UNIVERSIDADE ESTADUAL DE CAMPINAS

DEPARTAMENTO DE TELEMÁTICA - FEEC

CONSTRUCTION OF NEW TORIC QUANTUM CODES

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Presentation Outline

- Introduction
  - Quantum codes
  - Proposal
- Stabilizer Codes
- Known Toric Quantum Codes
- New Toric Quantum Codes
- Conclusions

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- Known Toric Quantum Codes
  - Kitaev's codes
  - Algebraic interpretation
  - Bombin and Martin-Delgado's codes
- New Toric Quantum Codes
- Conclusions

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- Known Toric Quantum Codes
- New Toric Quantum Codes
  - Lattices, quadratic forms and polyominoes
  - Toric quantum codes - An algebraic approach
  - Parameters of the codes
  - Shape of the polyomino
  - Reproduced known toric quantum codes
  - New class of toric quantum codes
- Conclusions

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- TORIC CODES:
  Kitaev proposes the class of *toric codes*, a subclass of the stabilizer quantum codes associated with the square lattice of torus $\mathbb{Z}^2$.

Proposal

- PROPOSAL:
  It is possible to generate toric quantum codes by means of tessellations of square lattice of torus by translations of a determined fundamental region.

Stabilizer Codes

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- **Pauli group of n qubits:**

$$\mathcal{P}_n = \pm\{I, \sigma_x, \sigma_y, \sigma_z\}^{\otimes n}.$$

$$I \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_x \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

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- **Properties:**
  - For each $M \in \mathcal{P}_n$, we have $M^2 = \pm I$;
  - If $M^2 = I$, then M is Hermitian; if $M^2 = -I$, then M is anti-Hermitian.
  - $M, N \in \mathcal{P}_n \Rightarrow MN = NM$ or $MN = -NM$.

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- The elements of $\mathcal{S}$ are called **stabilizer operators**.
- A stabilizer code $\mathcal{C}_S \subseteq \mathcal{H}$ associated to $\mathcal{S}$ is the simultaneous eigenspace, with eigenvalue $+1$, comprising all elements of an Abelian subgroup $\mathcal{S}$

$$\mathcal{C}_S = \{|\psi\rangle; \quad M|\psi\rangle = |\psi\rangle \quad \forall M \in S\}.$$

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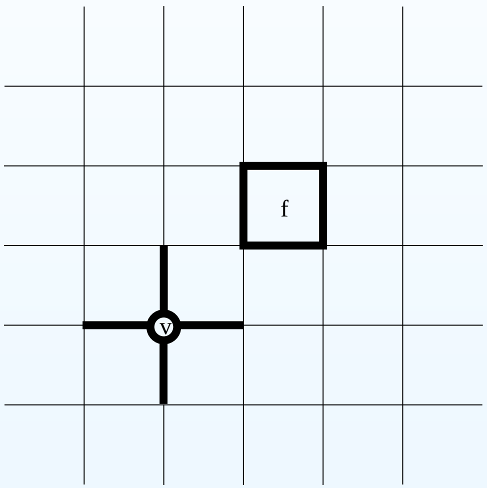
- The operators of $\mathcal{P}_n$ which commute with every $M_i \in S$ but do not belong to $\mathcal{S}$, preserve the coding space $\mathcal{C}_S$, by not acting trivially on it.
- The code distance is given by the least weight of $E \in \mathcal{P}_n$ such that E commutes with every $M_i \in \mathcal{S}$ but does not belong to $\mathcal{S}$.

Known Toric Quantum Codes

Kitaev's Toric Codes

- Qubits $\leftrightarrow$ edges of the tessellation $\{4, 4\}$ of the two dimensional torus.



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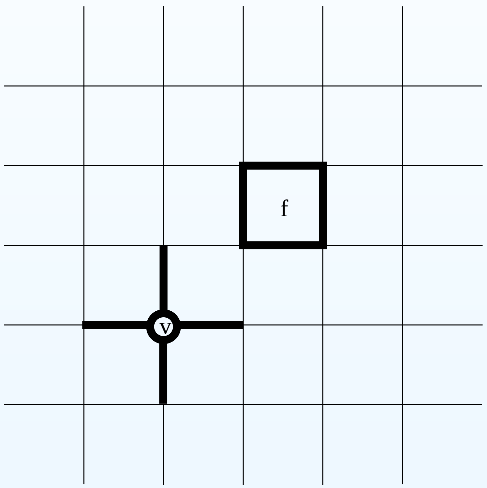

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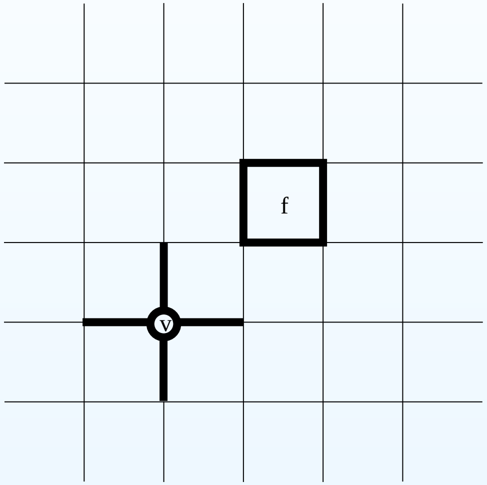

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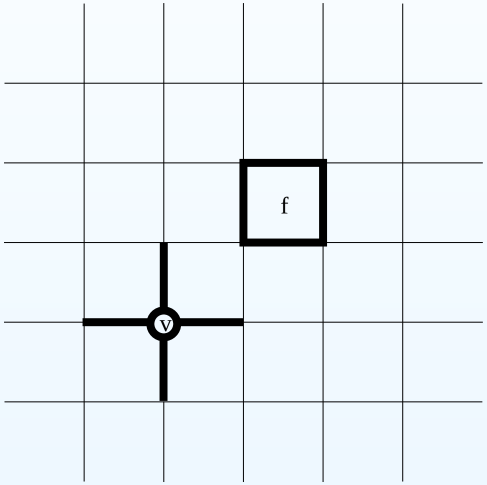

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- $\mathcal{C} = \{|\psi\rangle : A_v|\psi\rangle = |\psi\rangle,\ B_f|\psi\rangle = |\psi\rangle\ \ \forall\, v, f\}.$

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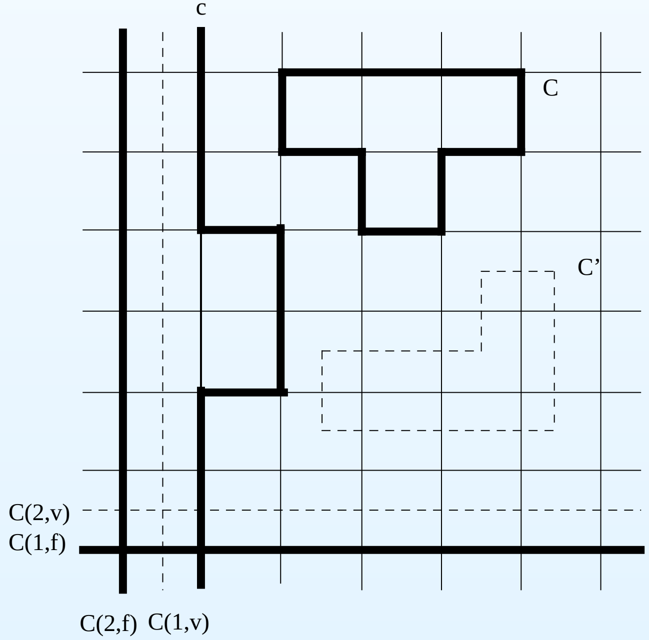

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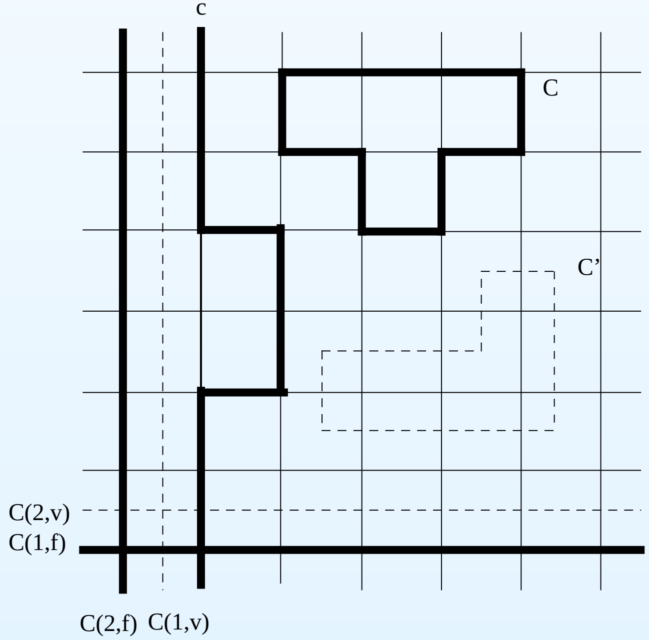

- $[[2m^2, 2, m]]$.

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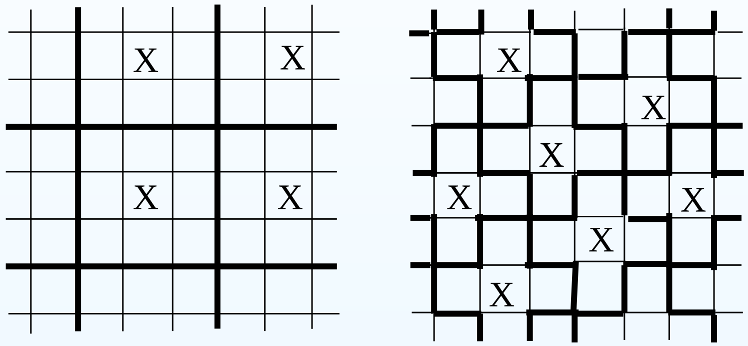
- The minimum distance of the code corresponds to the least number of edges in the dual lattice to be covered between the coset representatives, $d = m$.

Bombin and Martin-Delgado's Codes

- Another regular lattice of the torus:



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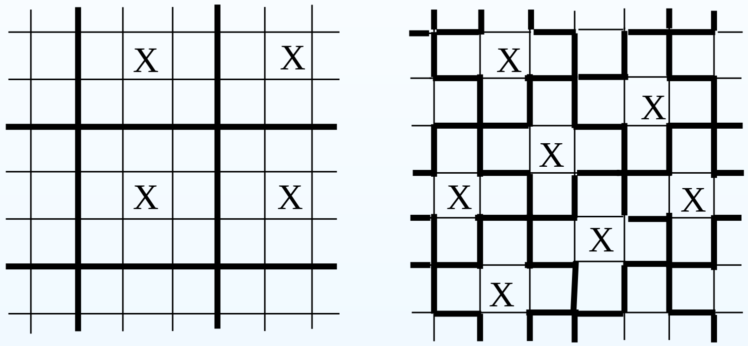

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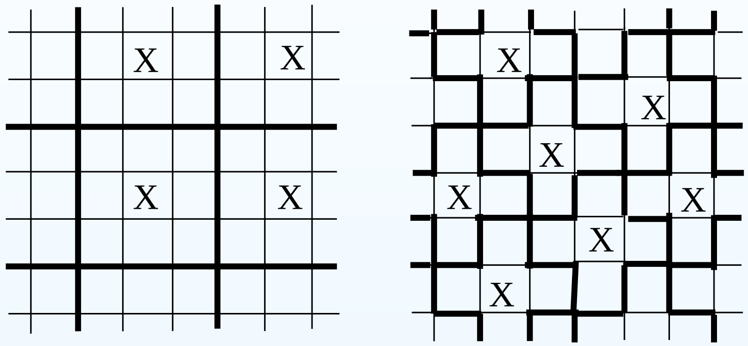

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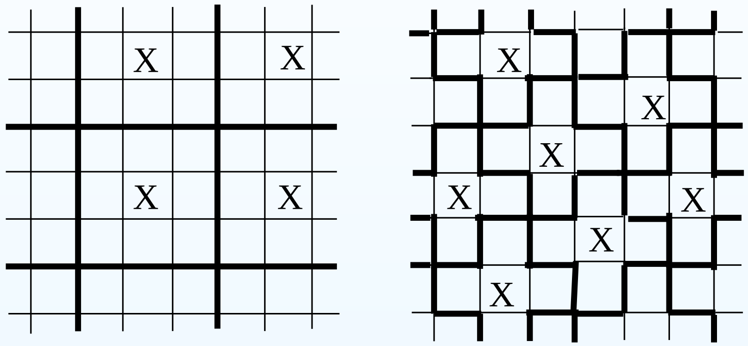

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- The length of the code n is the number of edges of the Lee spheres and the minimum distance of the code is the radius r of Lee spheres.
- This system of lattices supplies codes with parameters $[[d^2 + 1, 2, d]]$ and keep the same properties of the original Kitaev's code.

New Toric Quantum Codes

Lattices, Quadratic Forms and Polyominoes

- The lattice $\mathbb{Z}^2$ is generated by vectors $\nu_1 = (1, 0)$ and $\nu_2 = (0, 1)$, with fundamental region described by a square, with area equal to 1, and its corresponding quadratic form is $x^2 + y^2$.

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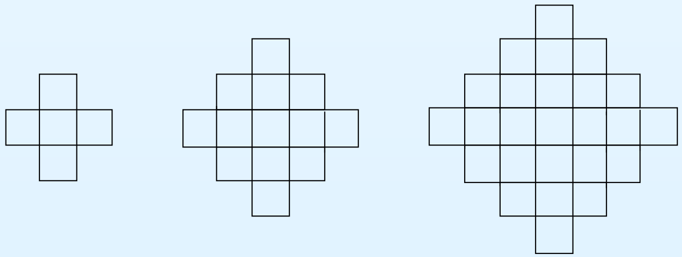
- Lee spheres are a special type of polyominoes that was used to generate perfect classical codes and the class of $[[d^2+1, 2, d]]$ quantum codes.



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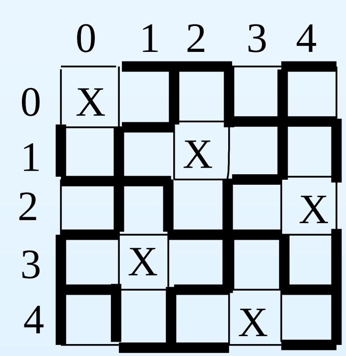
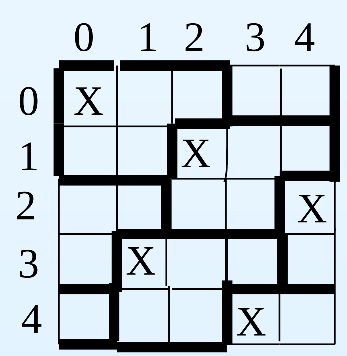
- X $\to (x, y) \in \mathbb{Z}_m \times \mathbb{Z}_m$ indicates the place of a polyomino.





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- Since we wish that $|\mathcal{A}| = m$, we have:
  - $gcd(x, y) = 1 \Rightarrow \mathcal{A} = \langle (x, y) \rangle$.
  - $gcd(x, y) = \delta \neq 0, 1 \Rightarrow \mathcal{A} = \langle (x, y), (-y, x) \rangle$.

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- The quantum code associated to this tessellation is defined as:
  - $n =$ number of edges of the polyomino $= 2m$.
  - $k = 2$.
  - The minimum code distance is given by the shortest distance between two representatives of the polyominoes. Therefore

    $$d = d_M = |x| + |y|,$$

    where d_M is the *Mannhein distance*.
  - $[[2m, 2, d_M]]$.

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- The shape of the polyomino influences the error correction pattern.
- The optimum shape for the polyomino depends on the type of graph associated with the discrete channel without memory.
- This shape may generally be considered as the union of a square $x \times x$ with a square $y \times y$.

$[[d^2+1, 2, d]]$ codes

- When $m = 2t^2 + 2t + 1$, for $t = 1, 2, 3, \ldots$, Bombin and Martin-Delgado's codes are reproduced.

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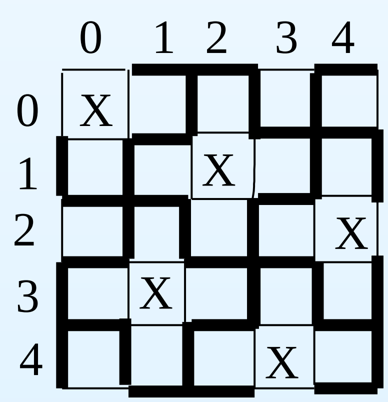
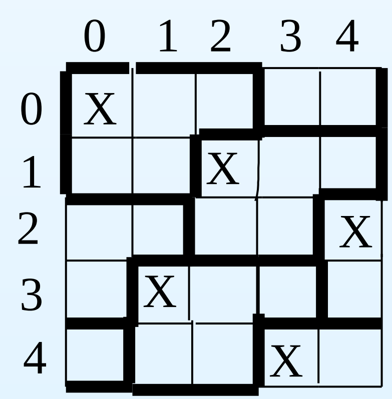
- $[[d^2+1,2,d]]$.
- **Example:** $m = 5 \Rightarrow \mathcal{A} = \langle (2,1) \rangle$. Code $[[10,2,3]]$ is obtained.





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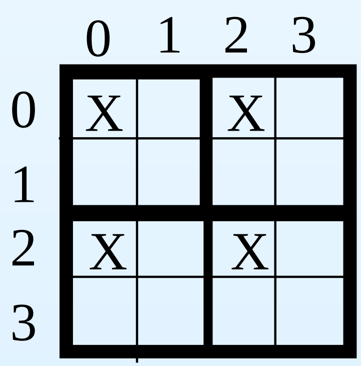
- **Example:** $m = 4 \Rightarrow \mathcal{A} = \{(0,0), (2,0), (0,2), (2,2)\}$. Code $[[8,2,2]]$ is obtained.



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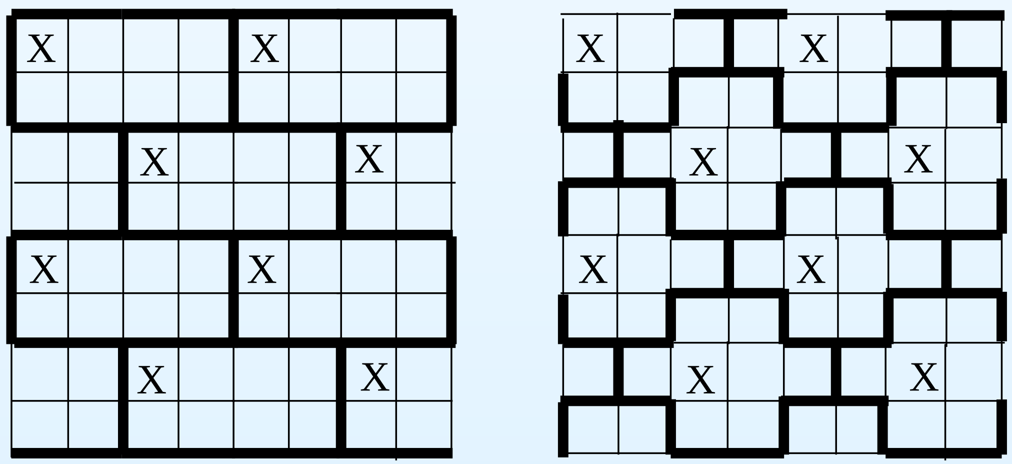
- $[[d^2, 2, d]]$.
- $k/n = 1/d$.
- **Example:** $m = 8 \Rightarrow$ $\mathcal{A} = \{(0,0), (2,2), (4,4), (6,6), (6,2), (2,6), (0,4), (4,0)\}$. Code $[[16, 2, 4]]$ is obtained.



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- The codes defined in this way keep the same properties of Kitaev's toric codes;
- Besides reproducing known classes of codes, new classes of toric codes are determined. For instance, the class $[[d^2, 2, d]]$, the best known so far.

Acknowledgment

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- FAPESP
- CNPQ