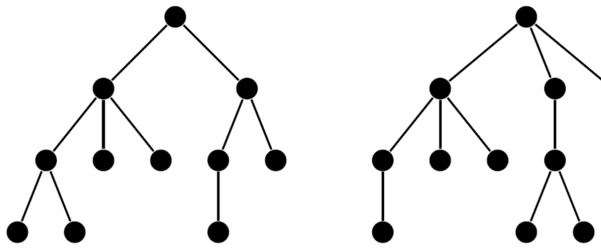
Problem sheet 8

1. Are the following trees isomorphic?



2. (a) Show on an example that the Euler formula for planar graph is not correct for non-connected graphs.
   (b) Produce an Euler formula that works for any type of planar graphs, even disconnected ones. (This formula will involve the number of components.)

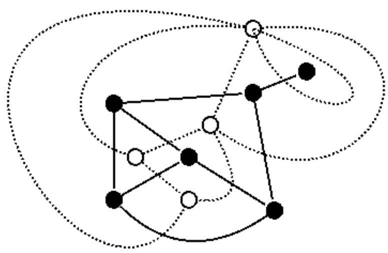
3. (a) Find a planar graph with degree sequence $(3, 3, 3, 3, 4, 4)$.
   (b) Find a non-planar graph with degree sequence $(3, 3, 3, 3, 4, 4)$. Hint: What are the main two graphs we know are not planar? (You will need to finish reading the chapter on planar graphs first.)

   So the degree sequence on its own does not always tell you if a graph is planar or not.

4. Let G be a planar representation of a graph (V, E), with set of faces F. The dual G^* of G is defined as follows (it is actually a pseudograph):

   For each face f of G, you put a vertex f^* of G^* in the face f. If e is an edge of G separating two faces f_1 and f_2 of G, you put an edge e^* between f_1^* and f_2^* (it helps to draw it in such a way that e^* intersects e). If e is totally inside a face f of G (e is a pendant edge), you put a a loop from f^* to f^*.

   Example, where G is in black and full lines, while G^* is in white and dotted lines.



   Using the two graphs below, show that two planar representations of the same graph may not have isomorphic duals: