Graphs and Networks (MATH20150) 2023/2024

Problem sheet 2

1. We say that a graph is k-regular if all its vertices have degree k. Draw two 3-regular graphs with the same same set of vertices, one connected, the other not. Hint: It can be helpful to start by constructing the smallest possible 3-regular graph, to have an example; then construct a simple 3-regular graph that is not connected.

2. Let V be a set of n points in the plane such that the distance between any two points is at least 1. The objective of this exercise is to show that there are at most $3n$ pairs (x, y) of elements of V with the property that the distance from x to y is 1.

   (a) Show that if $x \in V$, then there are at most 6 points of V in the circle of centre x and radius 1 (draw some pictures first, it should help). Hint: Think equilateral triangles.

   Define G to be the graph with vertex set V and where there is an edge between x and y if and only if the distance between x and y is 1.

   (b) Show that the degree of any vertex in G is at most 6.

   (c) Prove the result.

3. Let $G = (V, E)$ be a graph such that for every $v \in V$, $d(v) \geq k$. Show that there is a path of length at least k in G.
   Hint: Let $W = v_1 v_2 \cdots v_r$ be a path of maximal length in G (why is there such a path?). Show that we must have $r \geq k + 1$.

4. Is the sequence $(1, 1, 3, 3, 5, 5)$ graphic?
   Is the sequence $(1, 2, 2, 3, 4)$ graphic?