Forecast Factories Changing Climates Throbbing Triads and Swinging Springs

Peter Lynch

Meteorology & Climate Centre, Mathematical Physics Department

Outline of Presentation

This presentation will cover three separate topics

- ***[1] Digital Filter Initialization***

The failure of Richardson's forecast. The elimination of spurious high-frequency oscillations from forecast models.

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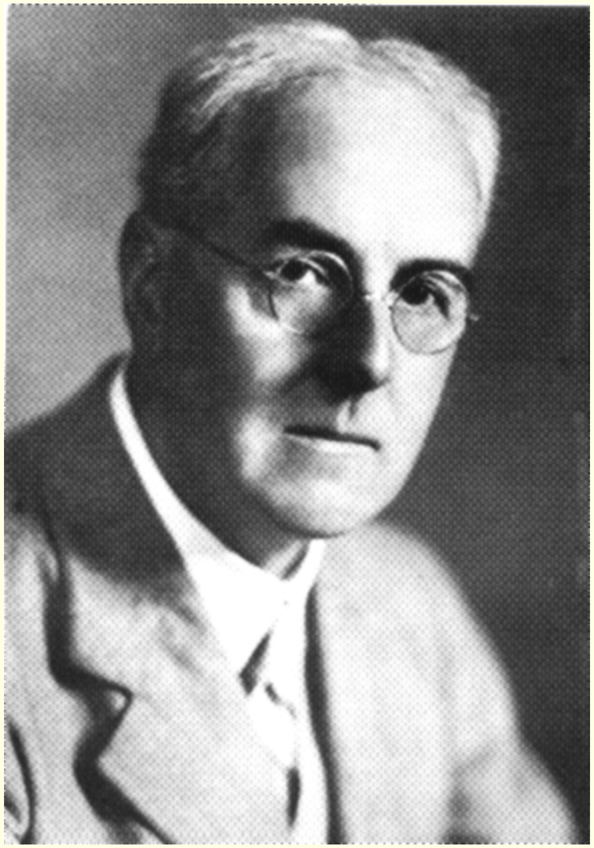
- ***[3] Springs and Triads***

The behaviour of resonant Rossby wave triads, and the insight gained by consideration of a simple mechanical system.

Lewis Fry Richardson, 1881–1953.



Richardson contributed to

- Meteorology
- Numerical Analysis
- Fractals
- Psychology
- Conflict Resolution

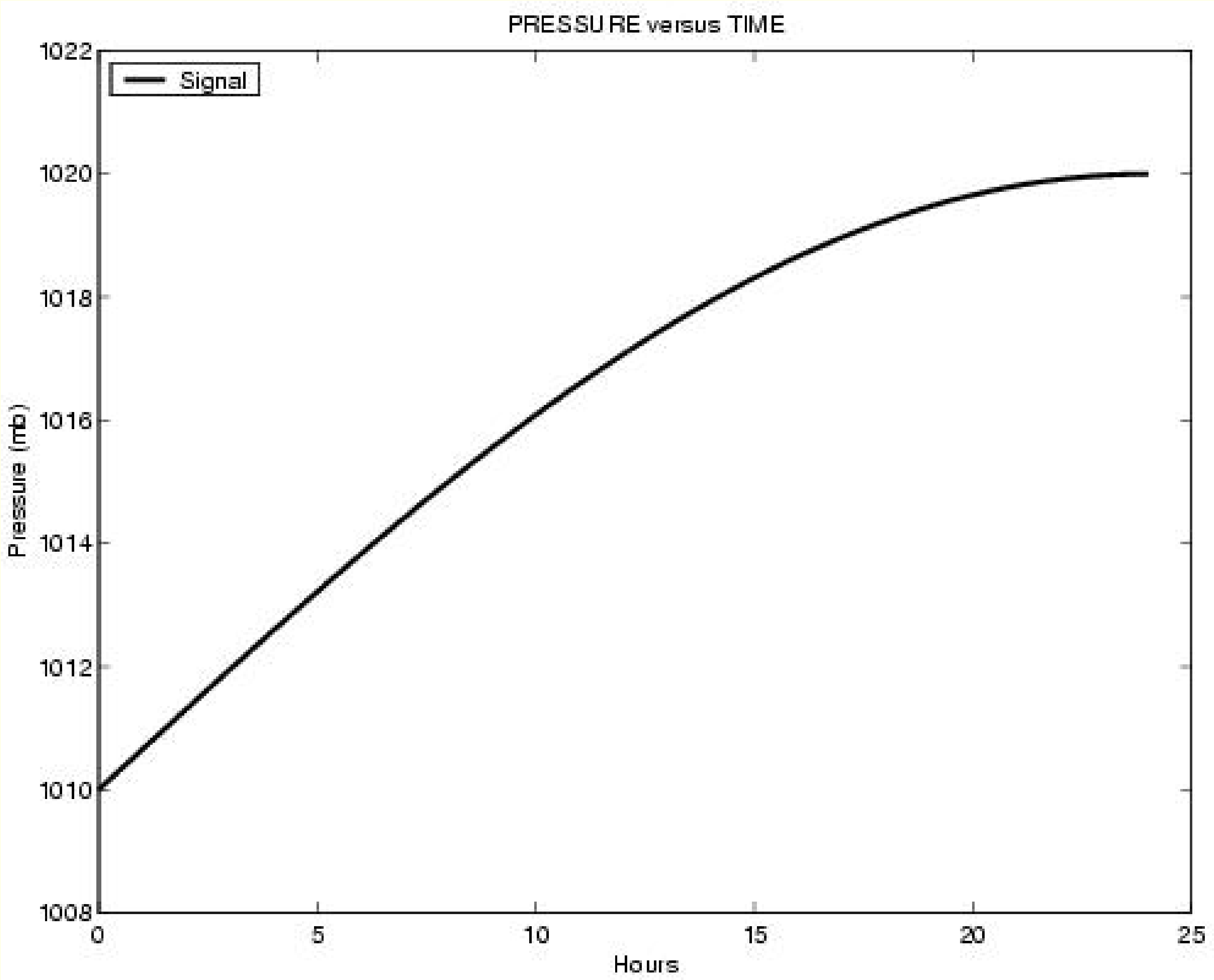
During the *First World War*, Richardson calculated, by hand, the change in pressure at a single point, using the equations of motion. The results were totally unrealistic.

Weather Prediction by Numerical Process published in 1922.

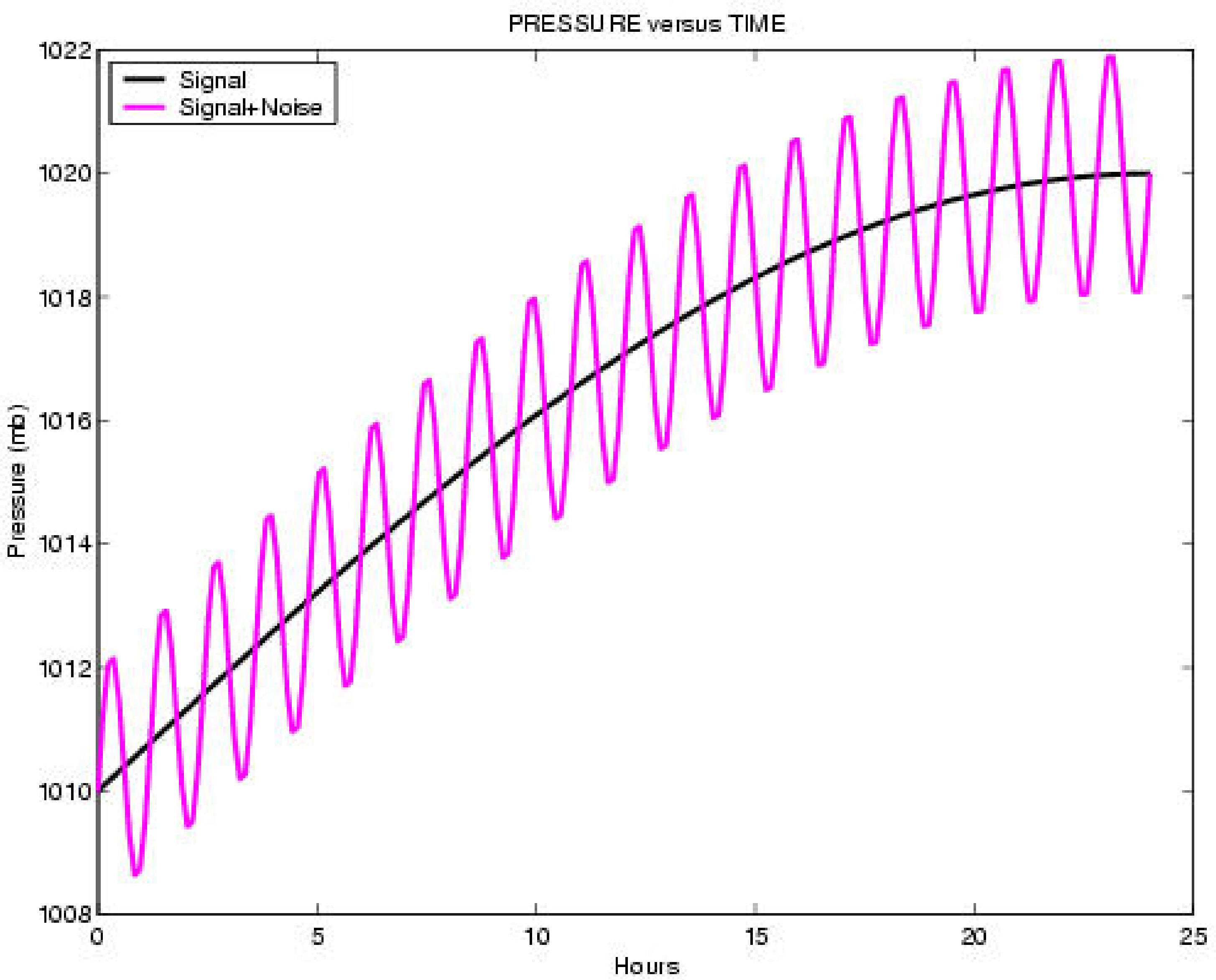
Richardson's ***Forecast Factory***

Illustration by the Belgian artist François Schuiten

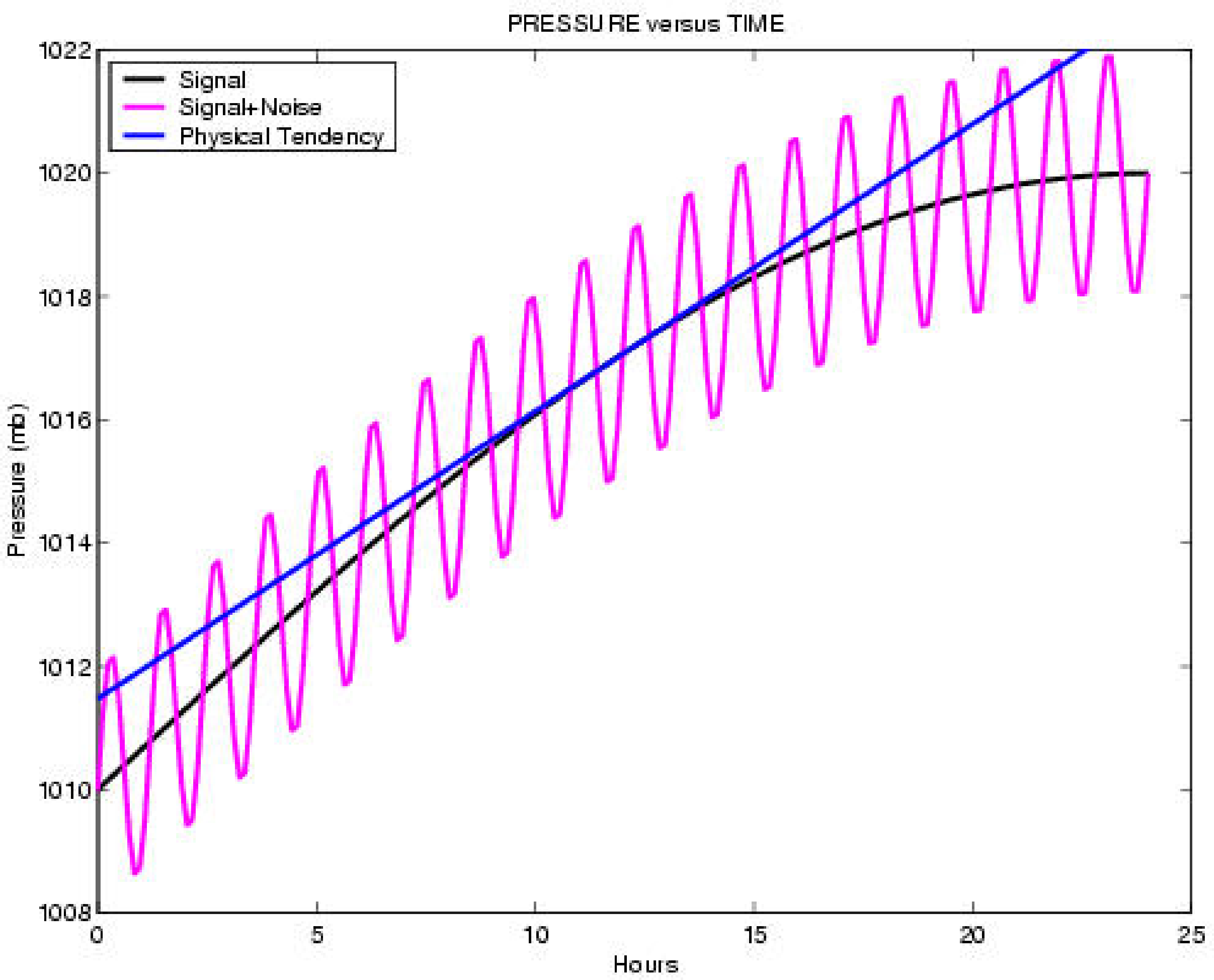
Smooth Evolution of Pressure



Noisy Evolution of Pressure



Tendency of a Smooth Signal



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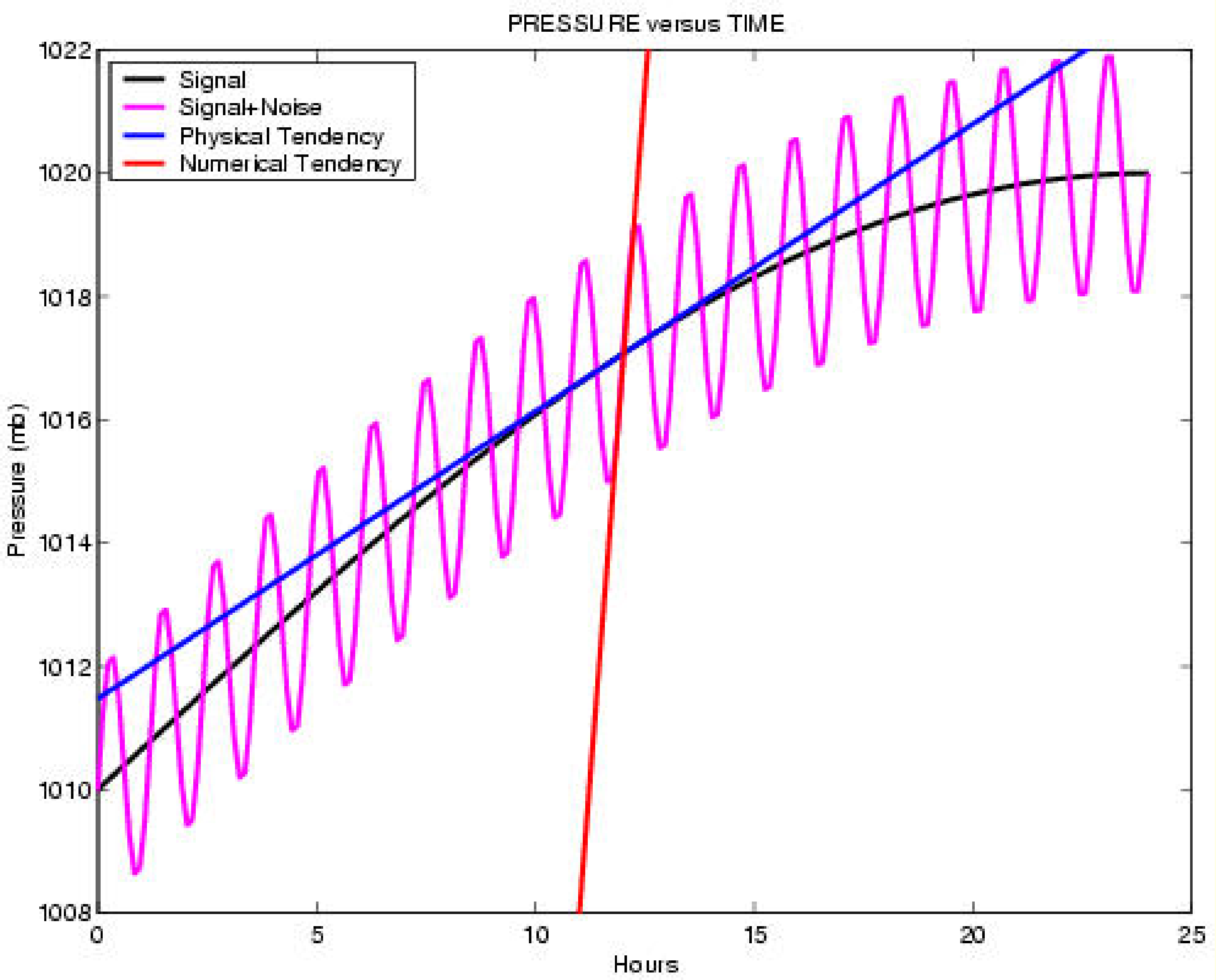


Richardson’s *Spread-sheet*

COMPUTING FORM P XIII. Divergence of horizontal momentum-per-area. Increase of pressure

The equation is typified by: $-\frac{\partial R_{86}}{\partial t} = \frac{\partial M_{E86}}{\partial e} + \frac{\partial M_{N86}}{\partial n} - M_{N86}\frac{\tan\phi}{a} + m_{H6} - m_{H8}^{*} + \frac{2}{a}M_{H86}$. (See Ch. 4/2 #5.)

* In the equation for the lowest stratum the corresponding term $-m_{G8}$ does not appear

Longitude *11° East*, $\delta e = 441 \times 10^5$ | Latitude *5400 km North*, $\delta n = 400 \times 10^5$ | Instant *1910 May 20ᵈ 7ʰ G.M.T.*, $a^{-1}.\tan\phi = 1{\cdot}78 \times 10^{-9}$ | Interval, δt *6 hours*, $a = 6{\cdot}36 \times 10^8$

REF.:—				previous 3 columns	previous column		Form P XVI	Form P XVI	equation above	previous column	previous column	previous column
h	$\frac{\delta M_E}{\delta e}$	$\frac{\delta M_N}{\delta n}$	$-\frac{M_N \tan\phi}{a}$	$\mathrm{div}'_{EN} M$	$-g\delta t\,\mathrm{div}'_{EN} M$	Leave the subsequent columns to be filled up after the vertical velocity has been computed on Form P XVI	m_H	$\frac{2M_H}{a}$	$-\frac{\partial R}{\partial t}$	$+\frac{\partial R}{\partial t}\delta t$	$g\frac{\partial R}{\partial t}\delta t$	$\frac{\partial p}{\partial t}\delta t$
	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$100\times$		$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$		$100\times$	$100\times$
h_0							0					0
	−61	−245	−6	−312	656				−229	49·5	483	
h_2							−83					483
	367	−257	2	112	−236			0·06	−136	29·4	287	
h_4							165					770
	93	−303	−16	−226	478			0·11	−124	26·8	262	
h_6							63					1032
	32	−55	−12	−35	74			0·07	−110	23·8	233	
h_8							138					1265
	−256	38	−8	−226	479			0·03	−88	19·0	186	
h_G												1451
	NOTE: $\mathrm{div}'_{EN}M$ is a contraction for $\frac{\delta M_E}{\delta e} + \frac{\delta M_N}{\delta n} - M_N\frac{\tan\phi}{a}$				SUM = 1451 $= \frac{\partial p_G}{\partial t}\delta t$							check by $\Sigma - g\delta t\,\mathrm{div}'_{EN}M$

Richardson’s Computing Form $\mathrm{P_{XIII}}$

The figure in the bottom right corner is the forecast change in surface pressure: 145 mb in six hours!

Digital Filter Initialization

- ***Introduction of the principles of DFI***
- ***Application to Richardson's Forecast***
- ***Application to the*** HIRLAM ***Model and to the RUC Model at NCEP***
- ***Current and ongoing developments***

Some Applications of Digital Filters

- **Telecommunications**
  - **Digital Switching / Multiplexing / Touch-tone Dialing**
- **Audio Equipment**
  - **Compact Disc Recording / Hi-Fi Reproduction**
- **Speech Processing**
  - **Voice Recognition / Speech Synthesis**
- **Image Processing**
  - **Image Enhancement / Data Compression**
- **Remote Sensing**
  - **Doppler Radar / Sonar Signal Processing**
- **Geophysics**
  - **Seismology / Initialization for NWP.**

Non-recursive Digital Filter

Consider a discrete time signal,

$$\{\cdots, x_{-2}, x_{-1}, x_0, x_1, x_2, \cdots\}$$

For example, x_n could be the value of surface pressure at time $n\Delta t$ at a specific location, say, Belfield.

Nonrecursive Digital Filter:

A *nonrecursive* **digital filter is defined by**

$$y_n = \sum_{k=-N}^{+N} h_k x_{n-k}$$

The inputs are $\{x_n\}$. The outputs are $\{y_n\}$. The outputs are <u>weighted sums of the inputs</u>.

Frequency Response of FIR Filter

Let x_n be the input and y_n the output.
Assume $x_n = \exp(in\theta)$ and $y_n = H(\theta)\exp(in\theta)$.

The transfer function $H(\theta)$ is then

$$H(\theta) = \sum_{k=-N}^{N} h_k e^{-ik\theta} \,.$$

This gives H once the coefficients h_k have been specified.

However, what is really required is the opposite: to derive coefficients which will yield the desired response.

This *inverse problem* has no unique solution, and numerous techniques have been developed.

Optimal Filter Design

This method uses the Chebyshev alternation theorem to obtain a filter whose maximum error in the pass- and stop-bands is minimized. Such filters are called Optimal Filters.

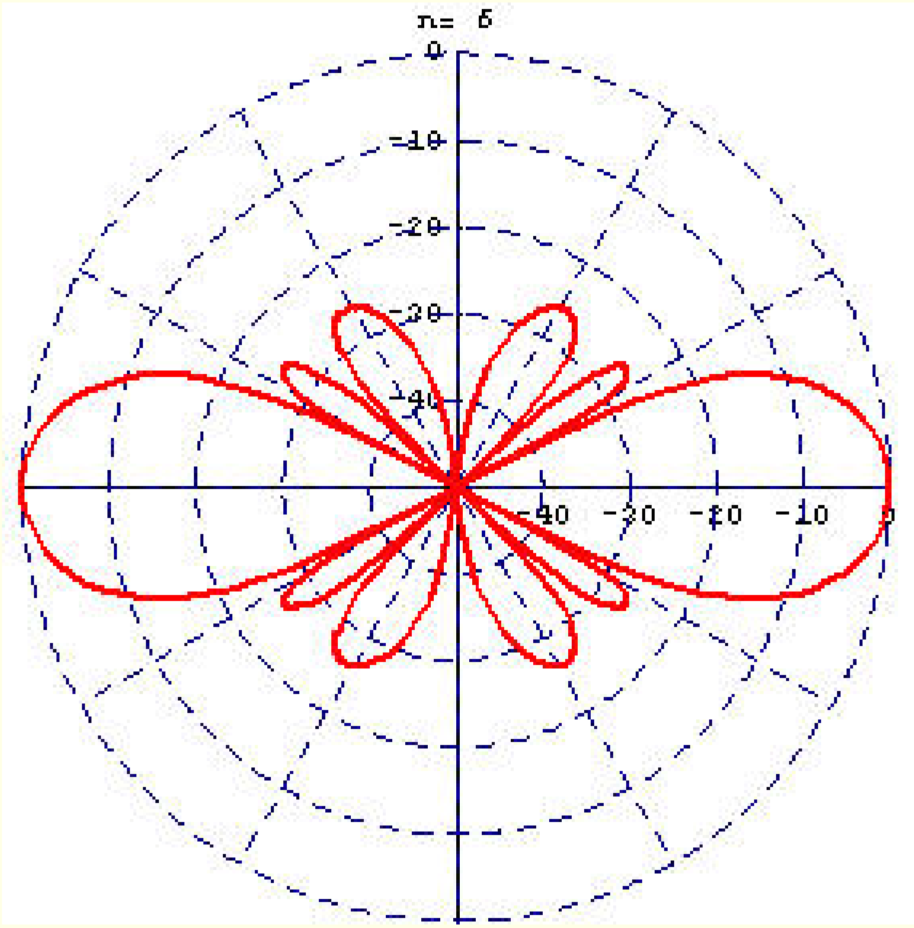
References:

- Hamming (1989)
- Oppenheim and Schafer (1989)

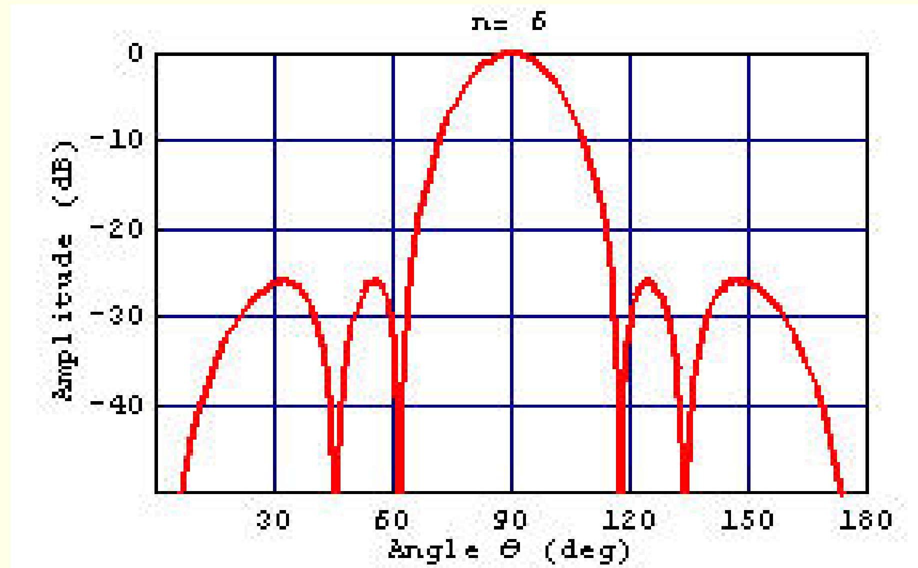
Optimal Filters require solution of complex nonlinear systems of equations. The algorithm for calculation of the coefficients involves about one thousand lines of code.

The Dolph Filter is a special optimal filter, which is much easier to calculate.

Response of Dolph Antenna



Polar Antenna Response



Linear Antenna Response

The Dolph-Chebyshev Filter

This filter is constructed using Chebyshev polynomials:

$$
\begin{aligned}
T_n(x) &= \cos(n \cos^{-1} x), \qquad & |x| \le 1 \\
T_n(x) &= \cosh(n \cosh^{-1} x), \qquad & |x| > 1\,.
\end{aligned}
$$

Clearly, $T_0(x) = 1$ **and** $T_1(x) = x$**. Also:**

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \quad n \ge 2\,.$$

Now define a function in the frequency domain:

$$H(\theta) = \frac{T_{2M}\left(x_0 \cos\left(\theta/2\right)\right)}{T_{2M}(x_0)}$$

where $x_0 > 1$**. Let** θ_s **be such that** $x_0 \cos(\theta_s/2) = 1$**. The form of** $H(\theta)$ **is that of a low-pass filter with a cut-off at** $\theta = \theta_s$**.**

$H(\theta)$ **can be written as a** ***finite expansion***

$$H(\theta) = \sum_{n=-M}^{+M} h_n \exp(-in\theta).$$

The coefficients $\{h_n\}$ **may be evaluated:**

$$h_n = \frac{1}{N}\left[1 + 2r\sum_{m=1}^{M} T_{2M}\left(x_0 \cos\frac{\theta_m}{2}\right)\cos m\theta_n\right],$$

where $|n| \le M$**,** $N = 2M+1$ **and** $\theta_m = 2\pi m/N$**.**
The coefficients h_n **are real and** $h_{-n} = h_n$**.**

The weights $\{h_n : -M \le n \le +M\}$ **define the Dolph-Chebyshev or, for short, Dolph filter.**

An Example of the Dolph Filter

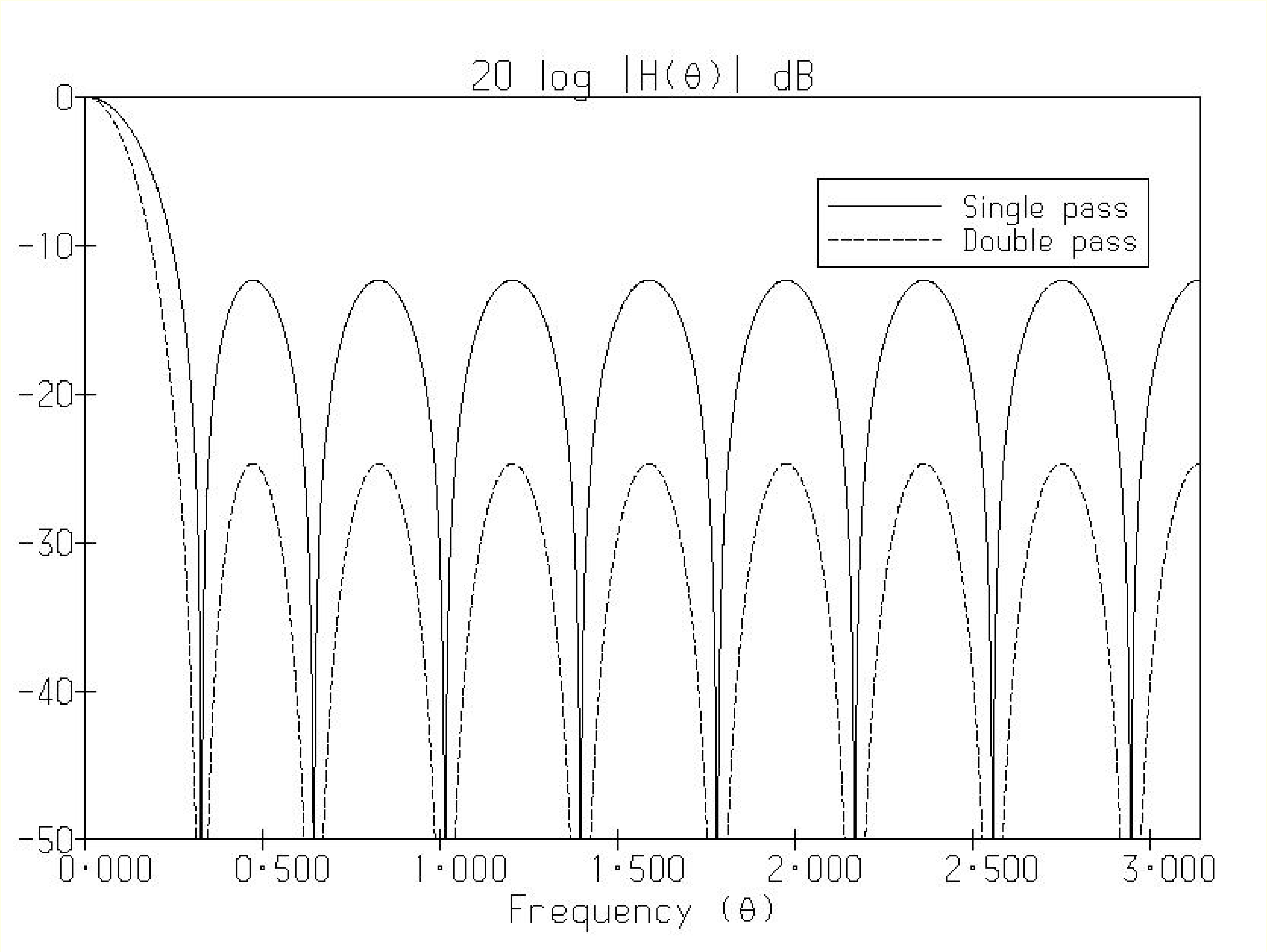
We choose the following parameters:

- **Cut-off period:** $\tau_s = 3\,\mathrm{h}$
- **Time-step:** $\Delta t = \frac{1}{8}\,\mathrm{h} = 7\frac{1}{2}\,\mathrm{min}$.
- **Filter span:** $T_{\mathrm{S}} = 2\,\mathrm{h}$.
- **Filter order:** $N = 17$.

Then the digital cut-off frequency is

$$\theta_s = 2\pi\Delta t/\tau_s \approx 0.26\,.$$

This filter attenuates high frequency components by more than $12\,$dB. Double application gives $25\,$dB attenuation.



Frequency response for Dolph filter with span $T_S = 2\text{h}$, order $N = 2M + 1 = 17$ and cut-off $\tau_s = 3\text{h}$. Results for single and double application are shown.

Application to Initialization

- ***Model integrated forward for N steps:***

$$y_{\mathrm{FORWARD}} = \frac{1}{2}h_0 x_0 + \sum_{n=1}^{N} h_{-n} x_n$$

- ***N-step 'hindcast' is made:***

$$y_{\mathrm{BAKWARD}} = \frac{1}{2}h_0 x_0 + \sum_{n=-1}^{-N} h_{-n} x_n$$

- ***The two sums are combined:***

$$y_0 = y_{\mathrm{FORWARD}} + y_{\mathrm{BAKWARD}}$$

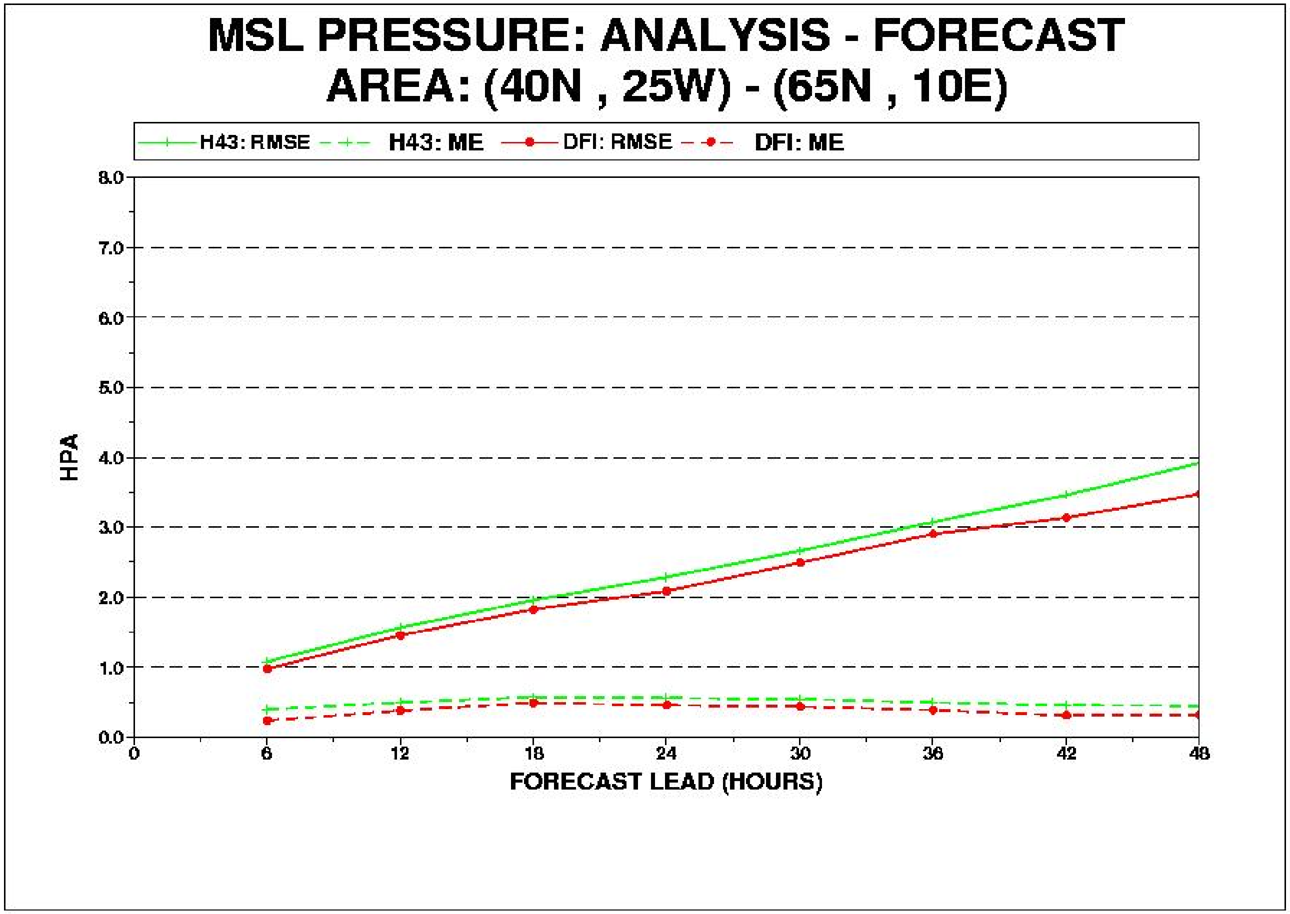
Application to Richardson Forecast

I have reproduced Richardson's results using a specially constructed computer model.

The atmospheric observations for 20 May, 1910, compiled by Hugo Hergessel and analysed by Vilhelm Bjerknes, were *recovered from original sources*.

- ***NIL:*** $$\frac{dp_s}{dt} = +145\,\mathrm{hPa}/6\,\mathrm{h}.$$
- ***LANCZOS:*** $$\frac{dp_s}{dt} = -2.3\,\mathrm{hPa}/6\,\mathrm{h}.$$
- ***DOLPH:*** $$\frac{dp_s}{dt} = -0.9\,\mathrm{hPa}/6\,\mathrm{h}.$$

Observations: Barometer steady!



Root-mean-square (solid) and bias (dashed) errors for mean sea-level pressure. Average over thirty Fastex forecasts. Green(+): reference run (NMI); Red(•): DFI run.

[Work done in collaboration with Ray McGrath]

Rapid Update Cycle @ NCEP

Application of DFI in the RUC Model at the
National Centre for Environmental Prediction
(NCEP) Washington, D.C.

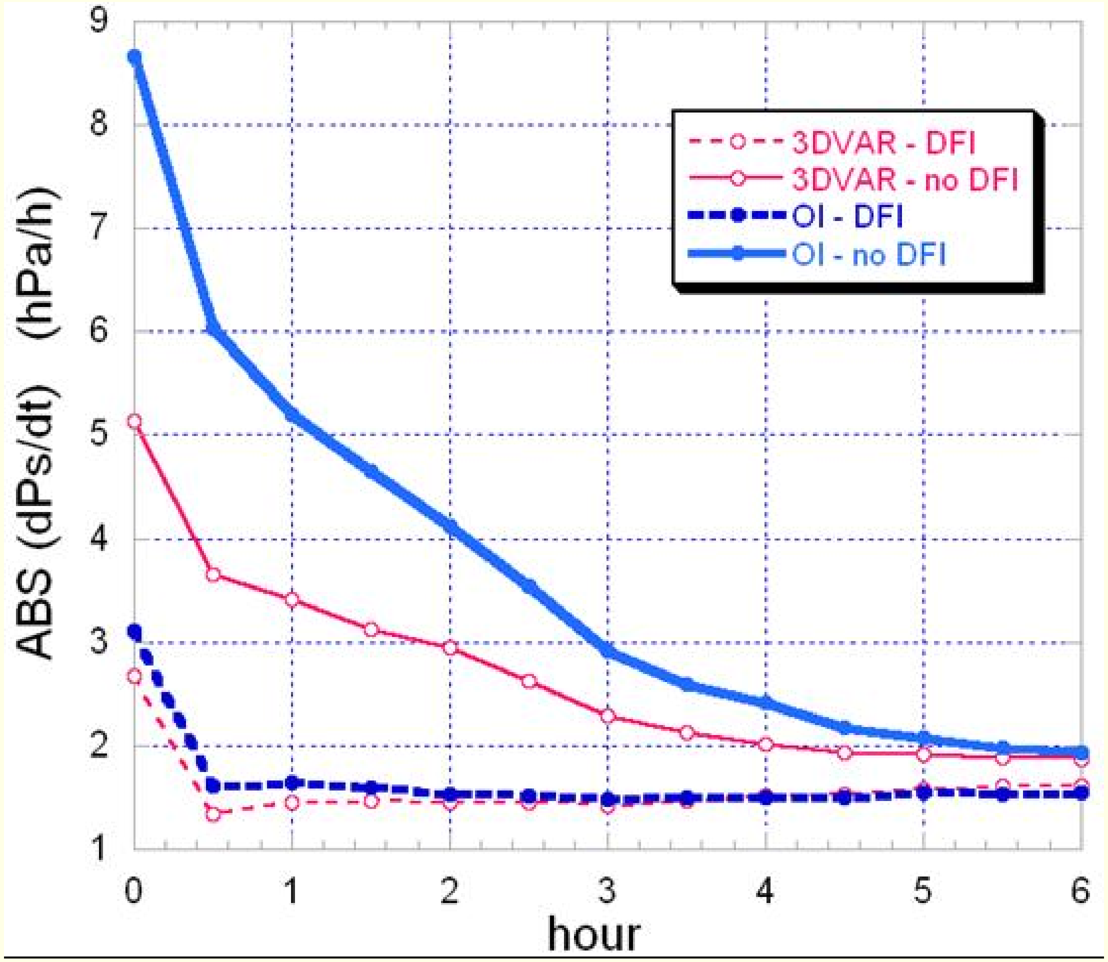


Illustration of the effective suppression of spurious noise by DFI

Advantages of DFI

1. **No need to compute or store normal modes;**
2. **No need to separate vertical modes;**
3. **Complete compatibility with model discretisation;**
4. **Applicable to exotic grids on arbitrary domains;**
5. **No iterative numerical procedure which may diverge;**
6. **Ease of implementation and maintenance;**
7. **Applicable to all prognostic model variables;**
8. **Applicable to non-hydrostatic models.**
9. **Economic and effective Constraint in 4D-Var Analysis.**

DFI is now used in the operational systems at several international weather prediction centres.

Goal of Current Work

- ***Elimination of the backward integration by application of a boundary filter, using***
  - Combined Half-sinc Filters
  - Padé Approximations
  - Laplace and Z Transforms

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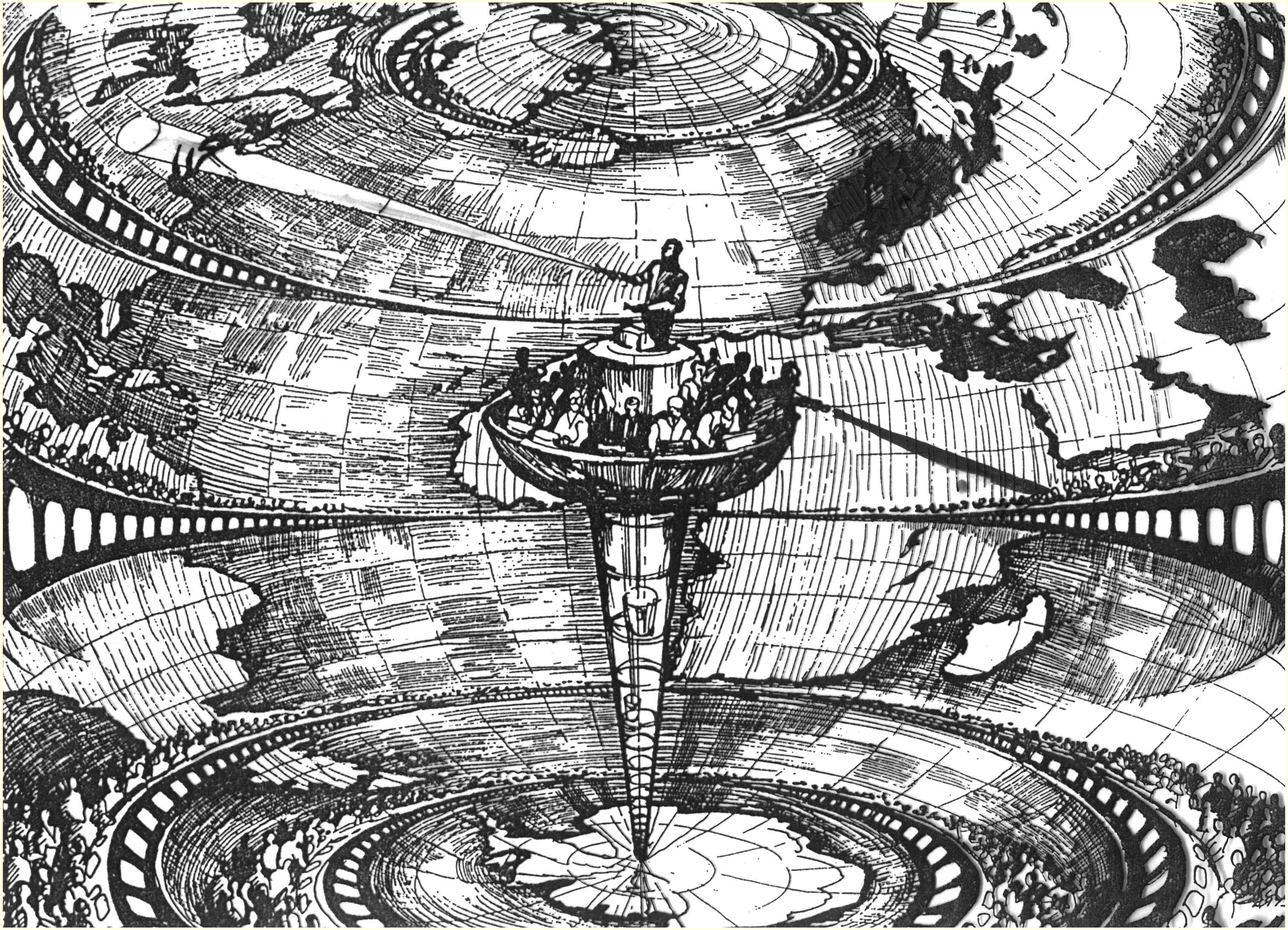
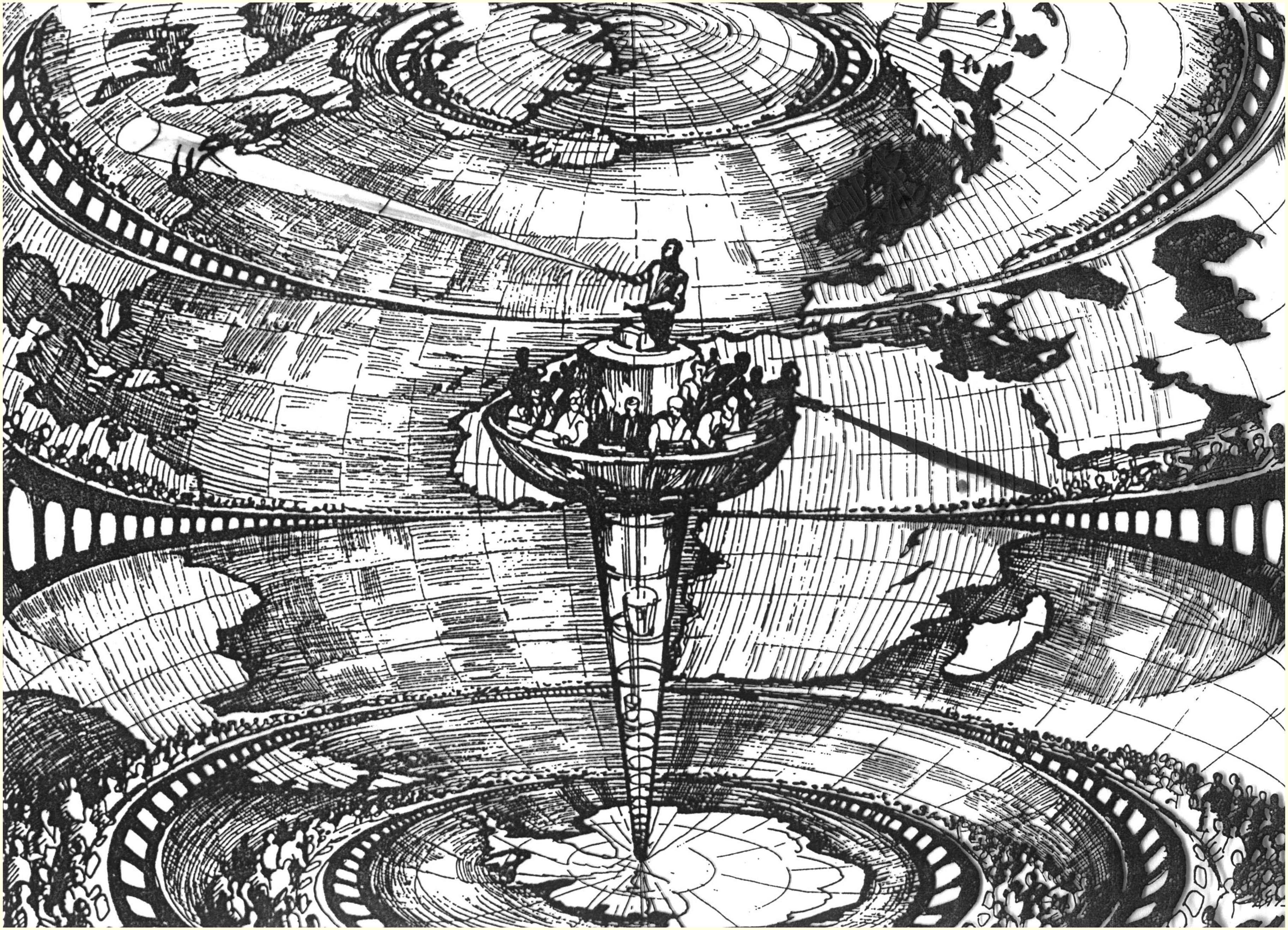
- ***Elimination of the backward integration by application of a boundary filter, using***
  - Combined Half-sinc Filters
  - Padé Approximations
  - Laplace and Z Transforms
- ***The Best is Yet to Come!***



Richardson's Forecast Factory (A. Lannerback).

Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984



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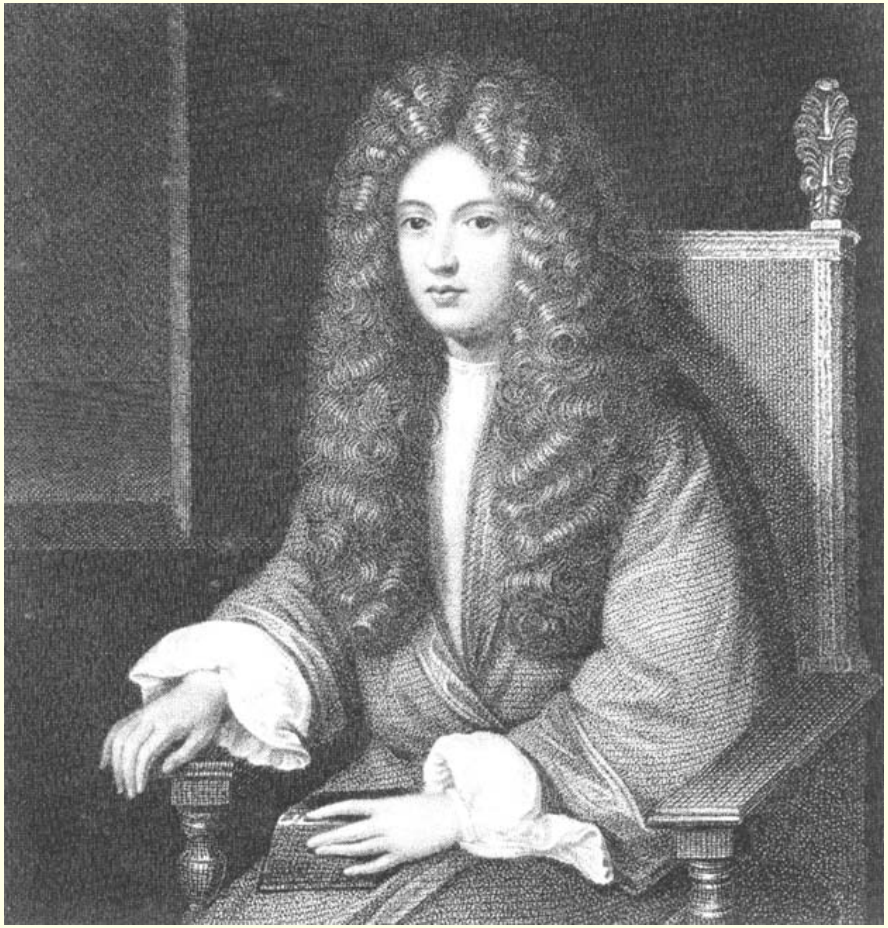
64,000 Computers: The first Massively Parallel Processor

Conclusion of Part I

Irish Scientists who have made Contributions to Meteorology

Robert Boyle (1627-1691)



Robert Boyle was born in Lismore, Co. Waterford.

He was a founding fellow of the Royal Society.

Boyle formulated the relationship between pressure and volume of a fixed mass of gas at fixed temperature.

$$p \propto 1/V$$

Richard Kirwan (1733–1812)

Richard Kirwan was born in Co. Galway. He grew up at Cregg Castle, which was built in 1648 by the Kirwan family.

He was a noted Chemist, Mineralogist, Meteorologist and Geologist

He was an early President of the Royal Irish Academy

He anticipated the concept of air-masses

He believed that the Aurora Borealis resulted from combustion of equatorial air.

Francis Beaufort (1774–1857)

Born near Navan in Co. Meath.

Served in the Royal Navy in the Napoleonic wars.

Helped to establish a telegraph line from Dublin to Galway.

Appointed Hydrographer to the Royal Navy in 1829, a post he held until the age of 81.

Promoted Rear Admiral in 1846.

Knight Commander of the Bath two years later.

Best remembered for his scale for estimating the force of the winds at sea — the Beaufort scale.

John Tyndall (1820–1893)

One of the great scientists of the 19th century.

Born in 1820 at Leighlinbridge, Carlow.

Studied with Robert Bunsen in Marburg, 1848.

Associated with the Royal Institution from 1853. Assistant to Michael Faraday.

Published more than 16 books and 145 papers.

Tyndall and the Greenhouse Effect

Tyndall wrote that, without water vapour, the Earth's surface would be *held fast in the iron grip of frost*.

He showed that water vapour, carbon dioxide and ozone are strong absorbers of heat radiation.

This is what we now call the Greenhouse Effect.

Tyndall speculated how changes in water vapour and carbon dioxide could be related to climate change.

The Tyndall Centre for Climate Change Research, recently established at the University of East Anglia in Norwich, is named in his honour.

George G Stokes, 1819–1903

George Gabriel Stokes, founder of modern hydrodynamics.

Some Contributions of Stokes to Meteorological Science

- ***Stokes' Theorem***
- ***Stokes Drag and Stokes' Law***
- ***Stokes Drift***
- ***Stokes Waves***
- ***Campbell-Stokes Sunshine Recorder***
- ***Navier-Stokes Equations***

William Thompson (1824–1907)

Sir William Thompson, 1st Baron Kelvin of Largs, born in Belfast.

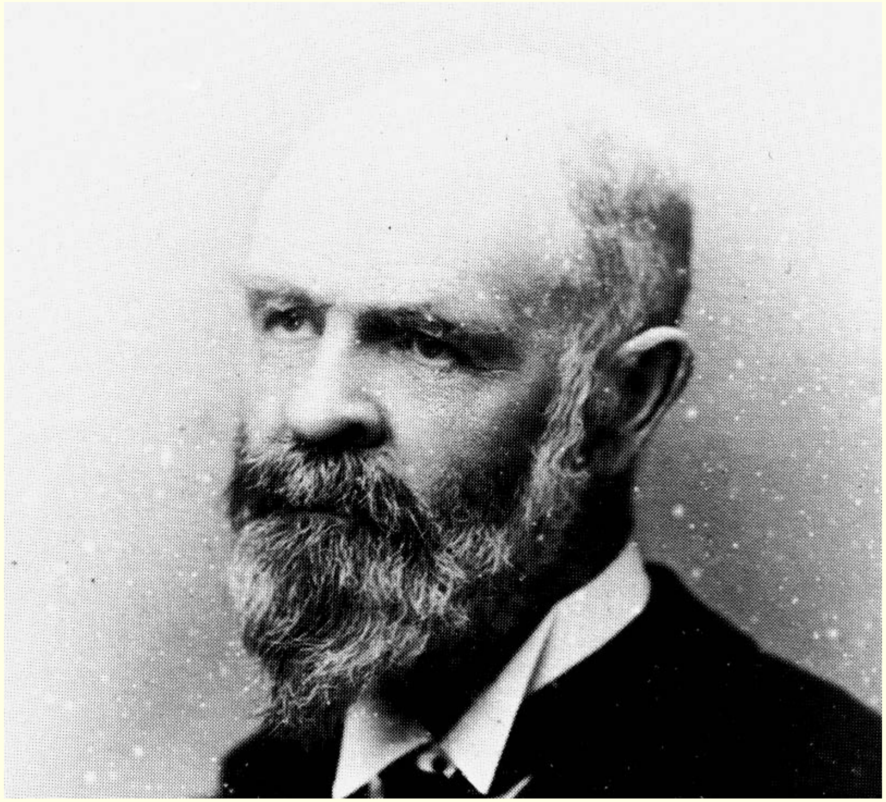
Among the most brilliant scientists of the 19th century.

Professor of Natural Philosophy in Glasgow, 1846 (aged 22) – 1899.

Developed the foundations of thermodynamics.

Introduced the absolute scale of temperature, with the zero at -273°.

Robert Henry Scott (1833–1916)



Robert Scott, born in Dublin, 1833.

Founder of Valentia Observatory

First Director of the British Meteorological Office.

With the exception of Kirwan, all these scientists, though born in Ireland, made their names abroad.

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All that has now changed!!!

Met Éireann-UCD Link

In October, Met Éireann and University College, Dublin signed an agreement to collaborate in establishing a Centre for Meteorology and Climatology at UCD.

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In October, Met Éireann and University College, Dublin signed an agreement to collaborate in establishing a Centre for Meteorology and Climatology at UCD.

- **The Centre is based in the Department of Mathematical Physics in Belfield**
- **A Professor and Lecturer have been appointed**
- **A Post-graduate course commenced in September**
- **Two students are undertaking PhD studies.**
- **An Undergraduate syllabus will be developed later.**

The C4I Project

Community Climate Change Consortium for Ireland (C4I)

C4I Objectives (2002–2006)

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- ***Build a Capability** for Climate Change Research in Ireland*
- ***Establish a Regional Climate Analysis, Modelling and Prediction Centre (RCAMPC)***
- ***Provide a User Interface between the Modelling Centre and Irish Researchers***
- ***A Computer Network, interfacing the RCAMPC to users***
- ***A Powerful Educational Facility for Students of Environmental Science***
- ***Greatly Improved Opportunities for Participation in European Programmes***
- ***A Comprehensive Plan for Continuation of C4I.***

C4I Funding

Funding is from four sources:

- *The Higher Education Authority, under the PRTLI Programme*
- *The Environmental Protection Agency, under the ERTDI Programme*
- *Sustainable Energy Ireland*
- *Met Éireann*

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C4I is linked to the CosmoGrid Project, coordinated by the Dublin Institute for Advanced Studies, and funded by HEA under PRTLI/Cycle 3.

Elements of C4I

- ***Modelling Centre:***

  A Regional Climate Analysis, Modelling and Prediction Centre (RCAMPC), based at Met Éireann in Glasnevin.

The C4I Project Manager is also Director of RCAMPC.

- ***System Work:***

  Specialized work on parallelization and on Grid Computation at UCD

The aim of the UCD Work Package is to develop a 'Grid Computing' climate model, re-engineered from the HIRLAM model.

RCAMPC

The Regional Climate Analysis, Modelling and Prediction Centre is a centre of excellence in climate modelling, with expertise in

- **Climatology**
- **Dynamical meteorology**
- **Numerical analysis**
- **Advanced parallel computing**

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- **Numerical analysis**
- **Advanced parallel computing**

Detailed, high-resolution simulations of the climate of Ireland are being carried out using the HIRLAM Model.

Project Staffing

The staff complement of the Regional Climate Analysis, Modelling and Prediction Centre is as follows:

- ***Project Manager (Ray McGrath)***
- ***Deputy Manager (Tido Semmler)***
- ***Technical Assistant (Elisa Nishimura)***
- ***3 Post-doctoral Scientists***

  (Conor Sweeney, Shiyu Wang, J.Venkata Ratnam)
- ***2 Post-graduate Students***

  (Paul Nolan and Michael Clark)

Applications of RCM outputs

- **Energy:** impact of changes on power requirements
- **Hydrology:** river catchment models; Flood-risk models
- **Agriculture:** Crop management; Animal disease models
- **Fisheries:** Marine currents; level of nutrients
- **Forestry:** Carbon storage potential; forest stress
- **Natural Environment:** habitat changes; conservation
- **Wind and wave climatology:** Coastal erosion, etc.
- **Tourism:** Length of season; Ultra-violet radiation levels
- **Insurance:** storm damage risk; commercial losses
- **Human health:** disease occurrence; indoor comfort

Digression:

Modelling the Atmosphere

Isaac Newton (1642-1727)

Sir Isaac Newton
(1642-1727)

Newton established the fundamental principles of Dynamics.

He formulated the basic law of Gravitation.

He produced monumental results in Celestial Mechanics.

He laid the foundation for differential and integral Calculus.

He made fundamental contributions to Optics.

Newton was arguably the greatest scientist the world has ever known.

Newton: the Inventor of Science

The Irish writer John Banville, in his work *The Newton Letters*, **go so far as to write that 'Newton invented science'.**

This is a debatable and thought-provoking claim.

There is a recent biography of Newton by James Gleick.

Newton's Law of Motion

The rate of change of momentum of a body is equal to the sum of the forces acting on the body.

Let m be the mass of the body and $\mathbf{V}$ its velocity. Then the momentum is $\mathbf{p} = m\mathbf{V}$.

If $\mathbf{F}$ is the total applied force, Newton's Second Law gives

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}\,.$$

The acceleration $\mathbf{a}$ is the rate of change of velocity, that is, $\mathbf{a} = d\mathbf{V}/dt$. If the mass m is constant, we have

$$\mathbf{F} = m\mathbf{a}\,.$$

$$\textbf{Force} = \textbf{Mass} \times \textbf{Acceleration}\,.$$

Edmund Halley (1656–1742)

Edmund Halley was a contemporary and friend of Isaac Newton.

He was largely responsible for persuading Newton to publish his *Principia Mathematica*.

Edmund Halley (1656–1742)

- **Edmund Halley attended Queen's College, Oxford.**
- **In 1683, he published his theory of the variation of the magnet.**
- **In 1684, he conferred with Newton about the inverse square low in the solar system.**
- **In 1686, he wrote on the trade winds and the monsoons.**
- **He undertook three voyages during 1698–1701, to test his magnetic variation theory.**
- **Then he became professor of Geometry at Oxford.**
- **At the age of 64, he invented the diving bell.**
- **Halley died in Greenwich in 1742.**

Halley and his Comet

Halley's analysis of what is now called Halley's comet is an excellent example of the scientific method in action.

Observation:

The comets of 1456, 1531, 1607, and 1682 followed similar orbital paths around the Sun. Each appearance was separated from the previous one by about 76 years.

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These events were due to the reappearance of one object on an orbit which brought it close to the Sun every 76 years.

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Further Confirmation:

Appearances of the comet have since been found in the historic record as far back as 2000 years.

Physical Laws of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)
Relates the pressure, temperature and density
CONTINUITY EQUATION
Conservation of mass; air neither created nor distroyed
WATER CONTINUITY EQUATION
Conservation of water (liquid, solid and gas)
EQUATIONS OF MOTION: Navier-Stokes Equations
Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction
THERMODYNAMIC EQUATION
Determines changes of temperature due to heating or cooling, compression or rarifaction, etc.

Seven equations; seven variables (u, v, w, ρ, p, T, q).

The Primitive Equations

$$\frac{du}{dt} - \left(f + \frac{u\tan\phi}{a}\right)v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$
$$\frac{dv}{dt} + \left(f + \frac{u\tan\phi}{a}\right)u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$
$$p = R\rho T$$
$$\frac{\partial p}{\partial y} + g\rho = 0$$
$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$
$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$
$$\frac{\partial\rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [\mathrm{Sources} - \mathrm{Sinks}]$$

Seven equations; seven variables $(u, v, w, p, T, \rho, \rho_w)$**.**

Scientific Weather Forecasting in a Nut-Shell

- **The atmosphere is a physical system**
- **Its behaviour is governed by the laws of physics**
- **These laws are expressed quantitatively in the form of mathematical equations**
- **Using observations, we can specify the atmospheric state at a given initial time: “Today’s Weather”**
- **Using the equations, we can calculate how this state will change over time: “Tomorrow’s Weather”**

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- **Using the equations, we can calculate how this state will change over time: "Tomorrow's Weather"**

- **The equations are very complicated (non-linear) and a powerful computer is required to do the calculations**
- **The accuracy decreases as the range increases; there is an inherent limit of predictibility.**

Outputs and Deliverables of C4I

The C4I Project will:

- Provide a capability to address climate change questions specific to National needs
- Bring cohesion to currently disparate research effort in Ireland
- Provide a research focus which will greatly enhance opportunities for Irish scientists to participate in EU environmental programmes
- Improve coordination of existing research by establishing research groups with continuity and critical mass
- Ensure the future viability of environmental research in Ireland by supporting post-graduate students

Regional Climate Models (RCM)

Idea:

- **RCM puts focus on a limited geographical area. The RCM is driven by boundary fields provided by a global model.**
- **The RCM dynamically downscales the global data.**
- **C4I uses the Rossby Centre RCM — a version of the HIRLAM numerical weather prediction model used for operational forecasting in Met Éireann.**

Current Work at the RCAMPC

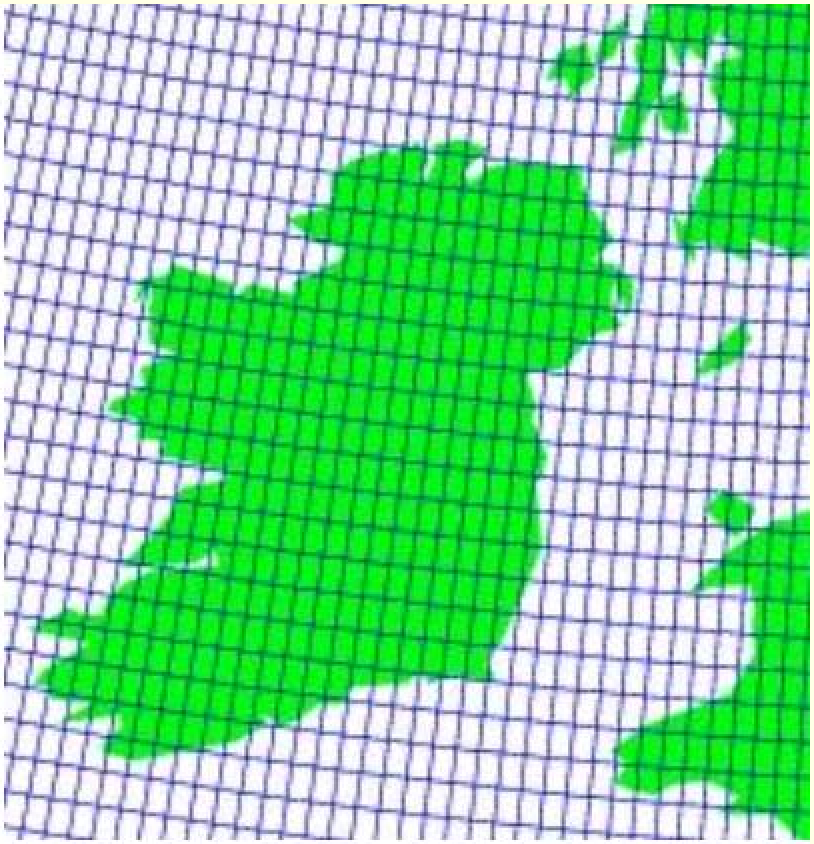
- **Validation of RCM using ECMWF ERA-40 re-analysis data (120km global data)**
- **Sensitivity experiments (area, resolution, etc.)**
- **Simulations of past climate (1960–2000)**
- **Simulations of future climate (2021-2030; 2051-2060)**

Target resolution:

- **horizontal resolution 15km**
- **40 vertical levels**

Global and Regional Grids

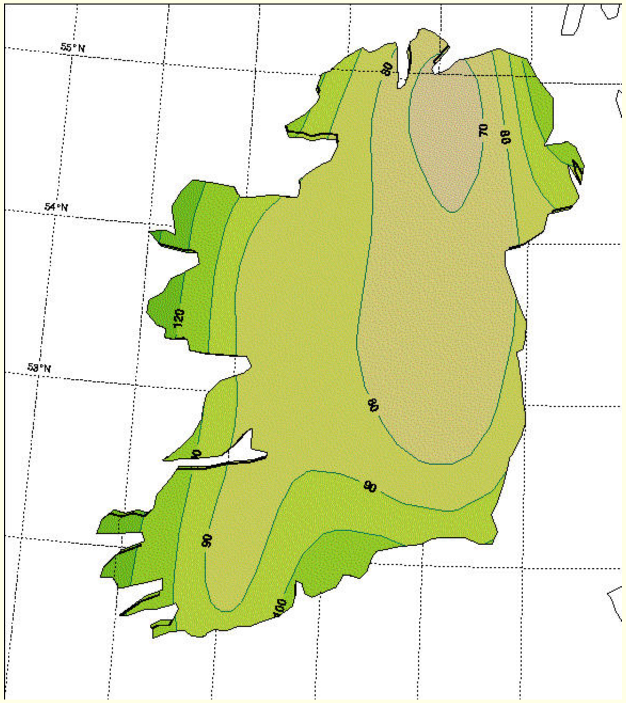
Global Grid (300 km)

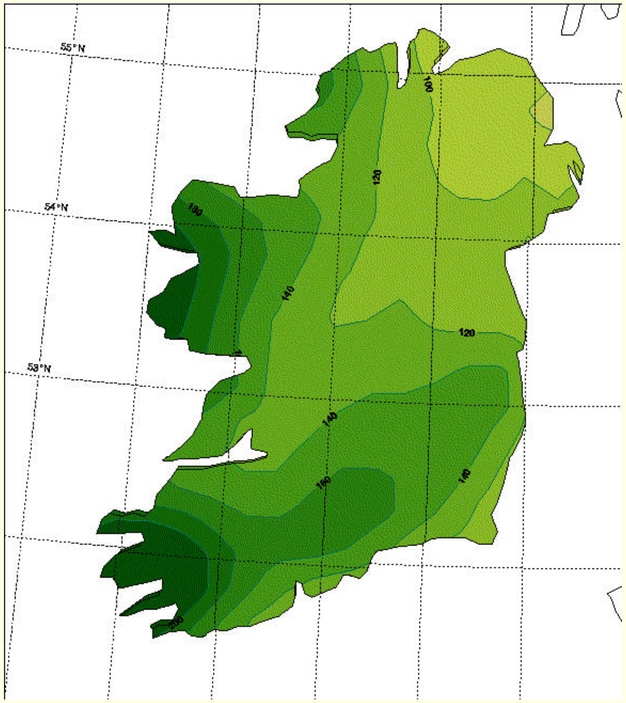


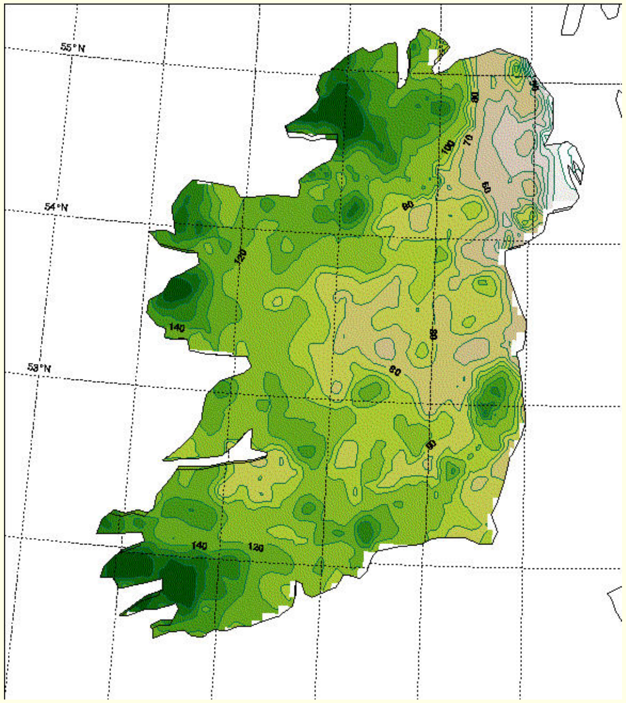
Regional Grid (15 km)

Validation of the RCM

Impact of model resolution on precipitation







ERA-40 (120 km) **RCM (20 km)** **Observed.**

Total precipitation for the month of January, 1991

Conclusion of Part II

In a Nutshell

A mathematical equivalence with a
simple mechanical system
sheds light on the dynamics of
resonant Rossby waves
in the atmosphere.

The Swinging Spring

Two distinct oscillatory modes with two distinct restoring forces:

- ***Elastic or springy modes***
- ***Pendular or swingy modes***

The Swinging Spring

Two distinct oscillatory modes with two distinct restoring forces:

- ***Elastic or springy modes***
- ***Pendular or swingy modes***

Take a peek at the Java Applet

In a paper in 1981, *Breitenberger and Mueller* **made the following comment:**

> **This simple system looks like a toy at best, but its behaviour is astonishingly complex, with many facets of more than academic lustre.**

I hope to convince you of the validity of this remark.

Original Reference

First comprehensive analysis of the elastic pendulum:

Oscillations of an Elastic Pendulum as an Example of the Oscillations of Two Parametrically Coupled Linear Systems.

Vitt and Gorelik (1933).

Inspired by analogy with Fermi resonance of CO_2.

Translation of this paper available as

Historical Note #3 (1999)

Published by Met Éireann.

Available at URL:

http://www.maths.tcd.ie/~plynch

The Exact Equations of Motion

In Cartesian coordinates the Lagrangian is

$$L = T - V = \underbrace{\frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{Z}^2\right)}_{K.E} - \underbrace{\frac{1}{2}k(r - \ell_0)^2}_{E.P.E} - \underbrace{mgZ}_{G.P.E}$$

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The equations of motion are (with $\omega_Z^2 \equiv k/m$):

$$\begin{aligned}
\ddot{x} &= -\omega_Z^2 \left(\frac{r - \ell_0}{r}\right) x \\
\ddot{y} &= -\omega_Z^2 \left(\frac{r - \ell_0}{r}\right) y \\
\ddot{Z} &= -\omega_Z^2 \left(\frac{r - \ell_0}{r}\right) Z - g
\end{aligned}$$

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\end{aligned}$$

There are two constants, the energy and the angular momentum:

$$E = T + V \qquad h = x\dot{y} - y\dot{x}\,.$$

The system is <u>not integrable</u> (two invariants, three DOF).

The Canonical Equations

We consider the case of planar motion. The canonical equations of motion (in polar coordinates) are:

$$\begin{aligned}
\dot{\theta} &= p_\theta / m r^2 \\
\dot{p}_\theta &= -m g r \sin\theta \\
\dot{r} &= p_r / m \\
\dot{p}_r &= p_\theta^2 / m r^3 - k(r - \ell_0) + m g \cos\theta
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\end{aligned}$$

These equations may also be written symbolically in vector form

$$\dot{\mathbf{X}} + \mathbf{L}\mathbf{X} + \mathbf{N}(\mathbf{X}) = 0$$

The state vector X specifies a point in 4-dimensional phase space:

$$\mathbf{X} = (\theta, p_\theta, r, p_r)^{\mathrm{T}}.$$

Linear Normal Modes

Suppose that amplitude of motion is small:

$$\frac{d}{dt}\begin{pmatrix}\theta \\ p_\theta \\ r' \\ p_r\end{pmatrix} = \begin{pmatrix} 0 & 1/m\ell^2 & 0 & 0 \\ -mg\ell & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/m \\ 0 & 0 & -k & 0 \end{pmatrix}\begin{pmatrix}\theta \\ p_\theta \\ r' \\ p_r\end{pmatrix}$$

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The matrix is block-diagonal:

$$\mathbf{X} = \begin{pmatrix}\mathbf{Y}\\ \mathbf{Z}\end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix}\theta\\ p_\theta\end{pmatrix}, \qquad \mathbf{Z} = \begin{pmatrix}r'\\ p_r\,.\end{pmatrix}$$

Linear dynamics evolve independently:

$$\dot{\mathbf{Y}} = \begin{pmatrix}0 & 1/m\ell^2\\ -mg\ell & 0\end{pmatrix}\mathbf{Y}\,, \qquad \dot{\mathbf{Z}} = \begin{pmatrix}0 & 1/m\\ -k & 0\end{pmatrix}\mathbf{Z}\,.$$

SLOW **FAST**

Application of KAM Theory

Ratio of rotational and elastic frequencies:

$$\epsilon \equiv \left(\frac{\omega_R}{\omega_Z}\right) = \sqrt{\frac{mg}{k\ell}} \, .$$

For $\epsilon = 0$, there is no coupling between the modes.

For $\epsilon \ll 1$ the coupling is weak.
We can apply classical *Hamiltonian perturbation theory.*

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We can apply classical *Hamiltonian perturbation theory.*

The consequences of the Kolmogorov-Arnol'd-Moser (KAM) Theorem for the swinging spring are discussed in:

Lynch, Peter, 2002: *The Swinging Spring*:
A Simple Model for Atmospheric Balance.
Pp. 64-108 in Large-Scale Atmosphere-Ocean Dynamics, Vol II: Geometric Methods and Models. Ed. J. Norbury and I. Roulstone. Cambridge University Press.

Regular and Chaotic Motion

We wish to discuss the phenomenon of ***Resonance*** for the spring, and its ***Pulsation*** and ***Precession***.

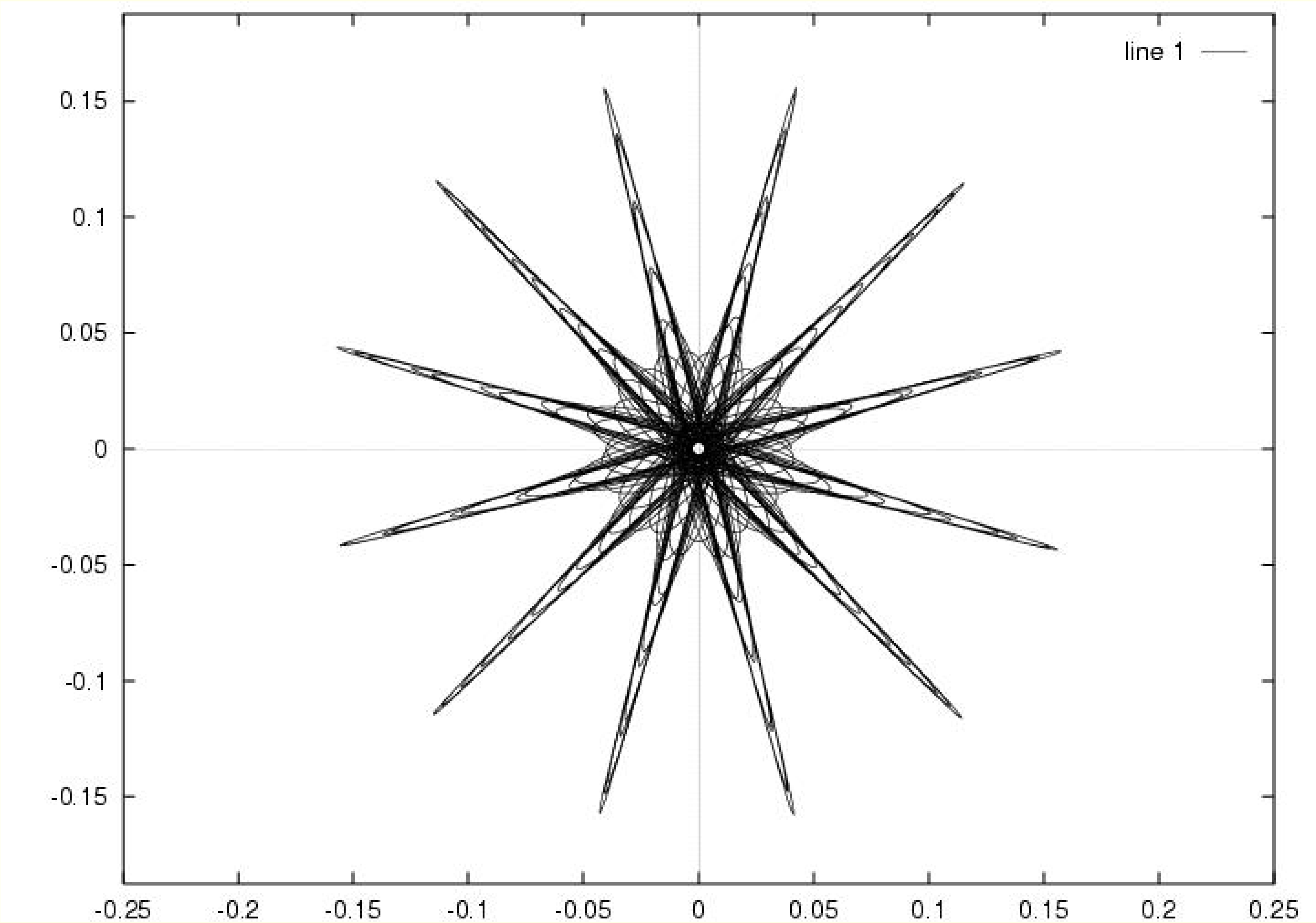
Resonance occurs for

$$\epsilon = \frac{1}{2} .$$

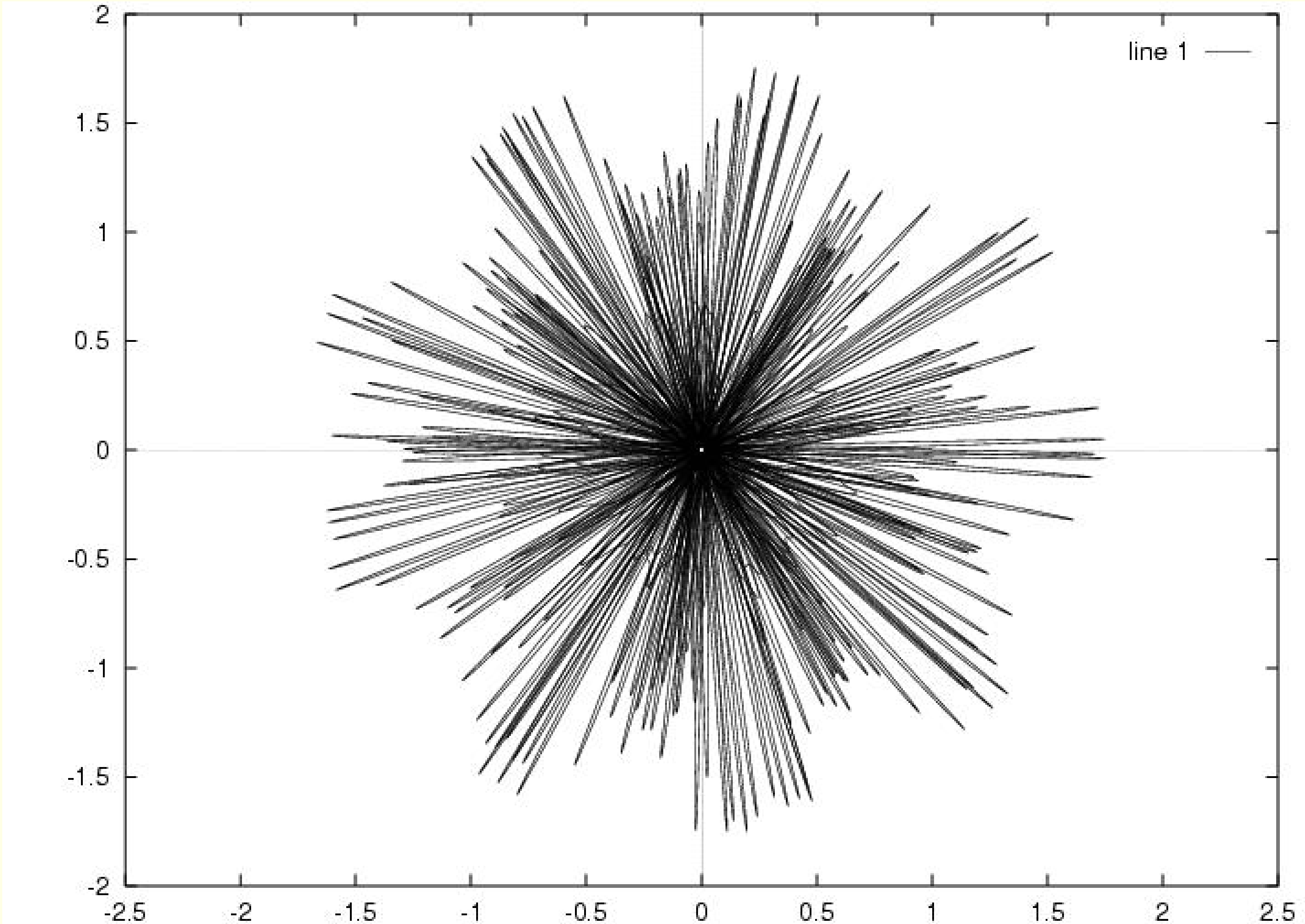
Clearly, this is far from the quasi-integrable case of small ϵ.

However, for ***small amplitudes***, the motion is also quasi-integrable. We look at two numerical solutions, one with small amplitude, one with large.

Horizontal plan: Low energy case



Horizontal plan: High energy case



The Resonant Case

The Lagrangian, to cubic order (3 dimensions again) is:

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{1}{2}\left(\omega_R^2(x^2 + y^2) + \omega_Z^2 z^2\right) + \frac{1}{2}\lambda(x^2 + y^2)z\,,$$

We study the <u>resonant case:</u>

$$\omega_Z = 2\omega_R\,.$$

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We study the resonant case:

$$\omega_Z = 2\omega_R\,.$$

Using the Averaged Lagrangian Technique, the equations for the modulation amplitudes may be found:

$$i\dot{A} = B^*C\,,$$
$$i\dot{B} = CA^*\,,$$
$$i\dot{C} = AB\,,$$

These are the *three-wave interaction equations*.

Ubiquity of Three-Wave Equations

- ***Modulation equations for wave interactions in fluids and plasmas.***
- ***Three-wave equations govern envelop dynamics of light waves in an inhomogeneous material; and phonons in solids.***
- ***Maxwell-Schrödinger envelop equations for radiation in a two-level resonant medium in a microwave cavity.***
- ***Euler's equations for a freely rotating rigid body (when $H = 0$).***

Analytical Solution of the 3WE

We can derive complete analytical expressions for the amplitudes and phases.

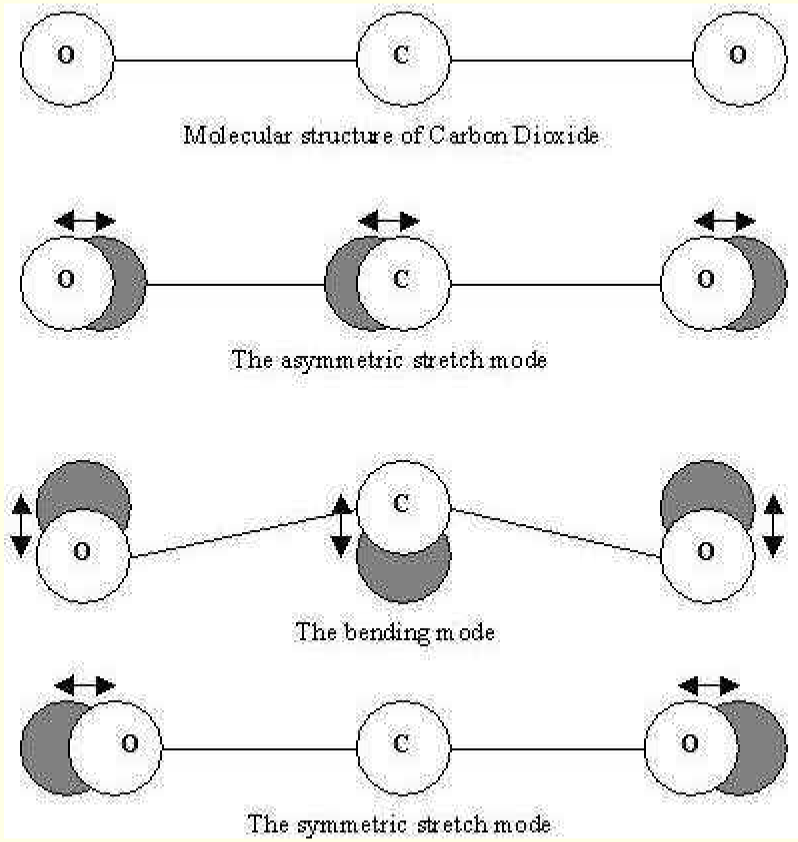
The amplitudes are expressed as elliptic functions.

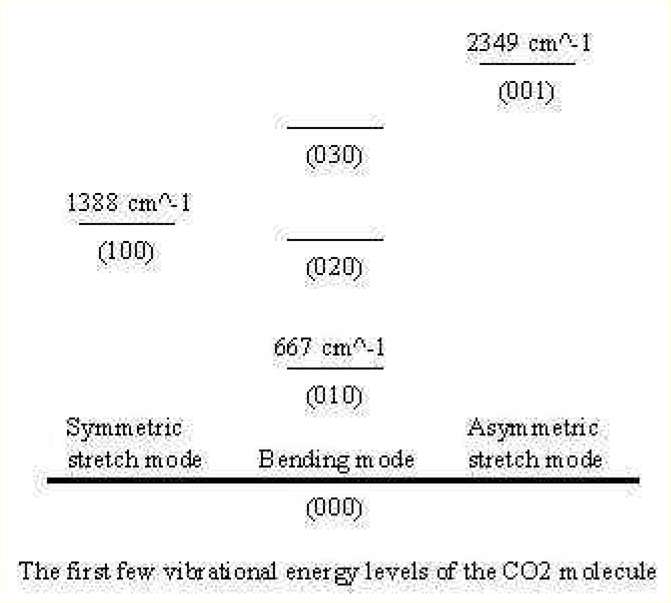
The phases are expressed as elliptic integrals.

The complete details are given in:

Lynch, Peter, and Conor Houghton, 2004:
Pulsation and Precession of the Resonant Swinging Spring.
Physica D, 190,1-2, 38-62
(see web-site `http://www.maths.tcd.ie/~plynch`**)**

Vibrations of CO_2 Molecule





Monodromy in Quantum Systems

It is 70 years since the work of Vitt and Gorelik.

" Remarkably, the swinging spring still has something interesting to offer to the quantum study of the Fermi resonance."

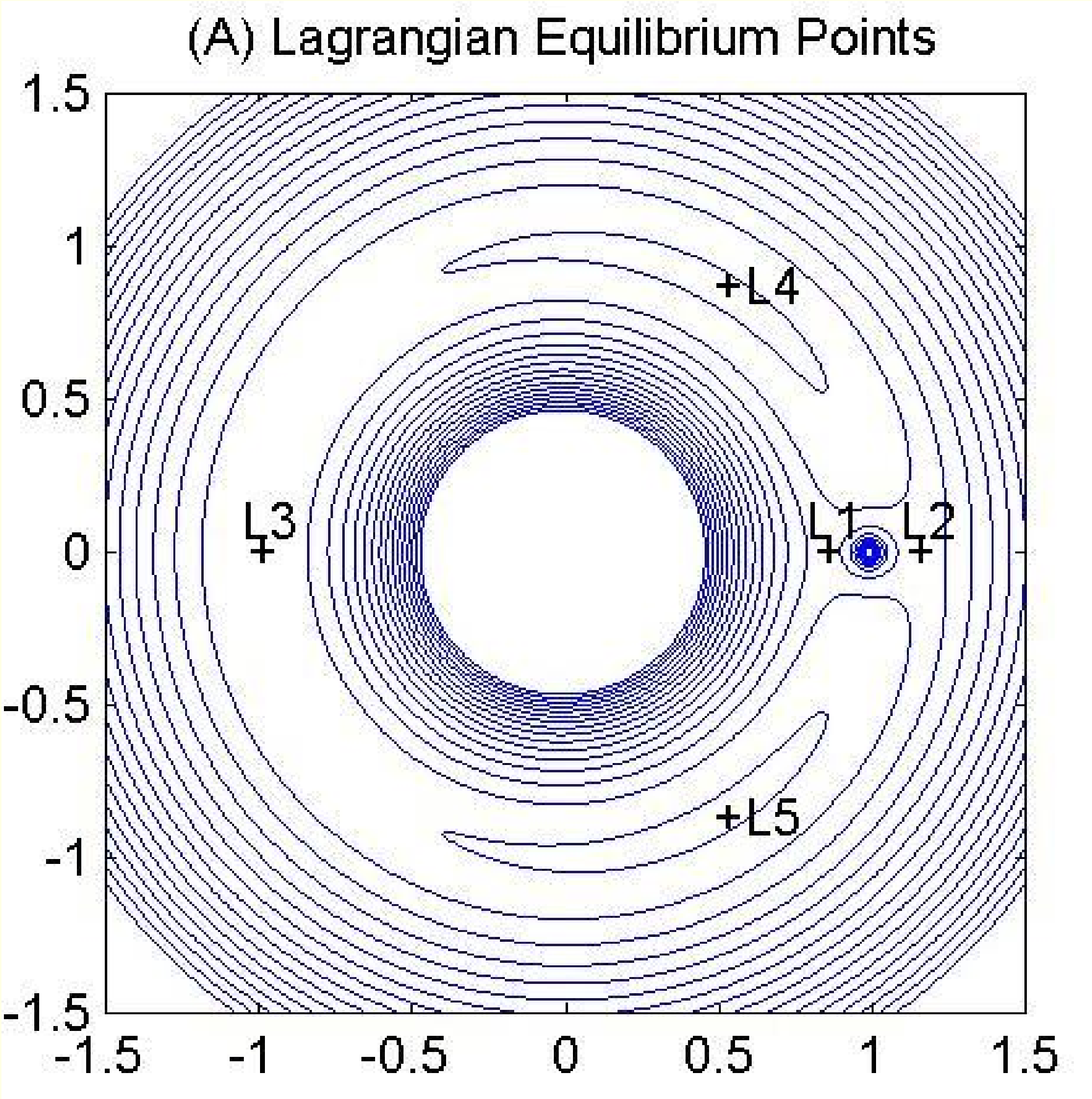
The CO_2 molecule as a quantum realization of the 1:1:2 resonant swing–spring with monodromy

Richard Cushman, Holger Dullin, Andrea Giacobbe, Darryl Holm, Marc Joyeux, Peter Lynch, Dmitrií Sadovskií, and Boris Zhilinskií

Accepted for publication in *Phys. Rev. Lett.* (2004)

"It is now tempting to think of experimental quantum dynamical manifestations of monodromy."

The Lagrange Equilibrium Points



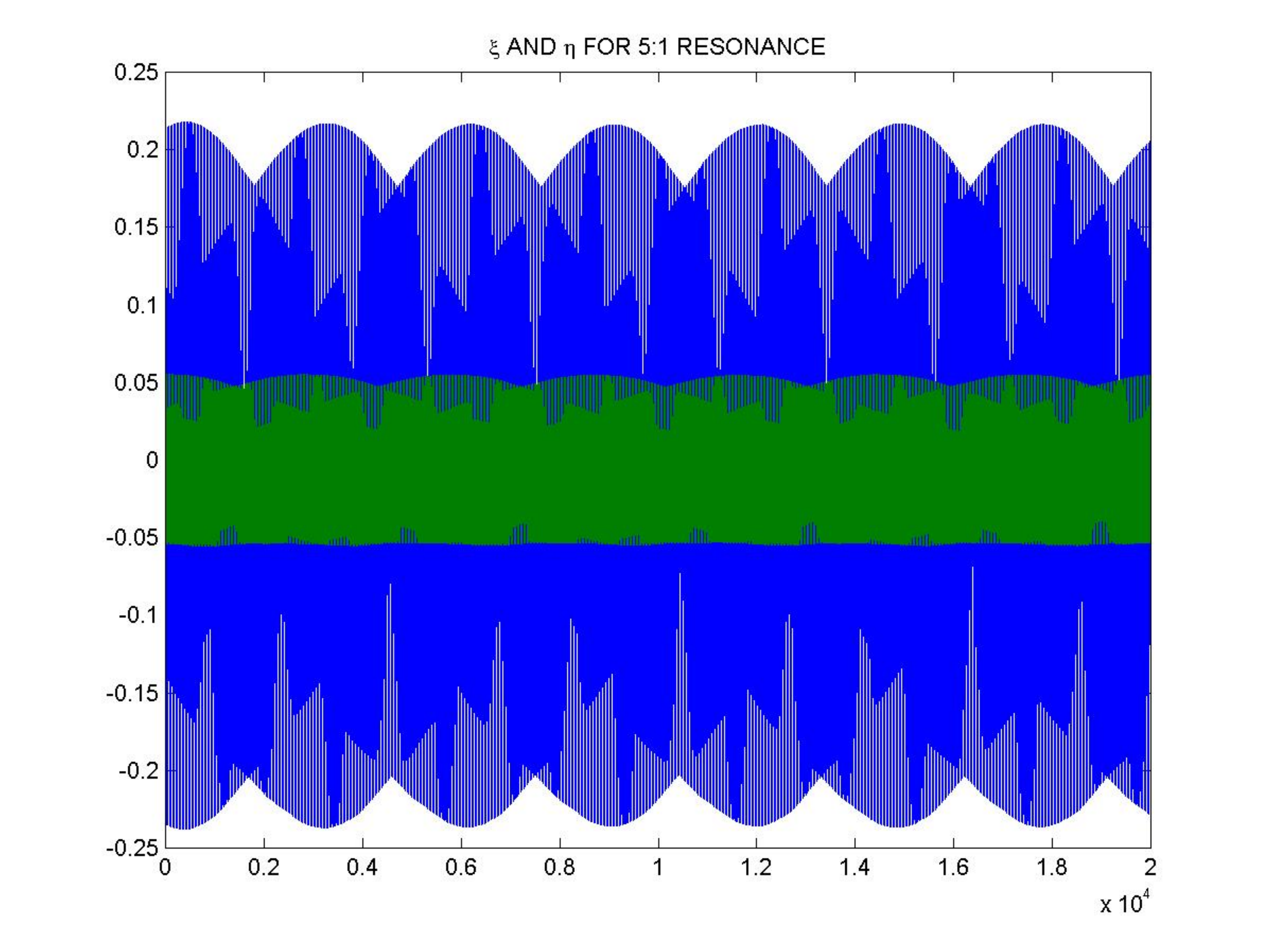
The five Lagrange equilibrium points for $\mu_2 = 0.01$.

Resonant Dynamics near L_4

[In collaboration with Conor Houghton, Trinity College, Dublin]

- ***We investigate dynamics near the stable Lagrange points in the restricted three-body problem.***
- ***There is resonant interaction when the eigenfrequencies are commensurate.***
- ***For 2:1 resonance, the modulation equations are the 'explosive interaction' three-wave equations.***
- ***We have found numerical evidence of a stable 5:1 resonance.***

Numerical Evidence of Resonance



Plot of ξ and η for a 20,000 year integration

The Earth/Moon System

For the Earth/Moon system, $\mu_2 = 0.012153$.

The eigenfrequencies are

$$\sigma_1 = 0.29824\,, \qquad \sigma_2 = 0.95449\,,$$

giving a frequency ratio:

$$\frac{\sigma_2}{\sigma_1} = 3.2004 \approx 3.2000 \equiv \frac{16}{5}\,.$$

It is possible that an object close to the L_4-point of the Earth/Moon system may be found to be in $16:5$ resonance.

It is also possible that such an object may fall within the unstable domain of the $3:1$ resonance.

Of course, this is speculative. Near-resonances are ubiquitous in the solar system, and many are without dynamical consequence. We shall see!

Potential Vorticity Conservation

We define

$$\begin{aligned} \zeta &= \textbf{Relative Vorticity,} \\ f &= \textbf{Planetary Vorticity,} \\ h &= \textbf{Fluid Depth.} \end{aligned}$$

From the *Shallow Water Equations*, we derive the principle of conservation of potential vorticity:

$$\frac{d}{dt}\left(\frac{\zeta + f}{h}\right) = 0\,.$$

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From the *Shallow Water Equations*, we derive the principle of conservation of potential vorticity:

$$\frac{d}{dt}\left(\frac{\zeta+f}{h}\right)=0\,.$$

Under the assumptions of quasi-geostrophic theory, the dynamics reduce to an equation for ψ alone:

$$\frac{\partial}{\partial t}[\nabla^2\psi-F\psi]+\left\{\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}\right\}+\beta\frac{\partial\psi}{\partial x}=0$$

This is the barotropic quasi-geostrophic potential vorticity equation (BQGPVE).

Rossby Waves

Wave-like solutions of the vorticity equation:

$$\psi = A\cos(kx + \ell y - \sigma t)$$

satisfies the equation provided

$$\sigma = -\frac{k\beta}{k^2 + \ell^2 + F}\,.$$

This is the celebrated Rossby wave formula

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The nonlinear term vanishes for a single Rossby wave:
A pure Rossby wave is solution of nonlinear equation.

When there is more than one wave present, this is no longer true: the *components interact* with each other through the nonlinear terms.

Resonant Rossby Wave Triads

Case of special interest: **Two wave components produce a third such that its interaction with each generates the other.**

By a multiple time-scale analysis we derive the modulation equations for the wave amplitudes:

$$
\begin{aligned}
i\dot{A} &= B^*C\,, \\
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This is the canonical form of the *three-wave equations*.

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$$

This is the canonical form of the *three-wave equations*.

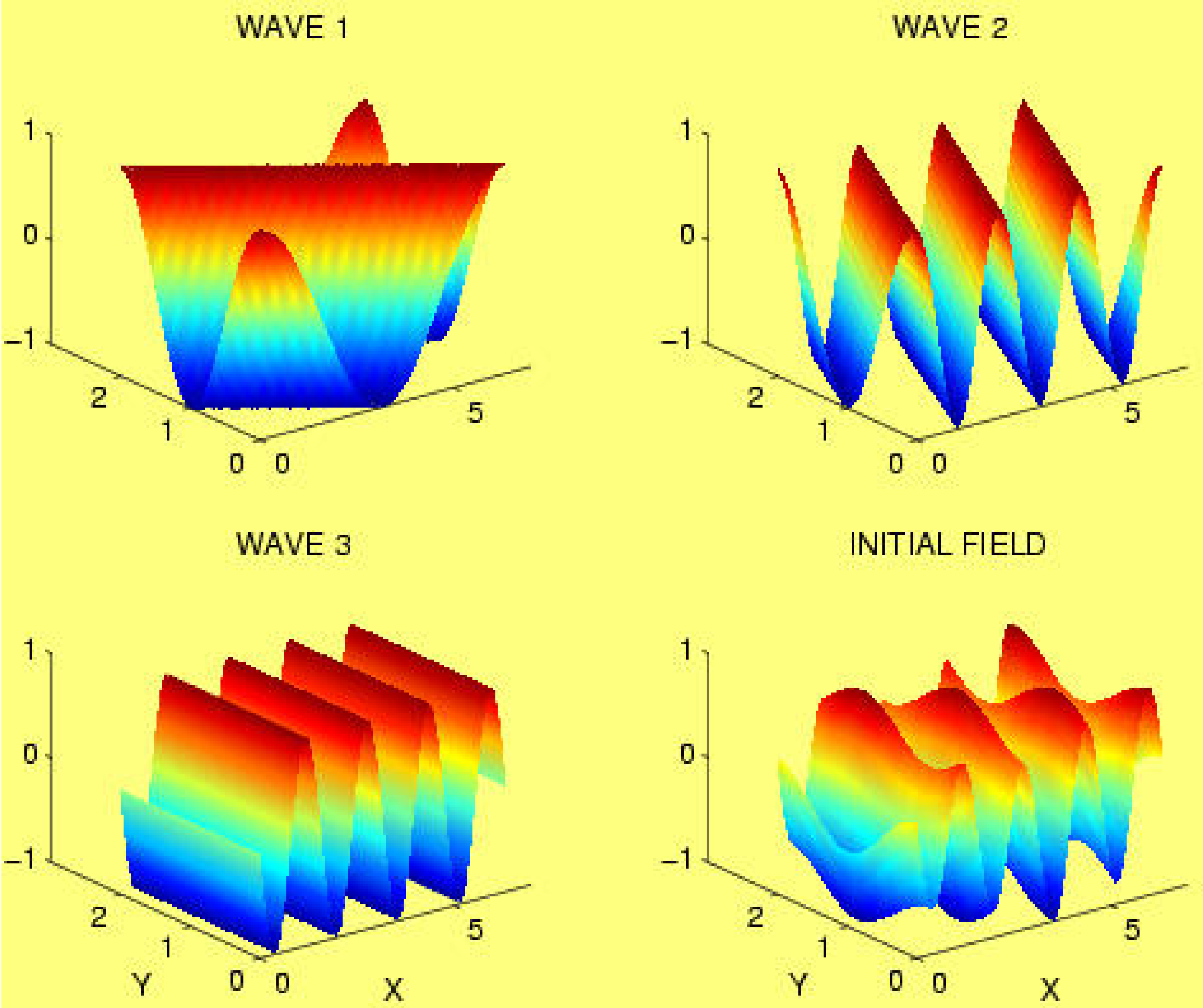
The Spring Equations
and the
Triad Equations are
are
Mathematically Identical!

Numerical Example of Resonance

Method of numerical solution of the Partial Differential Equation:

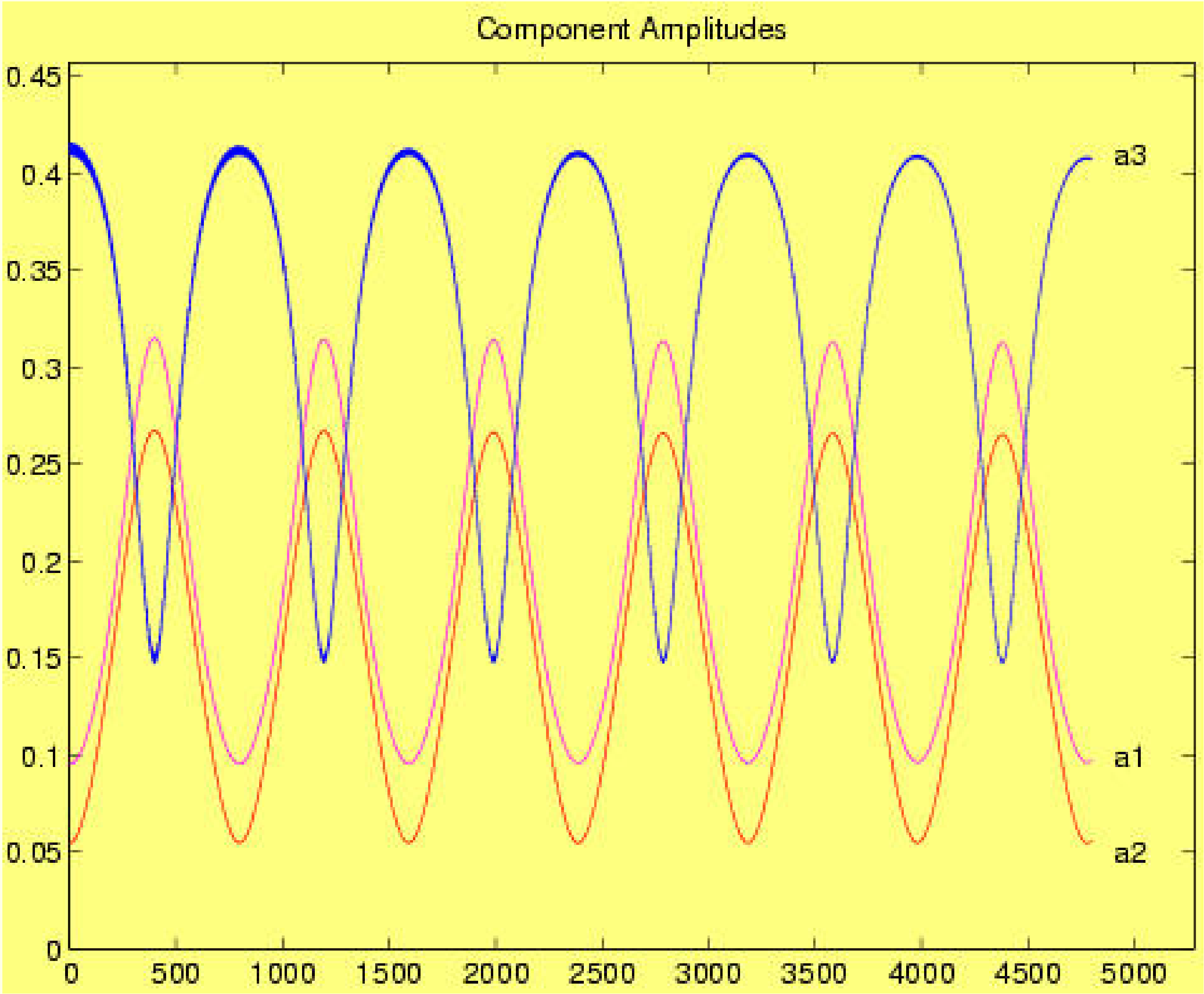
$$\frac{\partial}{\partial t}[\nabla^2\psi - F\psi] + \left\{\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}\right\} + \beta\frac{\partial\psi}{\partial x} = 0$$

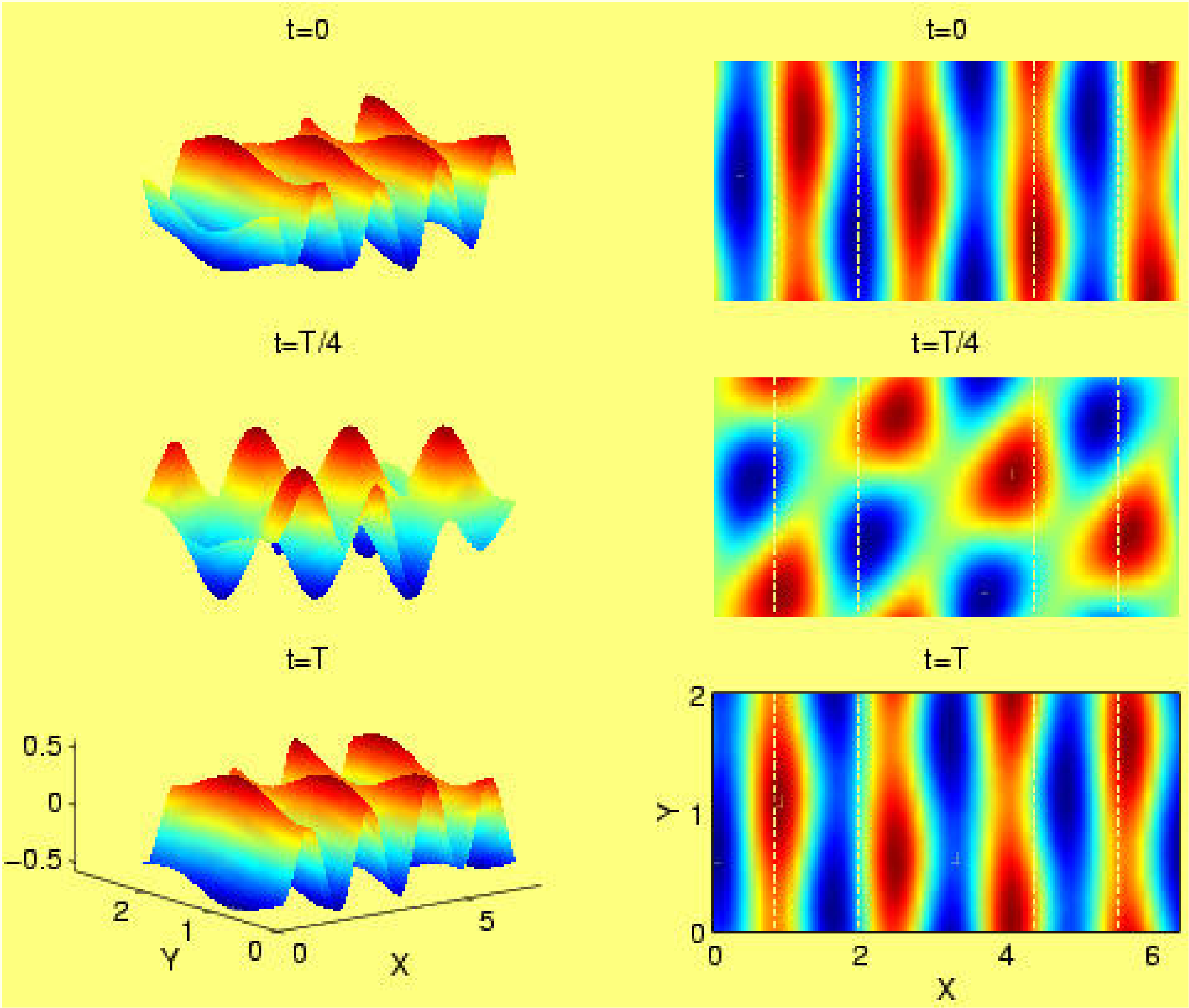
- **Potential vorticity, $q = [\nabla^2\psi - F\psi]$ is stepped forward (using a leap-frog method)**
- **ψ is obtained by solving a Helmholtz equation with periodic boundary conditions**
- **The Jacobian term is discretized following Arakawa (to conserve energy and enstrophy)**
- **Amplitude is chosen very small. Therefore, interaction time is very long.**



Components of a resonant Rossby wave triad
All fields are scaled to have unit amplitude.

Variation with time of the amplitudes of three components of the stream function.





Stream function at three times during an integration of duration $T = 4800$ days.

Precession of Triads

- **Analogies are Interesting — Equivalences are Useful!**

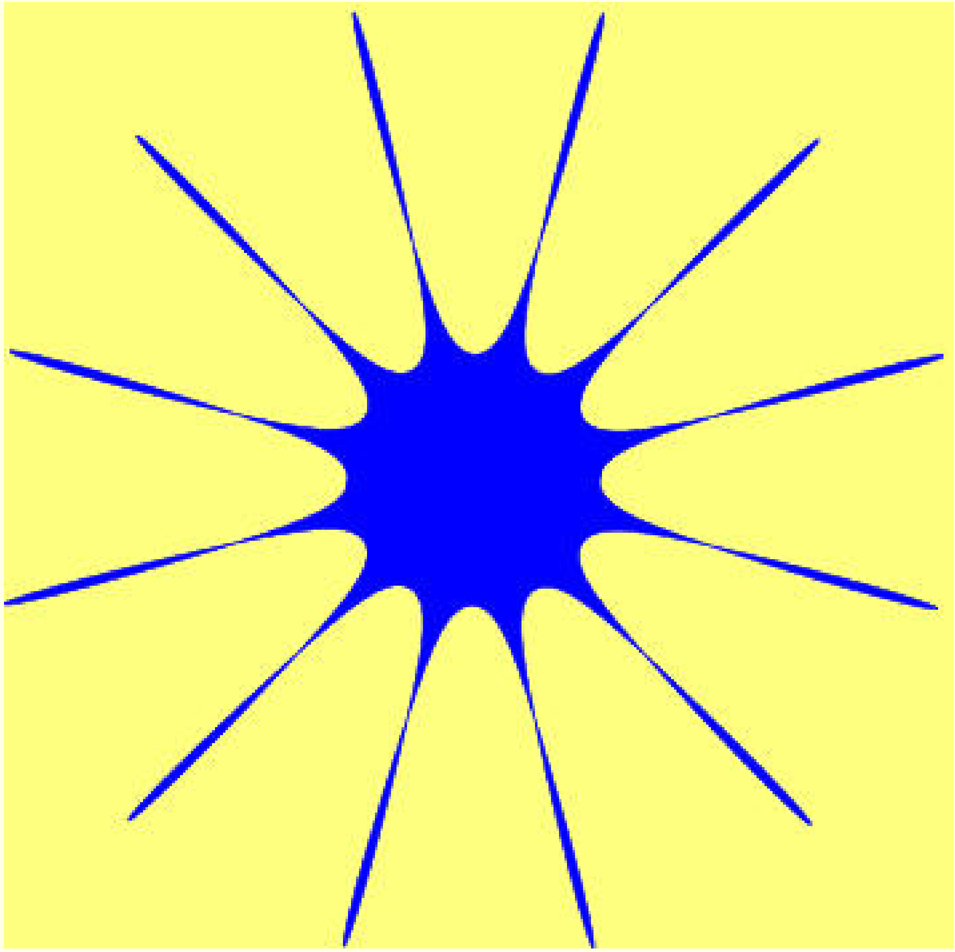
Precession of Triads

- **Analogies are Interesting — Equivalences are Useful!**

Since the same equations apply to both the spring and triad systems, the stepwise precession of the spring must have a counterpart for triad interactions.
In terms of the variables of the three-wave equations, the semi-axis major and azimuthal angle θ are

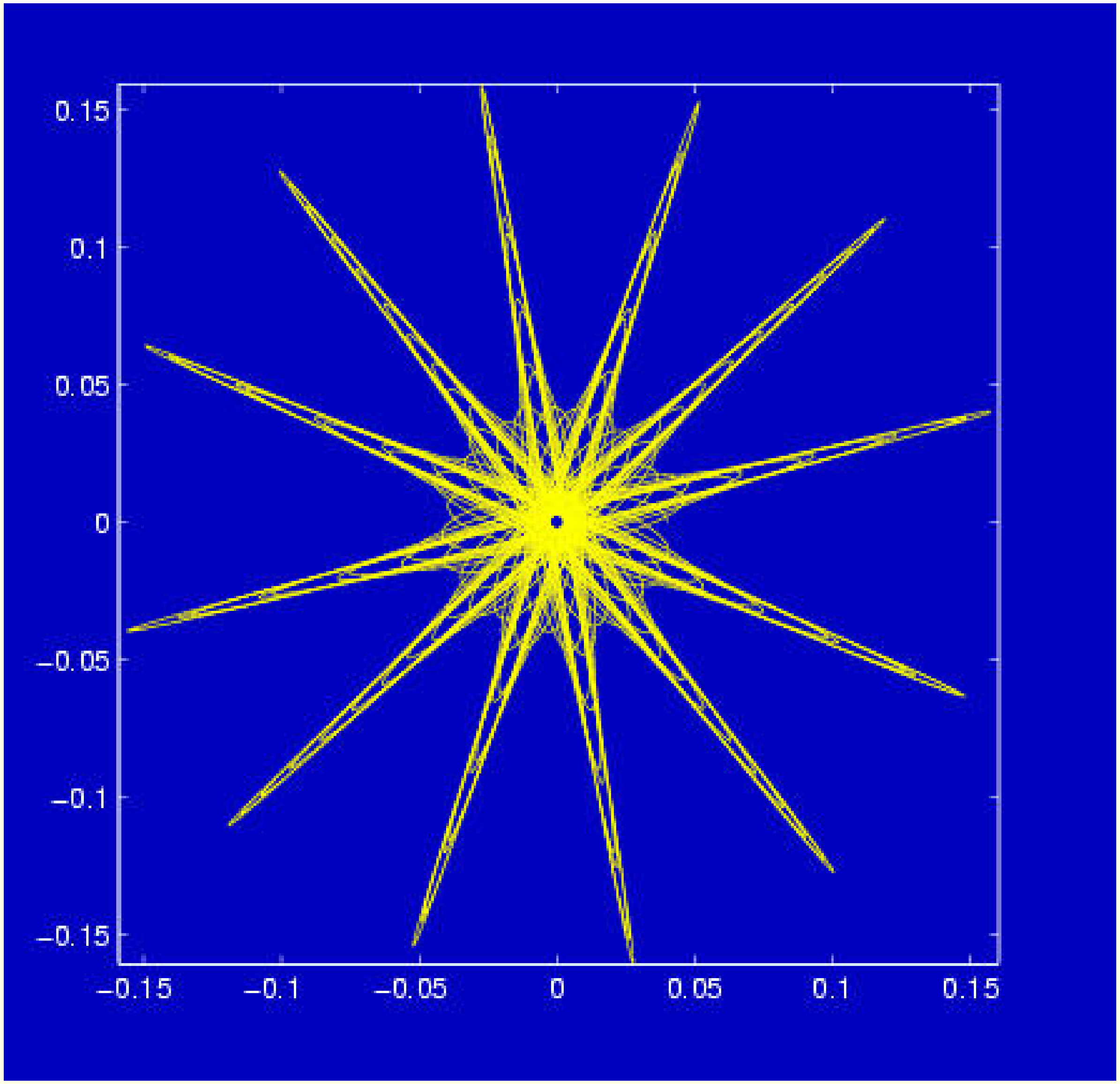
$$A_{\mathrm{maj}} = |A_1| + |A_2| \,, \qquad \theta = \frac{1}{2}(\varphi_1 - \varphi_2)\,.$$

Initial conditions chosen as for the spring (by means of the transformation relations).
Initial field scaled to ensure that small amplitude approximation accurate



Polar plot of A_{maj} versus θ for resonant triad.

Take a peek at the Applet, if available!



Horizontal projection of spring solution, y vs. x.

Polar plots of A_{maj} versus θ.

(These are the quantities for the Triad, which correspond to the horizontal projection of the swinging spring.)

- **The Star-like pattern is immediately evident.**
- **Precession angle again about 30°.**

This is remarkable, and illustrates the value of the equivalence:

Phase precession for Rossby wave triads has not been noted before.

Resonant interactions are important for energy distribution in the atmosphere. They play a central rôle in *Wave Turbulence Theory* [*e.g.*, Alan Newell et al., *Physica D*, 152, 520–550 (2001)].

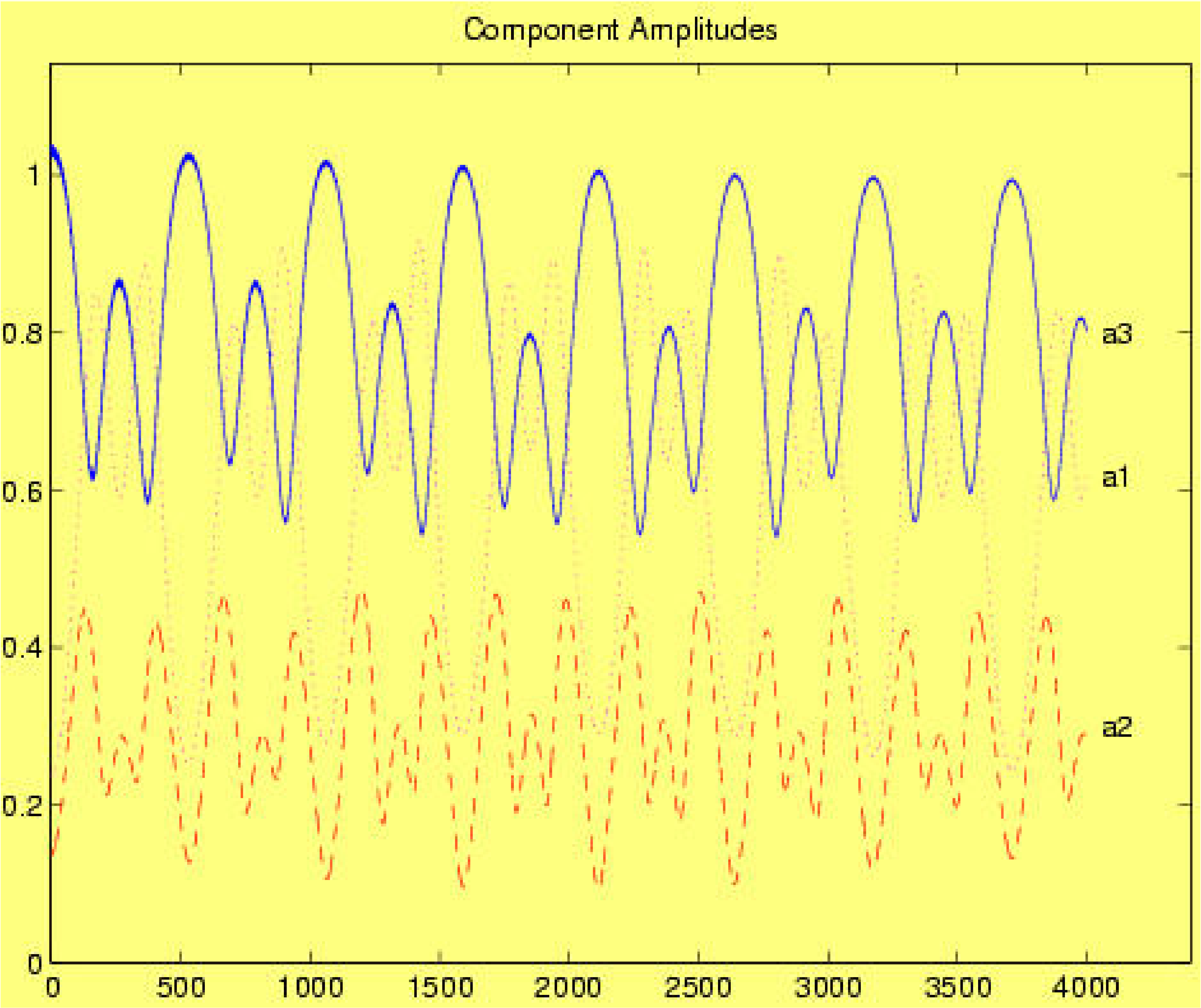
Prima facie evidence for a period-doubling route to chaos.

For large amplitudes, the periodic exchange of energy between the modes is replaced by a more irregular evolution.

As the amplitudes are increased slightly, there appears to be evidence of the period-doubling phenomenon.

Period-doubling is one of the transition routes from regular to chaotic motion. (Also from laminar to turbulent flow?)

This issue requires "further investigation". A UCD student is studying this question for his M.Sc. Project.


Component Amplitudes
a3
a1
a2
0
0.2
0.4
0.6
0.8
1
0
500
1000
1500
2000
2500
3000
3500
4000

Conclusion

I hope I have convinced you that:

> **This simple system looks like a toy at best, but its behaviour is astonishingly complex, with many facets of more than academic lustre . . .**
> (*Breitenberger and Mueller, 1981*)

. . . and that the Swinging Spring is a valuable model of some important aspects of atmospheric dynamics.

Conclusion of Part III.

Thank You for Listening