University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER II EXAMINATION 2009/2010

MAPH 40520

Numerical Weather Prediction

Extern examiner: Prof Keith Shine

Head of School: Prof Mícheál Ó Searcóid

Examiner: Prof Peter Lynch*

Time Allowed: 3 hours

Instructions for Candidates

Answer **four (4)** questions.
Question 1 **must** be answered, and carries 40 marks.
Three additional questions, each carrying 20 marks, must be answered.
Please use separate answer book for each question.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1 [mandatory]

Trace the development of numerical weather prediction (NWP) through the twentieth century, addressing each of the following topics/questions.

(a) (*12 marks*) Describe in one or two sentences each of the main components of a conventional operational NWP system: Automatic Data Extraction; Objective Analysis; Initialization; Numerical Prediction; Post-processing; Visualization; Verification.

(b) (*4 marks*) What was Vilhelm Bjerknes' "manifesto" and the two necessary conditions that he identified for achieving his goal of scientific weather forecasting. Describe the progress he made in achieving this goal.

(c) (*4 marks*) What were the four crucial technical and/or scientific developments in the period 1920–1950 that made NWP feasible.

(d) (*4 marks*) How has the S1 Score of the operational 500 hPa forecast from NMC/NCEP evolved over the past fifty years? Include a diagram to illustrate (qualitatively) the skill of the 36 hour and 72 hour forecasts.

(e) (*4 marks*) What is the main factor leading to the substantial improvement of operational NWP in the past two decades?

(f) (*4 marks*) List four types of satellite-based observations, and the difficulties associated with their assimilation into NWP systems.

(g) (*4 marks*) Give a description of the ECMWF Ensemble Prediction System, including the main products generated from the EPS.

(h) (*4 marks*) Describe the inter-dependency of global and regional NWP models and how they are used together in operational weather forecasting. Give a specific example to illustrate your response.

Question 2

Write a description of the WRF numerical weather prediction model. Include brief comments on each of the following (*2 marks each*):

1. Overall purpose of model. Main applications.
2. Basic dynamical core (governing equations).
3. Horizontal and vertical discretization.
4. Available map projections.
5. Time integration schemes.

6. Upper and lower boundary conditions.
7. WRF Preprocessing System (WPS).
8. Initialization scheme used.
9. Post-processing and graphics used.
10. Other model system components.

Question 3

(a) (*5 marks*) Discuss the difficulties encountered when unbalanced initial data are used to make a numerical forecast. Itemize five specific problems that can arise when uninitialized data is used.

(b) (*5 marks*) Describe each of the following methods of dealing with spurious gravity-wave noise (one or two sentences for each):

- Use of filtered equations
- Static initialization
- Dynamic initialization
- Variational initialization

(c) (*10 marks*) Assume that the system of model equations is written in the form

$$\dot{\mathbf{X}} + i\mathbf{L}\mathbf{X} + \mathbf{N}(\mathbf{X}) = 0\,,$$

where $\mathbf{X}$ is the state vector, $\mathbf{L}$ is a matrix and $\mathbf{N}$ is a nonlinear vector function. Show how this system can be written in normal mode form using the eigenvector matrix of $\mathbf{L}$. Decribe the procedures of linear and nonlinear normal mode initialization for this system.

Question 4

(a) (*5 marks*) Consider the barotropic absolute vorticity equation

$$\frac{\mathrm{d}}{\mathrm{d}t}(\zeta + f) \qquad (1)$$

where ζ is the relative vorticity and f the Coriolis parameter.

Assuming that the wind is non-divergent and can be expressed in terms of a streamfunction, $\mathbf{V} = \mathbf{k} \times \nabla\psi$, show that equation (1) can be written as an equation involving only ψ as a dependent variable.

(b) (*12 marks*) Give a step-by-step method of numerically integrating the equation for ψ. Assume that the values of ψ on the lateral boundaries remain fixed at their initial values. Include a method for solving the Poisson equation relating $\partial\psi/\partial t$ and $\partial\zeta/\partial t$, assuming that $\partial\psi/\partial t = 0$ on the lateral boundaries.

(c) (*3 marks*) Outline the use of equation (1) in the performance of the first computer weather forecast on the ENIAC in 1950.

Question 5

In three-dimensional variational assimilation (3D-Var), we define the analysis to be the state vector $\mathbf{x_a}$ that minimizes the *cost function*

$$J(\mathbf{x}) = \frac{1}{2}\left\{(\mathbf{x} - \mathbf{x_b})^T\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x_b}) + (\mathbf{y_o} - H(\mathbf{x}))^T\mathbf{R}^{-1}(\mathbf{y_o} - H(\mathbf{x}))\right\}$$

where all symbols have their conventional meanings.

- (a) (*6 marks*) Derive an expression for the gradient of J with respect to the state vector $\mathbf{x}$.
- (b) (*5 marks*) Setting the gradient to zero, show that the 3D-Var analysis may be written
$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{W}[\mathbf{y_o} - H(\mathbf{x_b})]$$
where the *gain matrix* $\mathbf{W}$ is given by
$$\mathbf{W} = \left[\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}$$
- (c) (*5 marks*) Show that $\mathbf{W}$ is equal to the gain matrix
$$\mathbf{W}_{\mathrm{OI}} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}$$
that is obtained in optimal interpolation analysis.
- (d) (*4 marks*) Outline the changes needed to convert 3D-Var to 4D-Var, that is, to assimilate observations at their time of validity within a fixed time window.

Question 6

Consider the linear shallow water equations on an f-plane:

$$\begin{aligned}
\frac{\partial u}{\partial t} - fv + \frac{\partial}{\partial x}\left(\frac{p}{\rho_0}\right) &= 0 \\
\frac{\partial v}{\partial t} + fu + \frac{\partial}{\partial y}\left(\frac{p}{\rho_0}\right) &= 0 \\
\frac{\partial}{\partial t}\left(\frac{p}{\rho_0}\right) + gH\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) &= 0\,.
\end{aligned}$$

Assume that the density ρ_0 is constant.

(a) (*4 marks*) Introduce characteristic scales: Length (L), velocity (V), advective time (L/V), pressure variations (P) and assuming f, g and H are constant, indicate the scale of each term in the system (*e.g.*, $\partial u/\partial t \sim V^2/L$).

(b) (*5 marks*) Assuming typical mid-latitude synoptic values for the scales, evaluate the order of magnitude of each term. Show how the Rossby Number arises naturally in the comparison of terms of the momentum equations. Show how, to lowest order, the flow is geostrophically balanced.

(c) (*6 marks*) Show how the scaling must be modified to ensure balance in the continuity equation. Discuss briefly the relevance of the Dines compensation mechanism in this context.

(d) (*5 marks*) Show how a 10% error in the wind speed may result in a 100% error in the wind tendency. Discuss the implications of this for numerical weather prediction and the manner in which the problem is alleviated.

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