University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SUMMER EXAMINATIONS 2006

SCMXF0028
SCMXP0028
M.Sc. in Meteorology

Numerical Weather Prediction

MAPH P313

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Time Allowed: 3 hours

Instructions for Candidates

Full marks will be awarded for complete answers
to **four** questions.
All questions carry equal marks.
Please do not use red pen on the answer books.
Please use separate answer book for each question.

P.T.O.

Question 1.

- (a) Given two independent, unbiassed temperature observations T_1 and T_2 with (unknown) errors ε_1 and ε_2, and (known) variances σ_1^2 and σ_2^2, derive an expression for the best linear unbiassed estimate (BLUE) of the temperature. State all assumptions you make.

- (b) Assuming $T_1 = 22°\text{C}$, $T_2 = 24°\text{C}$, $\sigma_1 = 1°\text{C}$ and $\sigma_2 = 3°\text{C}$, find the temperature that minimizes the least squares error, and also find its variance.

- (c) Show that the temperature that minimizes the *cost function*

$$J(T) = \frac{1}{2}\left[\left(\frac{T - T_1}{\sigma_1}\right)^2 + \left(\frac{T - T_2}{\sigma_2}\right)^2\right]$$

  yields the same estimate of T as the least squares estimate.

- (d) Briefly discuss the consequences of this equivalence in the context of optimal interpolation (OI) analysis and variational assimilation.

Question 2.

In three-dimensional variational assimilation (3D-Var), we define the analysis to be the state vector $\mathbf{x_a}$ that minimizes the *cost function*

$$J(\mathbf{x}) = \frac{1}{2}\left\{(\mathbf{x} - \mathbf{x_b})^T\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x_b}) + (\mathbf{y_o} - H(\mathbf{x}))^T\mathbf{R}^{-1}(\mathbf{y_o} - H(\mathbf{x}))\right\}$$

where all symbols have their conventional meanings.

- (a) Derive an expression for the gradient of J with respect to the state vector $\mathbf{x}$.

- (b) Setting the gradient to zero, show that the 3D-Var analysis may be written

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{W}[\mathbf{y_o} - H(\mathbf{x_b})]$$

  where the *gain matrix* $\mathbf{W}$ is given by

$$\mathbf{W} = \left[\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}$$

- (c) Show that $\mathbf{W}$ is equal to the gain matrix

$$\mathbf{W}_{\text{OI}} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}$$

  that is obtained in optimal interpolation analysis.

- (d) Outline the changes needed to convert 3D-Var to 4D-Var, that is, to assimilate observations at their time of validity within a fixed time window.

Question 3.

Consider the advection equation, the partial differential equation (PDE)

$$\frac{\partial Y}{\partial t} + c\,\frac{\partial Y}{\partial x} = 0\,.$$

- (a) Write a finite difference equation (FDE) approximation to this PDE using the *upstream method* (you may assume that the advection speed c is positive). Define "consistency" and "local truncation error" for a numerical scheme. Show that the FDE is consistent with the PDE by deriving the local truncation error.

- (b) State the von Neumann criterion for computational stability. Use the von Neumann method to study the stability of the FDE and derive a condition for the numerical stability of the upstream scheme.

- (c) Show that this condition is equivalent to the condition obtained using the "criterion of the maximum".

Question 4.

The one-dimensional advection equation may be written in either the Eulerian form or the Lagrangian form:

$$\frac{\partial Y}{\partial t} + u\,\frac{\partial Y}{\partial x} = 0 \qquad \text{or} \qquad \frac{dY}{dt} = 0\,.$$

- (a) Describe the procedure used to develop a finite difference approximation to the Lagrangian form of the equation.

- (b) Assuming that linear spatial interpolation is used to evaluate the solution at the departure point, show that the semi-Lagrangian scheme is stable irrespective of the time step.

- (c) Discuss the consequences of unrestricted stability for operational numerical weather prediction. Does unconditional stability of the scheme enable an arbitrarily large time step to be used in practice, or are there other considerations limiting it?

Question 5.

- (a) Discuss the difficulties encountered when unbalanced initial data are used to make a numerical forecast. Explain how the process of initialization alleviates the difficulties.
- (b) Briefly describe each of the following methods of dealing with spurious gravity-wave noise:
  - Use of filtered equations
  - Static initialization
  - Dynamic initialization
  - Variational initialization
- (c) Assume that the system of model equations is written in the form

$$\dot{\mathbf{X}} + i\mathbf{L}\mathbf{X} + \mathbf{N}(\mathbf{X}) = 0\,,$$

where $\mathbf{X}$ is the state vector, $\mathbf{L}$ is a matrix and $\mathbf{N}$ is a nonlinear vector function. Show how this system can be written in normal mode form using the eigenvector matrix of $\mathbf{L}$. Decribe the procedures of linear and nonlinear normal mode initialization for this system.

Question 6.

- (a) Write the six-point Crank-Nicholson or centered implicit finite difference equation (FDE) for the linear advection equation in one dimension.
- (b) Assuming that the FDE has a solution of the form

$$Y_m^n = A\exp[ik(m\Delta x - Cn\Delta t)]\,,$$

show that the physical phase speed c and the computational phase speed C are related by the equation

$$C = \left(\frac{2}{k\Delta t}\right)\tan^{-1}\left[\left(\frac{c\Delta t}{2\Delta x}\right)\sin k\Delta x\right]$$

- (c) Interpret this relationship in terms of propogation of wave components of the numerical solution and discuss its implications for the stability and accuracy of forecasts using the scheme.

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