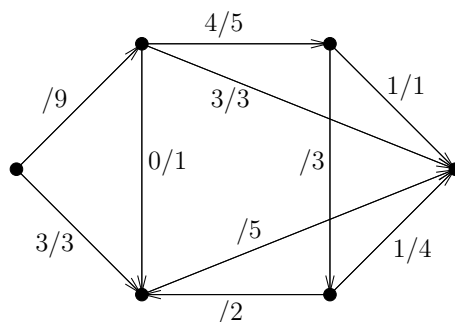## Problem sheet 9

1. Assume that the edges of K_{11} are coloured red and blue. Show that the blue graph and the red graph cannot both be planar. Hint: We know something about the number of vertices and edges in a planar graph.

2. In the following network, we have indicated the capacity, and some of the values of a map $f : E \to \mathbb{R}_+$. Determine the missing values so that f is a flow.


3. Let N be a network in which the capapcity of every arc is even. Show that a maximum flow has even value.

4. (a) Prove that a bipartite graph with an odd number of vertices is not Hamiltonian.

   (b) Show that the following graph is not Hamiltonian
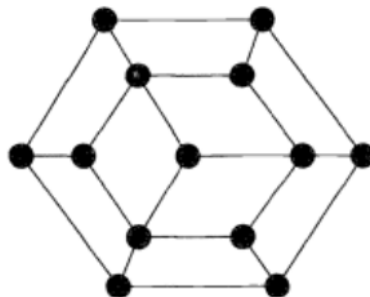
   (c) Show that, if n is odd, it is not possible for a knight to visit all the squares of an $n \times n$ chessboard exactly once and return to its starting point (using knight's moves). It is useful to observe that a knight's move always go from a square of one colour to a square of another colour.

5. Let $G = (V, E)$ be a graph. The complement of G is the graph $\bar{G}$ with set of vertices V, in which two vertices are adjacent if and only if they are not adjacent in G.

(a) Draw the complement of the following graph
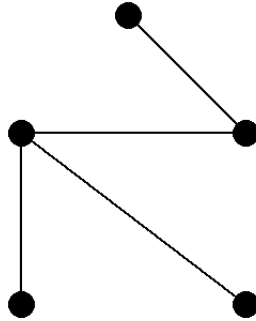
(b) Show that the complement of a complete bipartite graph is the union of two complete graphs.