Problem sheet 7

1. Which of the following are groups under (i) addition, (ii) multiplication:

   (a) Real numbers of the form $a+b\sqrt{2}$, where a, b are rational numbers;

   (b) Invertible 2×2 matrices with entries from $\mathbb{R}$.

2. Let G be a group. Show that each row in the Cayley table of G contains each element of G exactly once (we do not consider the first column and first row as part of the table, they are rather just the lists of the elements of the group). The same is true for the columns, with a very similar justification.

3. Let $G = \{a, b, c, d\}$ be a group with 4 elements. Complete the Cayley table of G:

$\cdot$	a	b	c	d
a			a	
b		d		a
c		b		
d			d	

   Which element is the identity element? What is the inverse of a? Is the group Abelian (how can you see it on the table?)?

4. Let G be an abelian group of order n and let $a_1, \ldots, a_n$ be all the elements of G. Let $a = a_1 a_2 \cdots a_n$. Show that a has order at most 2. Hint: If x is in G, then x^{-1} is also in G.