Graphs and Networks (MATH20150) 2023/2024

Problem sheet 7

1. Let f be the (potential) isomorphism from G_1 to G_2. 3 is the only vertex of G_1 of degree 2 so we must take $f(3) = 5$. Then 2 (being adjacent to 3) has to be sent to 1 or 4. Let's pick $f(2) = 1$. Then we must have $f(4) = 4$. Looking at the graphs, there is no obvious difference between the properties of the remaining vertices, so we take (at random) $f(5) = 2$ and $f(1) = 3$.

   We now must check, for every vertices u and v in G_1, that uv is an edge iff $f(u)f(v)$ is an edge. The most efficient way to do this is to chekc that for every edge uv of G_1 then $f(u)f(v)$ is an edge of G_2, and that for every edge xy of G_2 then $f^{-1}(x)f^{-1}(y)$ is an edge of G_1. It is a bit long but straightforward.

2. Since it is Eulerian there is a circuit that will go through all streets exactly once. Since he needs to go through every street at least once, the Euler circuit is a shortest possible route.

3. Let (X, Y) be a bipartition of G, and let f be the isomorphism from G to H. Then $V_H = f(X) \cup f(Y)$ with $f(X) \cap f(Y) = \emptyset$. Furthermore, if uv is an edge in H, we have $u = f(a)$ and $v = f(b)$ for some $a, b \in V_G$. Since G is bipartite, one of a, b is in X and the other in Y, so one of u, v is in $f(X)$ and the other in $f(Y)$.

   You can also use the isomorphism to show that H has no cycle of odd length, which will show that H is bipartite.

4. An isomorphism would have to send the unique vertex of degree one of the first graph to the unique vertex of degree one of the second graph, so a to 2, then d to 8, then $\{g, h\}$ to $\{3, 6\}$. But 3 and 6 are adjacent, while g and h are not.

5.