UNIVERSITY COLLEGE DUBLIN

NATIONAL UNIVERSITY OF IRELAND, DUBLIN

An Coláiste Ollscoile Baile Átha Cliath

Ollscoil na hÉireann, Baile Átha Cliath

SUMMER EXAMINATIONS 2005

SCMXF0028
SCMXP0028

MATHEMATICAL PHYSICS

Physical Meteorology

MAPH-P311

Professor F. Hodnett;
Professor A. Ottewill;
Professor P. Lynch*.

Instructions for Candidates

All questions carry equal marks
Full marks for complete answers to *four* questions.

Time: 3 hours.

Notes for Invigilators

Candidates will require *Printed Tephigram Sheets*
[There will be provided by Prof Lynch]

Non-programmable calculators are permitted

Question 1. (a) Consider a fixed volume V of moist air at temperature T and pressure p comprising a mass m_d of dry air and a mass m_v of water vapour. Define the mixing ratio w of the air mass. Indicate the typical range of w in the lower troposphere.

(b) By application of the universal gas law

$$pV = nR^*T$$

separately to the dry air and water vapour, show that the vapour pressure e is given in terms of the total pressure p by

$$\frac{e}{p} = \frac{w}{w + \varepsilon}$$

where $\varepsilon \approx 0.622$ is the ratio of the molecular weights of water vapour and dry air. If the mixing ratio is $5.5\,\mathrm{g\,kg^{-1}}$ and the total pressure is $1027\,\mathrm{hPa}$, calculate the vapour pressure.

(c) Show that the gas equation for the moist air may be written

$$p = R_d \rho T_v$$

where R_d is the gas constant for *dry air* and T_v is the virtual temperature, defined by

$$T_v = \frac{T}{1 - (e/p)(1 - \varepsilon)}$$

(d) Calculate the virtual temperature of moist air at 30°C having a mixing ratio of $20\,\mathrm{g\,kg^{-1}}$.

Question 2. (a) Let Z_g, T_g and p_g be the geopotential height, temperature and pressure at ground level and p_0 the pressure at sea-level ($Z_0 = 0$). Using the hypsometric equation, and making appropriate assumptions, show that the sea-level pressure may be estimated from

$$p_0 \approx \left(1 + \frac{Z_g}{H}\right) p_g$$

where $H = RT_g/g$. State all approximations you make. Deduce an estimate of the rate of decrease of pressure with height near the ground.

(b) Assuming that the temperature decreases linearly with height, $T = T_0 - \Gamma z$, derive the relationship

$$z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{p}{p_0}\right)^{R\Gamma/g}\right]$$

for the height of a pressure surface p in terms of the pressure p_0 and temperature T_0 at sea level.

(c) Using the standard values $T_0 = 288\,\mathrm{K}$, $p_0 = 1013\,\mathrm{hPa}$ and $\Gamma = 6.5\,\mathrm{K\,km^{-1}}$, use this relationship to estimate the height of the 500 hPa surface.

Question 3. (a) Define the specific heat of air at constant volume, c_v, and the specific heat at constant pressure, c_p. Explain in physical terms why $c_p > c_v$.

(b) Given the First Law of Thermodynamics in the form

$$dq = c_v\,dT + p\,d\alpha$$

use the equation of state and the hydrostatic relationship to write it in the alternative form

$$dq = c_p\,dT + g\,dz$$

(c) The specific enthalpy of a unit mass of air is defined as

$$h = u + p\alpha$$

where u is the internal energy, p the pressure and α the specific volume. By means of the First Law of Thermodynamics, show that

$$h = c_p T$$

Question 4. (a) Defining the increment in specific entropy of a unit mass of air as

$$ds = \frac{dq}{T}$$

show by application of the thermodynamic equation that

$$s = s_0 + c_p \log(\theta/\theta_0)$$

where s_0 is the entropy at a reference level θ_0.

(b) Describe the construction of the thermodynamic diagram called the *tephigram*, describing the four sets of isopleths (lines of constant values).

Question 5. (a) Using the First Law of Thermodynamics, show that the rate of change of temperature of a parcel of saturated air as it moves vertically upwards in the atmosphere is given by the *saturated adiabatic lapse rate*:

$$\Gamma_d \equiv \left(\frac{dT}{dz}\right) = \frac{g/c_p}{1 + \dfrac{L_v}{c_p}\left(\dfrac{\partial w_s}{\partial T}\right)_p}$$

(b) A parcel of air with an initial temperature of 15°C and dew point 2°C is lifted adiabatically from the 1000 hPa level. Determine its LCL and temperature at that level. If the air parcel is lifted a further 200 hPa above its LCL, what is its final temperature and how much liquid water is condensed during this rise? (A tephigram chart may be used to solve this problem).

Question 6. (a) The Stefan-Boltzmann Law for the blackbody flux density is

$$F = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}K^{-4}}$. Estimate the percentage increase in flux if the temperature increases by one degree from 250 K to 251 K.

(b) Calculate the equivalent blackbody temperature of the solar photosphere, the outermost visible layer of the Sun, based on the following information: The flux density of solar radiation reaching the earth is $1380\,\mathrm{W\,m^{-2}}$; the Earth-Sun distance is 1.50×10^{11}m; and the radius of the solar photosphere is 7.00×10^{8}m.

(c) Explain why the temperature value computed from the Stefan-Boltzman Law differs from the Sun's *colour temperature* estimated from Wien's Displacement Law.

Question 7. (a) The *solar constant*, the flux of solar radiation incident on the Earth, is $1380\,\mathrm{W\,m^{-2}}$. Assuming that the earth is in radiative equilibrium with the Sun, and that the planetary *albedo* is 0.30, calculate the flux density of terrestrial radiation.

(b) Using the Stefan-Boltzmann Law, with $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}K^{-4}}$, compute the equivalent blackbody temperature of the earth.

Question 8. (a) Assuming the solution corresponding to the Ekman spiral,

$$\begin{aligned} u &= u_g(1 - e^{-\gamma z}\cos\gamma z) \\ v &= u_g e^{-\gamma z}\sin\gamma z \end{aligned}$$

with $\gamma = \sqrt{f/2K}$, show that the vertical velocity at the top of the Ekman layer may be expressed as

$$w = \zeta_g / 2\gamma$$

where ζ_g is the geostrophic vorticity. Estimate the characteristic scale of w for typical values of ζ_g and γ.

(b) Show that the e-folding time for a barotropic vortex of depth h is

$$\tau_E = \sqrt{\frac{fK}{2h^2}}\,.$$

Estimate the value of τ_E for typical atmospheric values.

oOo