

Problem sheet 4

1. A saturated hydrocarbon is a molecule C_mH_n in which every carbon atom (C) has four bonds (=is linked to 4 other atoms) and every hydrogen atom (H) has one bond, and no sequence of bonds forms a cycle. Show that, for every positive integer m , C_mH_n can exist only if $n = 2m + 2$.
2. Let G be a graph with n vertices and let A be the adjacency matrix of G . Show that G is connected if and only if $(A + I_n)^{n-1}$ has no zero entries. Hint: Is $A + I_n$ the adjacency matrix of some graph (possibly pseudograph)?
3. Let G be a connected graph with n vertices and m edges. We remove from G the minimum number of edges such that the resulting graph T contains no cycle. What is the value of this minimum number of edges? Hint: Show first that T is a tree.
4. Let G be a graph. Show that the following two statements are equivalent:
 - (a) G is a forest.
 - (b) For every two vertices x, y in G with $x \neq y$, there is at most one path in G from x to y .