flows. The associated partial differential equations arising from these scenarios pose difficult problems for which standard methods in PDE theory and simulation do not apply. The aim of the proposed research is to develop an academic network to apply new techniques to analyse such equations using both rigorous analytical methods, and highly parallelized three-dimensional direct numerical simulation.
Discipline/subject area(s):	<i>Pure Mathematics, Applied Mathematics, Computational Fluid Dynamics</i>

4. DETAILED PROPOSAL DESCRIPTION (max. 1,000 words)***Aims and Objectives***

The aim of the research is the study of solutions of the following class of nonlinear fourth-order partial differential equation (PDE):

$$\frac{\partial u}{\partial t} = N(u) + \Delta^2 u, \quad t \in (0, T], \quad x \in \mathbb{R}^n, \quad n = 1, 2, 3,$$

where $u(x, t)$ is a scalar field, T is a positive real number, and where Δ^2 is the Bilaplace operator. Smooth initial data is supplied: $u(x, t=0) = u_0(x)$, where $u_0(x)$ is a function whose regularity will be prescribed in the research to follow. The following detailed objectives are proposed:

1. Use qualitative methods to understand the properties of the equilibrium solution. Obtain estimates for the norm of the solution in the relevant Sobolev space, for various forms of the nonlinearity.
2. Use qualitative methods to understand the dynamical problem. Investigate the circumstances in which the dynamical evolution involves a relaxation to the equilibrium solutions in (1). For certain standard nonlinearities (e.g. those of the Cahn-Hilliard or Allen-Cahn type), Lyapunov-functional theory can be used to determine rigorously the manner of convergence to equilibrium. For more atypical problems, new methods will be developed.
3. Develop numerical methods for quantitative predictions in support of the analysis in (1). Methods will be developed *ab initio* using Newton-Kantorovich iteration or some other scheme, and this approach will be supplemented by examination of off-the-shelf approaches, e.g. PETSC high-performance computing library.
4. For various nonlinear functions of practical relevance, develop and implement numerical methods for the transient problem, and investigate to what extent the equilibrium solutions in (1) and (2) inform the dynamics.

The research will be driven by a new collaboration between the PI and a researcher on the pure Mathematics side, Dr Iama Phibber (IP). The project is feasible because of the track record of the PI and Phibber in the modelling, simulation, and analysis of fourth-order nonlinear PDEs. The PI has 15 publications in fluid mechanics, fourth-order problems, and computational mathematics since 2006. Additionally, Phibber has an outstanding record of publication in the qualitative analysis of PDE problems (135 publications since 2002, and 5 books since 2008). Both researchers have a strong track record of participation in European collaborative research (a combined total of three ULYSSES grants, two HPC Europa FP7 grants, and one Marie Curie Fellowship).

Implementation

The research will be implemented by the formation of a new collaborative network directed by the PI and supported by Phibber. The research will be driven by regular meetings, attendance at conferences, and one workshop in Dublin bringing together the participants and interested parties.

Project management will be by regular meetings between Phibber and the PI. The aim of these meetings will be the management of dissemination and outcomes (see below) but more importantly, the close supervision of all strands of the problem so the interdisciplinary aims of the proposed research (bringing together engineers, physicists, pure and applied mathematicians) can be fulfilled.

Expected outputs and outcomes

1. The PI and one PhD student will make a number of academic visits to the collaborators on the numerical side, namely Dr Nemo Piccard (CNRS, Ecole Centrale de Bordeaux), and Dr Bertrand Zobrist (Computational Engineering, University of Edinburgh). *One publication in Physical Review E is targeted on the basis of these visits.*
2. Work under (1) will additionally be disseminated in two international conferences, the British Applied Mathematics Colloquium, and the American Physical Society Division of Fluid Dynamics Meeting. Funding for the PI's attendance at the BAMC is requested herein, but funding for the PhD student's conference attendance will be sought elsewhere.
3. On the qualitative/analytic side, *two publications in the Journal of Differential Equations are targeted.* The work will additionally be disseminated by Phibber in the European Conference on Elliptic and Parabolic Equations, Girona (Italy).
4. The keystone of the collaboration is a planned workshop bringing together workers on both the applied and the theoretical aspects. This will involve inviting relevant collaborators to Dublin, together with two high-level speakers on complex fluids to deliver a plenary seminar addressing interdisciplinary themes in the modelling and analysis of fourth-order nonlinear problems (Cuthbert Calculus, Leonid Gorbovsky, participation already confirmed).

Potential impacts and benefits

The project will have a strong impact because it targets a set of problems of industrial relevance in multiphase flow, polymer physics, and phase separation. The research will be disseminated in leading journals in Pure and Applied Mathematics and in Engineering, and at high-profile international conferences. Publication impact will be maximized by making the research freely available on institutional archives and/or individual researcher websites. Crucially, the workshop in Dublin will have a significant impact for a number of compelling reasons:

1. The bringing-together of experts on the modelling and simulation side of fourth-order problems, and on the rigorous analysis side, for an intensive two-day session of presentations and collaborative research.
2. The integration of a joint TCD-UCD analysis/applied mathematics seminar into the workshop programme, with two leading international experts to speak.
3. On the basis of the workshop (to involve participants from France, Italy, UK, Sweden, and Ireland), the formulation of an application to COST to create and fund a network of European researchers focused on fourth-order nonlinear problems in the Mathematical Sciences.
4. Involvement of other researchers in the workshop, e.g. members of the Xwave group of Asimov Tarkovsky (holder of ERC advanced grant Xwave, exploring analogies between Quantum Mechanics and fluid turbulence) will be invited, since further collaboration in this area is of great relevance to the proposed research.

A further benefit of the proposed research is to strengthen the economic impact of Pure Mathematics research in UCD, thereby making funding applications from Mathematics more competitive in future national and European funding initiatives.

Costs

The proposed cost of the project is €7,000. Costs of attendance at the workshop by two researchers up to €1,000 will be covered by funding from another source, PRTL, "Simulation Science". This funding is already in place (note: Simulation Science would not fund the present proposal in its entirety). Additionally, funding for the PhD student's conference attendance will be obtained from elsewhere (UCD seed funding – application in progress). Consequently, €6,000 is sought from the council.

5. DOES THE PROPOSAL OUTLINED IN THIS SUBMISSION REQUIRE APPROVAL BY THE RELEVANT UNIVERSITY/INSTITUTIONAL ETHICS COMMITTEE?	YES ()	NO (X)
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If there are ethical implications of proposal outlined above, please provide details of what they are and how they will be addressed:

In the event of a successful outcome to this application, where relevant, written evidence of such ethical approval must be received by the Research Council prior to confirmation of the grant.

APPLICANT'S DECLARATION

I declare that the above particulars are correct and understand that the circulated 'Terms and Conditions' apply. I accept that failure to abide by the 'Terms and Conditions' may disqualify me from this Scheme. I also authorise the Irish Research Council to verify, if necessary, any of the information supplied in this application.

Signature of Applicant:	
Date:	

ENDORSEMENT BY INSTITUTIONAL AUTHORITIES

I hereby endorse this application to the Irish Research Council in accordance with the circulated 'Terms and Conditions'.

Signature of Head of Department/School/Faculty or equivalent: (please indicate position held)	
Date:	
Signature of Proposed Mentor: (where required)	
Date:	
Signature of Vice-President for Research or authorised signatory: (please indicate position held)	
Date:	

Institutional Stamp:

