

# Atmospheric Fronts

The material in this section is based largely on  
*Lectures on Dynamical Meteorology*  
by Roger Smith.

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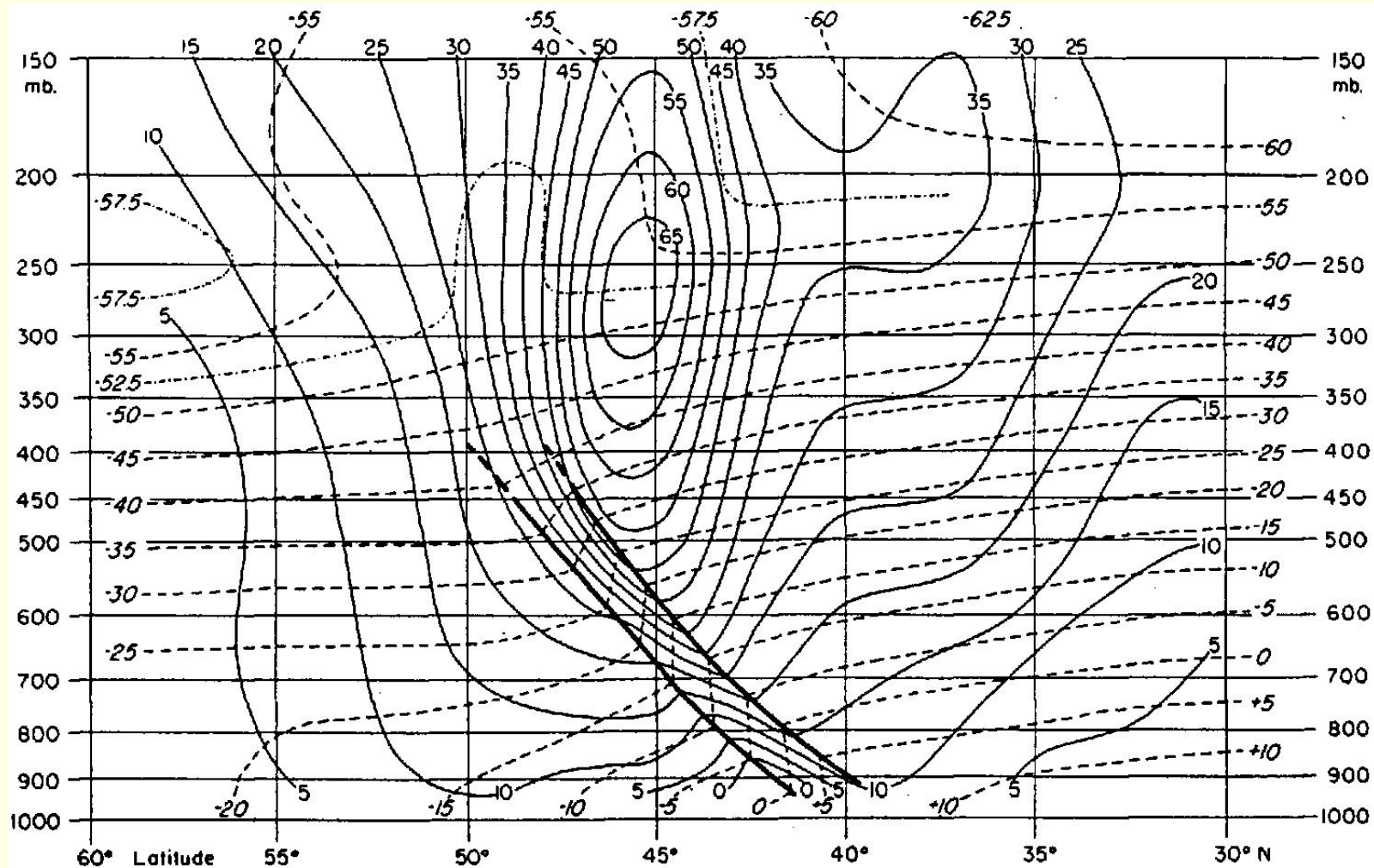
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Sharp temperature differences can occur across a frontal surface: several degrees over a few kilometres.

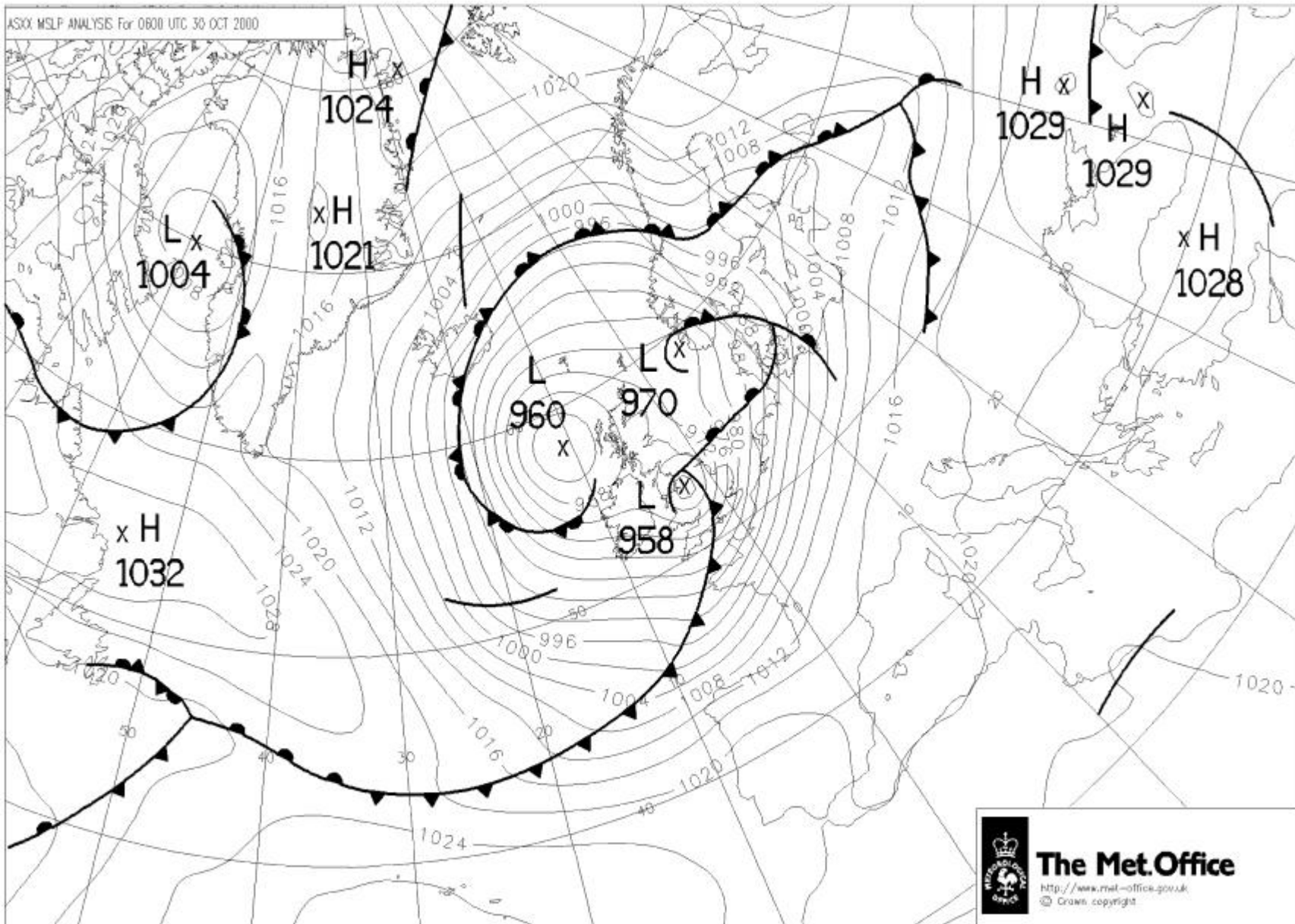
The following Figure shows the passage of a cold front.







Composite meridional cross-section at 80°W of mean temperature and the zonal component of geostrophic wind computed from 12 individual cross-sections. The means were computed with respect to the position of the polar front in individual cases (from Palmén and Newton, 1948).



Analysed surface pressure, storm in October, 2000.

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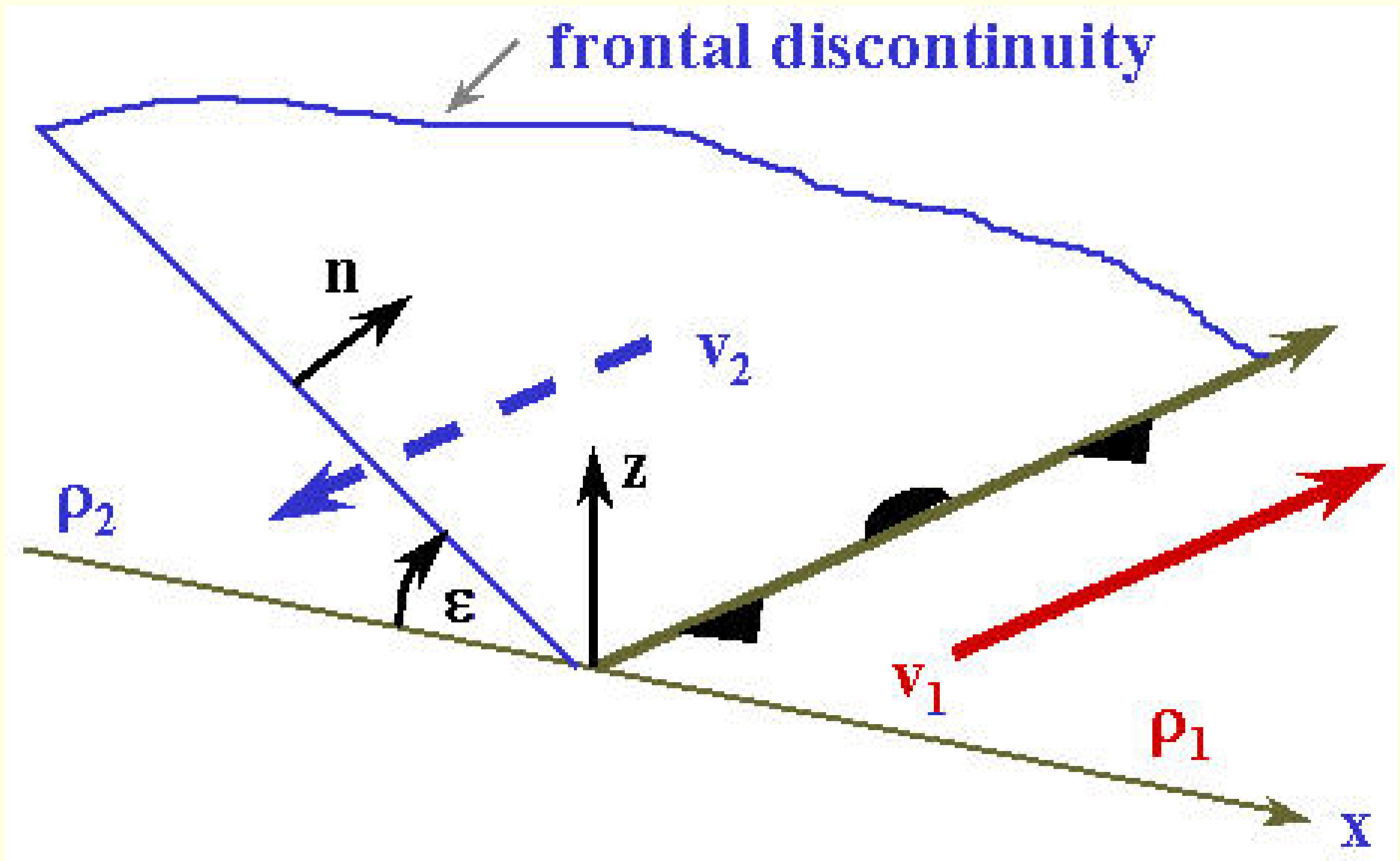


Max Margules (1856–1920)

The simplest model representing a *frontal discontinuity* is **Margules' model**.

In this model, the front is idealized as a sharp, plane, temperature discontinuity separating two inviscid, homogeneous, geostrophic flows.





Configuration of Margules' frontal model.  
Subscripts 1 and 2 refer to the warm and cold air masses.

We take the  $x$ -direction to be normal to the surface front and the  $y$ -direction parallel to it.

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We assume that the temperature difference between the air masses is small in the sense that  $(T_1 - T_2) / \bar{T} \ll 1$ , where  $\bar{T} = (T_1 + T_2) / 2$  is the mean temperature of the two air masses,  $T_1$  the temperature of the warm air and  $T_2$  the temperature of the cold air.

We assume the temperature and density are such that

$$\begin{array}{lcl} T = \bar{T} + T' & \text{where} & T' \ll \bar{T} \\ \rho = \bar{\rho} + \rho' & & \rho' \ll \bar{\rho} \end{array}$$

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The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

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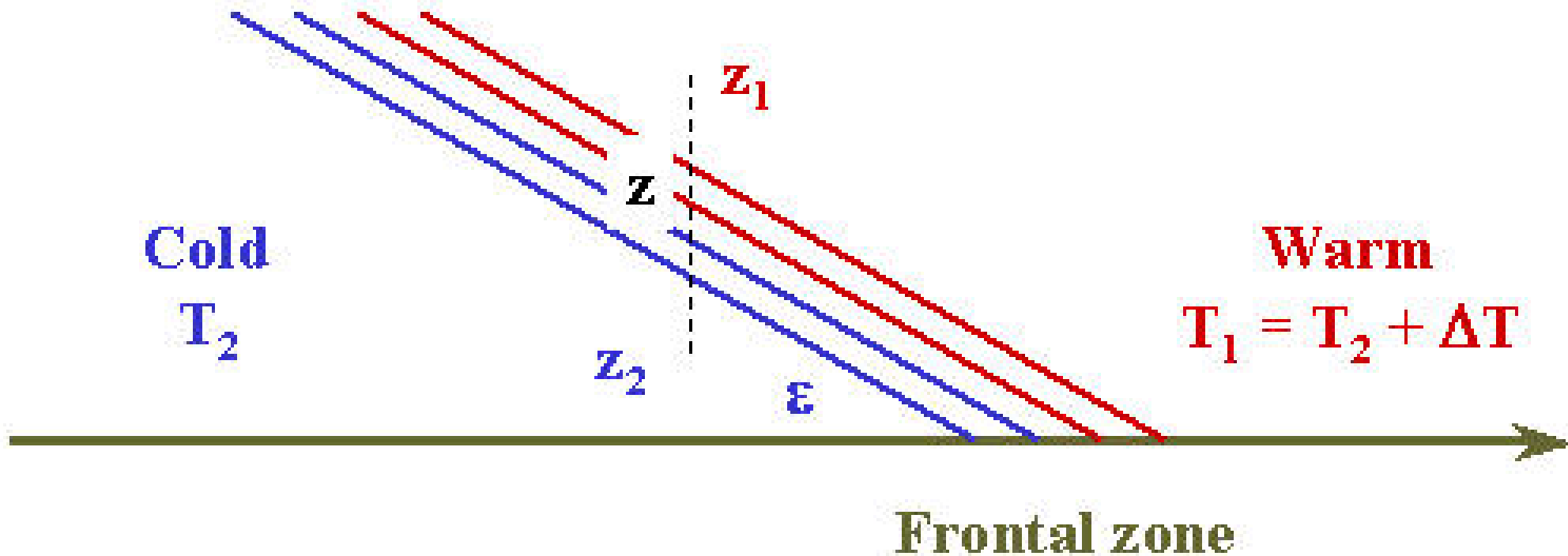
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We consider this to be the limiting case of the situation in which the temperature gradients are very small except across the frontal zone, where they are very large.

## Frontal zone



Vertical cross-section through a (smeared-out) front.  
The coloured lines indicate isotherms.

In the frontal zone  $T = T(x, z)$ . Otherwise,  $T$  is constant.

On any isotherm the temperature is constant, so that

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$$f \frac{\partial v}{\partial z} = \frac{1}{\bar{\rho}} \frac{\partial^2 p}{\partial x \partial z} = \frac{g}{\bar{T}} \frac{\partial T}{\partial x} = \frac{g}{\bar{T}} \frac{\partial T}{\partial z} \tan \varepsilon$$

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This is simply the *thermal wind equation* relating the vertical shear across the front to the horizontal temperature contrast across it.



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This is *Margules' formula* and relates the slope of the frontal surface to the change in geostrophic wind speed across it and to the temperature difference across it.

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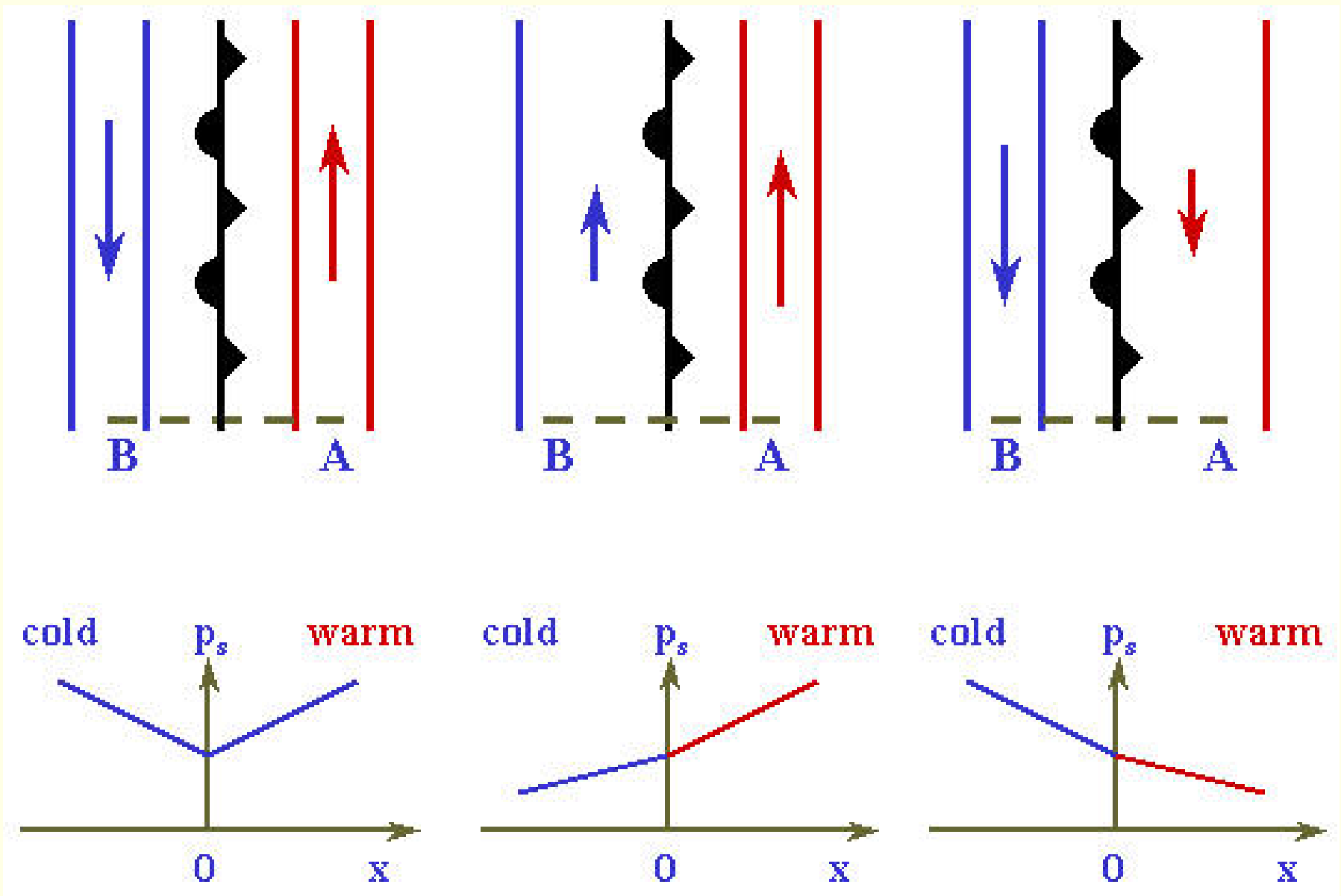
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There are three possible configurations as illustrated below.





Surface isobars in Margules' stationary front model hemisphere showing the three possible cases with the cold air to the left:

(left)  $v_1 > 0, v_2 < 0$ ; (centre)  $0 < v_2 < v_1$ ; (right)  $v_2 < v_1 < 0$ ;

The surface pressure variation along the line AB is also shown.

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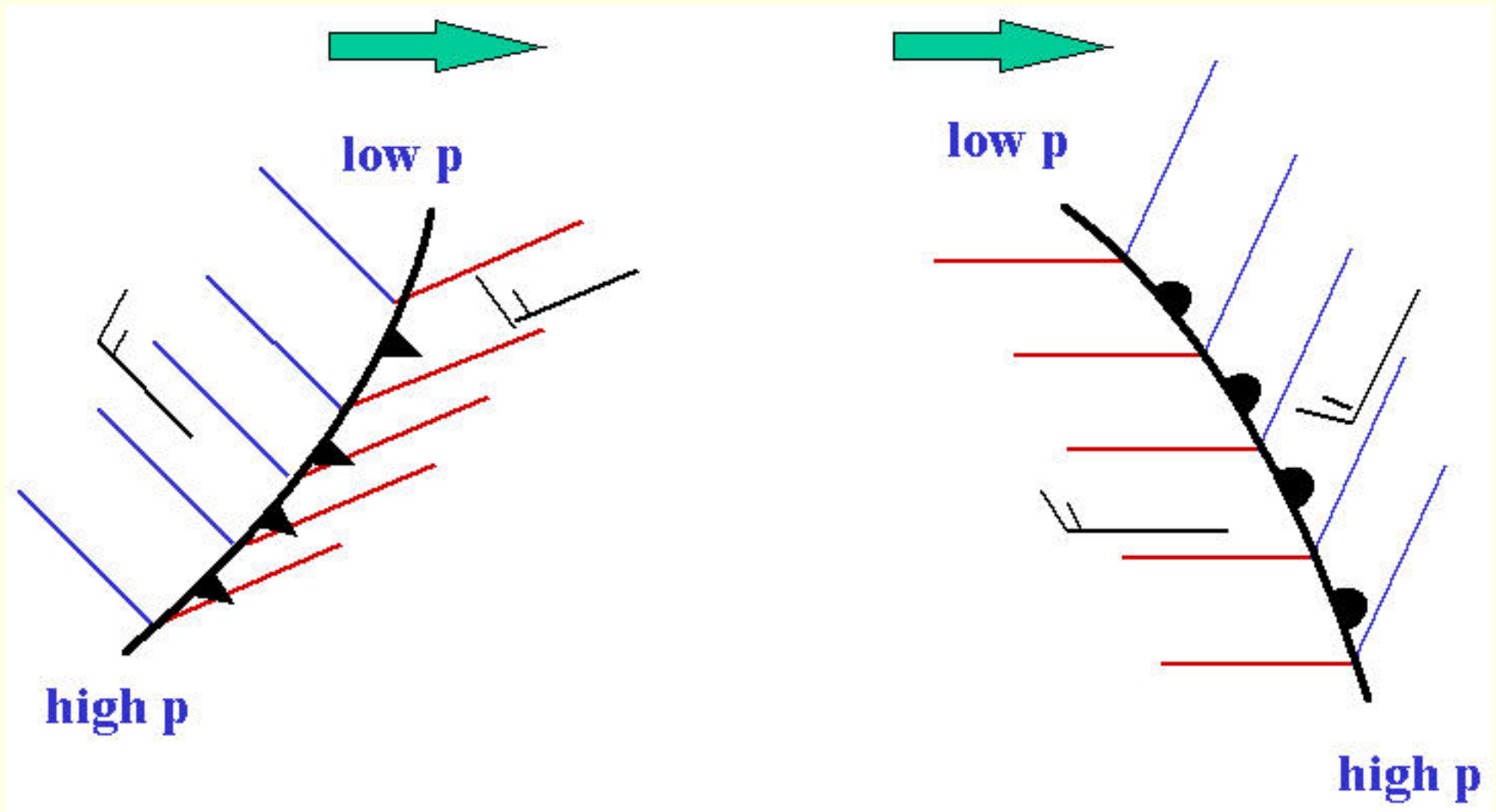
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Moreover, real cold and warm fronts are generally not stationary, but may have speeds comparable to the horizontal wind itself. We illustrate fronts in motion in the following figure.



Schematic representation of (left) a translating cold front and (right) a translating warm front as they might be drawn on a mean sea level synoptic chart for the northern hemisphere. Note the sharp cyclonic change in wind direction and the discontinuous slope of the isobars.

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It is interesting that Margules developed his model of an atmospheric discontinuity some fifteen years before the emergence of the frontal models of the **Norwegian School**. There were other precursors of frontal theory in Germany and in Britain.

**Exercise:** Check the dimensional consistency of Margules' Formula.

Calculate the frontal slope using Margules' Formula, assuming that the mean temperature is  $\bar{T} = 280 \text{ K}$ , the Coriolis parameter  $f = 10^{-4} \text{ s}^{-1}$ ,  $g = 10 \text{ m s}^{-2}$ , the difference in wind-speed across the front is  $\delta v = 12 \text{ m s}^{-1}$  and the difference in temperature is  $\delta t = 4 \text{ K}$ .

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**Solution:** ...

Answer:  $\epsilon \approx 1/120$ .