UNIVERSITY COLLEGE DUBLIN

NATIONAL UNIVERSITY OF IRELAND, DUBLIN

An Colaiste Ollscoile Baile Atha Cliath

Ollscoil na hEireann, Baile Atha Cliath

WINTER EXAMINATIONS 2004

SCMXF0028
SCMXP0028

M.Sc. in METEOROLOGY
MAPH P311
PHYSICAL METEOROLOGY

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Instructions for Candidates,

Candidates should attempt four (4) questions.
All questions carry equal marks.

Time: 3 hours

Notes for Invigilators

Candidates will require the following:
Printed Tephigram sheets.

Calculators are permitted

- **Question 1.**

  Show that, for a stationary atmosphere, balance between the upward pressure force and downward gravitational force may be expressed by the hydrostatic equation
$$\frac{dp}{dz} = -g\rho$$
  Define the geopotential height Z in terms of the geometric height z and the acceleration of gravity g_0 at the earth's surface.

  Using the hydrostatic equation and the gas law for dry air, derive the hypsometric equation
$$Z_2 - Z_1 = \bar{H} \log \frac{p_1}{p_2}$$
  where $\bar{H} = R\bar{T}/g_0$, for the thickness of the layer between pressure levels p_1 and p_2 having mean temperature $\bar{T}$. Using $R = 287\,\mathrm{J\,K^{-1}kg^{-1}}$, $g_0 = 9.81\,\mathrm{m\,s^{-2}}$ and a typical value for the mean tropospheric temperature, derive the characteristic size of the scale-height of the atmosphere.

  Given that the heights of the 1000 hPa and 500 hPa surfaces are 85 m and 5542 m, estimate the mean temperature of the layer.

- **Question 2.**

  Define what is meant by an *adiabatic* process and by an *isothermal* process.

  Using the First Law of Thermodynamics, show that the rate of change of temperature of a parcel of dry air as it moves vertically in the atmosphere without the addition or removal of heat is given by the *dry adiabatic lapse rate*:
$$\Gamma_d \equiv \left(\frac{dT}{dz}\right) = \frac{g}{c_p}$$
  Using characteristic values for the parameters, estimate the magnitude of Γ_d. How does Γ_d compare to typical values of the *observed* lapse rate in the troposphere and in the stratosphere?

  Define the *potential temperature* θ of a parcel of air. Using the thermodynamic equation and the equation of state, derive Poisson's relation
$$\theta = T\left(\frac{p}{p_0}\right)^{-R/c_p}$$
  for the potential temperature.

  Assuming that $\kappa = R/c_p \approx \frac{2}{7}$, find the potential temperature of a parcel of air at pressure level 750 hPa with temperature $T = -15°\mathrm{C}$.

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- **Question 3.**

Define the following quantities for a parcel of moist air: Vapour pressure e, mixing ratio w and specific humidity q. Define the saturation vaopur pressure e_s and saturation mixing ratio w_s. Show that e_s and w_s are related approximately by the relationship

$$w_s \approx \varepsilon \frac{e_s}{p}$$

where ε is the ratio of the gas constants for dry air and for water vapour.

Define the relative humidity RH and dew-point temperature T_d and show that they are related by

$$RH = 100 \times \frac{w_s(T_d, p)}{w_s(T, p)}$$

Define the Lifting Condensation Level (LCL) and illustrate by means of a sketch how it may be found on a tephigram if the temperature, pressure and dew-point are given.

- **Question 4.**

State *Normand's Rule* and illustrate it by means of a sketch.

An air parcel at 950 hPa has a temperature of 14°C and a mixing ratio of $8\,\mathrm{g\,kg^{-1}}$. What is the wet-bulb potential temperature of the air?

The air parcel is lifted to the 700 hPa level by passing over a mountain, and 70% of the water vapour that is condensed out by the ascent is removed by precipitation. Determine the temperature, potential temperature, mixing ratio, and wet-bulb potential temperature of the air parcel after it has returned to the 950 hPa level on the other side of the mountain.

Discuss the relevance of the above to the character of mountain winds.

(A tephigram chart may be used to solve this problem).

- **Question 5.**

A parcel of dry air at height $z = 0$, with density ρ_0 and temperature T_0, is lifted to height z, where it has density ρ' and temperature $T' = T_0 - \Gamma_d\, z$. The density and temperature of the ambient air at this height are ρ and $T = T_0 - \Gamma\, z$. Show that the downward force of the air parcel is

$$F = g(\rho' - \rho)$$

Using the gas equation, show that the downward acceleration is

$$a = g\left(\frac{\Gamma_d - \Gamma}{T}\right) z$$

Show that if $(\Gamma_d - \Gamma) > 0$ the parcel may execute bounded oscillations with squared frequency $N^2 = (g/T)(\Gamma_d - \Gamma)$. Find the period of oscillation when the ambient temperature and lapse-rate are $T = 250\,\mathrm{K}$ and $\Gamma = 6\,\mathrm{K\,km^{-1}}$.

- **Question 6.**

  Consider the Planck function which determines the radiation emitted by a black body,

$$B_\lambda = \frac{c_1}{\lambda^5(\exp(c_2/\lambda T) - 1)}$$

  with $c_1 = 3.74 \times 10^{-16}\,\mathrm{W\,m^{-2}}$ and $c_2 = 1.44 \times 10^{-2}\,\mathrm{m\,K}$. Assuming that the exponential term is much larger than unity, deduce Wien's Displacement Law for the wavelength λ_m of peak emission for a blackbody at temperature T:

$$\lambda_m \approx \frac{2.9 \times 10^{-3}}{T}$$

  where λ_m is expressed in metres and T in degrees kelvin.

  On the basis of this equation, and assuming that the wavelength of maximum solar emission is $0.475\,\mu\mathrm{m}$, estimate the "colour temperature" of the Sun.

  Discuss the separation of the spectrum into *solar* and *terrestrial* radiation in terms of Wien's Law.

- **Question 7.**

  Define the terms *absorbtivity* and *emissivity* for a radiating body. State Kirchhoff's Law for the relationship between these quantities.

  Calculate the radiative equilibrium temperature of the earth's surface and atmosphere assuming that the atmosphere can be regarded as a thin layer with an absorbtivity of 0.1 for solar radiation and 0.8 for terrestrial radiation. Assume that the earth's surface radiates as a blackbody at all wavelengths. Also assume that the net solar irradiance absorbed by the earth-atmosphere system is $F = 240\,\mathrm{W\,m^{-2}}$.

  Explain why the surface temperature computed above is considerably higher than the effective temperature in the absence of an atmosphere.

- **Question 8.**

  For steady, horizontally homogeneous, incompressible flow the momentum equations for the atmosheric boundary layer may be written

$$\begin{aligned} -fv + \frac{1}{\rho}\frac{\partial p}{\partial x} - K\frac{\partial^2 u}{\partial x^2} &= 0 \\ +fu + \frac{1}{\rho}\frac{\partial p}{\partial y} - K\frac{\partial^2 v}{\partial y^2} &= 0 \end{aligned}$$

  where K and f may be assumed to be constant.

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Defining $\gamma = \sqrt{f/2K}$ and assuming that the motion vanishes at $z = 0$ and tends to the zonal geostrophic value $\mathbf{V} = (u_g, 0)$ in the free atmosphere, derive the equations

$$\begin{aligned} u &= u_g(1 - e^{-\gamma z}\cos\gamma z) \\ v &= u_g e^{-\gamma z}\sin\gamma z \end{aligned}$$

corresponding to the Ekman spiral. Describe the solution in qualitative terms. Estimate the effective depth of the boundary layer if $f = 10^{-4}\,\mathrm{s}^{-1}$ and $K = 10\,\mathrm{m}^2\mathrm{s}^{-1}$.

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