## Problem sheet 7

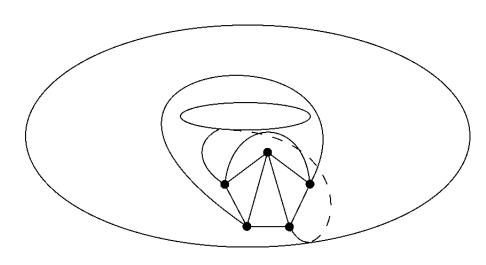
1. Let f be the (potential) isomorphism from  $G_1$  to  $G_2$ . 3 is the only vertex of  $G_1$  of degree 2 so we must take f(3) = 5. Then 2 (being adjacent to 3) has to be sent to 1 or 4. Let's pick f(2) = 1. Then we must have f(4) = 4. Looking at the graphs, there is no obvious difference between the properties of the remaining vertices, so we take (at random) f(5) = 2 and f(1) = 3.

We now must check, for every vertices u and v in  $G_1$ , that uv is an edge iff f(u)f(v) is an edge. The most efficient way to do this is to check that for every edge uv of  $G_1$  then f(u)f(v) is an edge of  $G_2$ , and that for every edge xy of  $G_2$  then  $f^{-1}(x)f^{-1}(y)$  is an edge of  $G_1$ . It is a bit long but straightforward.

- 2. Since it is Eulerian there is a circuit that will go through all streets exactly once. Since he needs to go through every street at least once, the Euler circuit is a shortest possible route.
- 3. Let (X, Y) be a bipartition of G, and let f be the isomorphism from G to H. Then  $V_H = f(X) \cup f(Y)$  with  $f(X) \cap f(Y) = \emptyset$ . Furthermore, if uv is an edge in H, we have u = f(a) and v = f(b) for some  $a, b \in V_G$ . Since G is bipartite, one of a, b is in X and the other in Y, so one of u, v is in f(X) and the other in f(Y).

You can also use the isomorphism to show that H has no cycle of odd length, which will show that H is bipartite.

4. An isomorphism would have to send the unique vertex of degree one of the first graph to the unique vertex of degree one of the second graph, so a to 2, then d to 8, then  $\{g, h\}$  to  $\{3, 6\}$ . But 3 and 6 are adjacent, while g and h are not.



5.