University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

WINTER EXAMINATIONS 2005/2006

SCMXF0028/SCMXP0028 MSc in Meteorology

Physical Meteorology

MAPH P311

Extern examiner: Prof. Frank Hodnett

Head of School: Prof. Adrian Ottewill

Examiner: Dr. Rodrigo Caballero*

Time Allowed: 3 hours

Instructions for Candidates

Answer **four (4)** of the following six questions. Each question carries 25 marks.

Instructions for Invigilators

Non-programmable calculators may be used during this examination. A tephigram chart will be handed to each candidate as part of the examination material.

Question 1

- (*10 marks*) By applying Newton's 2nd Law to a body of mass m moving in the Earth's gravitational field, show that

$$\frac{d}{dt}\left[\frac{1}{2}mv_z^2 + \frac{GmM_E}{R_E}\left(1 - \frac{R_E}{R_E + z}\right)\right] = 0,$$

  where z is vertical distance above Earth's surface, v_z is vertical velocity, G=6.67×10^{-11} N m^2 kg^{-2} is the universal gravitational constant, M_E=5.98×10^{-24} kg is Earth's mass and R_E=6.38×10^6 m is Earth's radius. What names are given to the two terms inside the square brackets?

- (*10 marks*) A can containing 1 kg of petrol (gasoline), initially at rest on the Earth's surface, is lifted to a height of 2.55×10^7 m above the surface. Compute the work (i.e. energy) required to do this. Ignore the can's mass and assume it is at rest in the final position.

- (*5 marks*) Burning 1 kg of petrol releases about 5×10^7 J. This energy comes from work done on the molecules by intermolecular forces during combustion. Assuming molecules move through a typical distance of 1 μm during combustion, give a *rough* estimate of the ratio (gravitational force)/(intermolecular force). Hint: remember work=force×distance.

Question 2

- (*10 marks*) Show that for a fluid at rest in a gravitational field

$$\frac{dp}{dz} = -\rho g,$$

  where p is pressure, z is vertical distance, ρ is density and g the acceleration due to gravity. (Use either a molecular or continuum approach, as you prefer).

- (*10 marks*) Consider an ocean in hydrostatic equilibrium. How deep must you go to find a pressure of 2000 hPa? Assume the ocean is incompressible and has a density of 1000 kg m^{-3}, and assume that atmospheric pressure at the ocean surface is 1000 hPa. Take the acceleration due to gravity as 10 m s^{-2}

- (*5 marks*) Imagine you are at 40 m depth in the ocean and are provided with a long, rigid tube that reaches the surface. Would you be able to breathe through this tube? Explain your reasoning. Hint: consider the difference between the pressure of air coming in to your lungs and the pressure of the ambient ocean.

Question 3

- (*10 marks*) Make a rough sketch of the emission spectrum as a function of wavelength for a blackbody at a temperature of 300 K, and (on the same diagram) for one at 6000 K. On the diagram, identify the curves as "solar" and "terrestrial" and indicate the approximate location of the peak in each spectrum.

- (*10 marks*) Imagine you are floating around in space. At what rate (in K s^{-1}) would you cool down if you removed your space suit? For simplicity, assume your body is cylindrical, with length 2 m and radius 10 cm, and you are made entirely of water with density 1000 kg m^{-3}. The sun is not striking you, and your initial body temperature is 37°C. The Stefan-Boltzmann constant σ=5.67$\times 10^{-8}$ W m^{-2} K^{-4} and the specific heat capacity of water is 4218 J kg^{-1} K^{-1}.

- (*5 marks*) To stay warmer, would you wrap yourself in black wool or in aluminium foil? Explain your reasoning. Hint: remember Kirchhoff's law.

Question 4

- (*10 marks*) Energy conservation for an ideal gas can be expressed as

$$c_p \frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} = J,$$

where c_p is the specific heat at constant pressure, T is temperature, ρ is density, p is pressure and J is the heating rate. Use this result to show that in an adiabatic process, the potential temperature

$$\theta = T\left(\frac{p}{p_0}\right)^{-R/c_p}$$

is constant.

- (*15 marks*) A cylinder of 1 m^2 cross-sectional area, fitted with a piston, contains 1 kg of dry air. The air is initially in equilibrium at a temperature of 300 K. External pressure is 1000 hPa. The air is instantaneously warmed to 310 K, and is then allowed to expand adiabatically until the internal and external pressures are once more in equilibrium. How far does the piston move? Assume the piston is massless and moves with no friction. The gas constant for dry air is 287 J kg^{-1} K^{-1} and the specific heat capacity at constant pressure is 1004 J kg^{-1} K^{-1}.

Question 5

- (*7 marks*) Write down a mathematical expression for the buoyancy force and briefly explain its meaning in words.
- (*8 marks*) Consider a balloon containing 1 kg of helium at a temperature of 300 K and pressure of 1000 hPa. The air surrounding the balloon is at the same temperature and pressure. Compute the ratio (density helium)/(density air) and the buoyancy force on the balloon. Take the molecular weight of helium as 4, that of dry air as 29 and the acceleration due to gravity as 10 m s^{-2}.
- (*10 marks*) Imagine the balloon above is released. If the atmosphere is isothermal (zero lapse rate), at which pressure level will the balloon stop rising? Assume there is no heat exchange with ambient air as the balloon rises, and the internal pressure of the balloon is always equal to the atmospheric pressure outside. Hint: the potential temperature of helium is

$$\theta_{He} = T \left(\frac{p}{p_0} \right)^{-2/5} .$$

Question 6

Consider the following idealized atmospheric sounding:

Point	p (hPa)	T (°C)	T_d (°C)
A	1000	30	10
B	740	5	5
C	600	-6	-6
D	300	-40	-50

where T is temperature and T_d is dew point temperature.

- (*8 marks*) Plot the sounding on the tephigram provided.
- (*12 marks*) Classify each of points A, B and C as stable, conditionally unstable, unstable, or neutral (i.e. not clearly stable or unstable). Briefly motivate your choices in terms of the behaviour of adiabatically displaced parcels.
- (*5 marks*) Under the atmospheric conditions specified by the above sounding, and with the sun directly overhead, what colour would you most likely see on looking upwards from the ground?

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