UNIVERSITY COLLEGE DUBLIN

NATIONAL UNIVERSITY OF IRELAND, DUBLIN

An Coláiste Ollscoile Baile Átha Cliath

Ollscoil na hÉireann, Baile Átha Cliath

SUMMER EXAMINATIONS 2005

SCMXF0028
SCMXP0028

MATHEMATICAL PHYSICS

Synoptic Meteorology

MAPH-P312

Professor F. Hodnett;
Professor A. Ottewill;
Professor P. Lynch*.

Instructions for Candidates

Please use *separate answer book* for each question
All questions carry equal marks
Full marks for complete answers to *four* questions.

Time: 3 hours.

Notes for Invigilators

Non-programmable calculators are permitted

Question 1. (a) Describe the Chapman reactions which are responsible for maintaining steady-state concentrations of O_3 in the stratosphere.

(b) Describe the four critical stages in the formation of the "ozone hole" over the Antarctic in the Austral Spring. Identify the key factors which are involved in this phenomenon.

(c) Why does the phenonemon of major ozone depletion occur primarily in the Southern hemisphere and why in the Springtime?

Question 2. A cloud is cylindrical in shape and has a cross-sectional area of $10\,\mathrm{km}^2$ and a height of 3 km. All of the cloud is initially supercooled and the liquid water content is $2\ \mathrm{g\,m^{-3}}$. If all of the water in the cloud is transferred onto ice nuclei present in a uniform concentration of 1 per liter, determine the total number of ice crystals in the cloud and the mass of each ice crystal produced. If all the ice crystals precipitate and melt before they reach the ground, what will be the total rainfall produced?

Question 3. Consider a "collector drop" of radius r_1 with a terminal fall speed v_1, falling in still air through a cloud of equal sized droplets of radius r_2 with terminal fall speed $v_2 \ll v_1$. The rate of increase in the mass M of the collector drop due to collisions may be shown to be

$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_\ell E_c$$

where w_ℓ is the LWC (in $\mathrm{kg\,m^{-3}}$) of the cloud droplets of radius r_2 and E_c is the collection efficiency.

(a) Show that the increase in the radius of the collector drop is governed by an equation of the form

$$\frac{dr_1}{dt} = K v_1$$

where K is constant.

(b) Assuming, following from Stokes' Drag Law, that v_1 is proportional to the square of the radius r_1, derive an expression for r_1 as a function of time and show that the solution diverges to infinity in a finite time.

(c) Explain the consequences of this result for the formation of rain drops by the process of growth by collection.

Question 4. Assume that the geopotential field Φ is described by the function

$$\Phi = \Phi_0(p) + cf_0\Big\{-y[1+\cos(\pi p/p_0)] + (1/k)\sin k(x-ct)\Big\}$$

where Φ_0 depends only on p, c is a constant speed, f_0 is the (constant) Coriolis parameter, k the zonal wavenumber and $p_0 = 1000\,\text{hPa}$.

(a) Derive expressions for the geostrophic wind components and relative vorticity field.

(b) Obtain an expression for the advection of relative vorticity.

(c) Deduce the horizontal divergence field by means of the quasi-geostrophic vorticity equation (assume $\beta = 0$).

(d) By integration of the continuity equation, obtain an expression for the vertical velocity ω (assume $\omega(p_0) = 0$).

(e) Sketch the geopotential fields at 250 hPa and 750 hPa. Indicate the regions of maximum divergence and convergence and of positive and negative vorticity advection.

Question 5. Two air masses, of uniform temperature T_1 and T_2, are moving with constant velocity V_1 and V_2 respectively, parallel to the plane frontal surface separating them, with no along-front variations.

(a) Show, assuming geostrophic flow and making the Boussinesq approximation, that the angle of slope ε of the frontal surface is given by

$$\tan\varepsilon = \frac{f\bar{T}}{g}\frac{V_1 - V_2}{T_1 - T_2}$$

where $\bar{T} = (T_1 + T_2)/2$. State any further approximations or assumptions that you make.

(b) Calculate the frontal slope assuming that the mean temperature is $\bar{T} = 280\,\text{K}$, the Coriolis parameter $f = 10^{-4}\,\text{s}^{-1}$, $g = 10\,\text{m}\,\text{s}^{-2}$, the difference in windspeed across the front is $\Delta V = 12\,\text{m}\,\text{s}^{-1}$ and the difference in temperature is $\Delta T = 4\,\text{K}$.

(c) Sketch the pressure pattern associated with this flow configuration and describe how it is modified by superposition of a constant drift perpendicular to the front. How is this used in synoptic analysis.

Question 6. As cold, continental air passes over a warm ocean on a winter day, the temperature rises by 10°C over a distance of 300 km. Within this interval, the mean mixed layer depth is 1 km and the average wind speed is $10\,\mathrm{m\,s^{-1}}$. Assuming that no condensation is taking place within the lowest km and that the radiative fluxes are negligible, calculate the sensible heat flux from the sea surface. [You may assume that the mean density of the 1 km column is $1.2\,\mathrm{kg\,m^{-3}}$ and take $c_p = 1004\,\mathrm{J\,kg^{-1}K^{-1}}$.]

Question 7. (a) Starting from the geopotential tendency equation

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) + \frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\mathbf{V_g}\cdot\nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

derive the quasi-geostrophic potential vorticity (QGPV) equation

$$\left(\frac{\partial}{\partial t} + \mathbf{V_g}\cdot\nabla\right)\left[\frac{1}{f_0}\nabla^2\Phi + f + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial \Phi}{\partial p}\right)\right] = 0$$

(b) Show that, for barotropic flow, this reduces to the equation

$$\frac{d_g}{dt}(\zeta_g + f) = 0\,.$$

(c) Describe in qualitative terms how this barotropic equation may be used to predict the evolution of atmospheric flows in midlatitudes. Outline the main steps in the numerical solution of the equation.

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