University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

WINTER EXAMINATIONS 2005

SCMXF0028
SCMXP0028
M.Sc. in Meteorology

Dynamical Meteorology

MAPH P310

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Time Allowed: 3 hours

Instructions for Candidates

Full marks will be awarded for complete answers
to **four** questions.
All questions carry equal marks.
Please do not use red pen on the answer books.
Please use separate answer book for each question.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1.

a) Let **A** be an arbitrary vector whose Cartesian components in a frame of reference rotating with angular velocity **Ω** are

$$\mathbf{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z \qquad (1.1)$$

Write down the relationship between the total derivative of **A** in an inertial reference frame (D_aA/Dt) and the corresponding total derivative in the rotating system (DA/Dt) .

b) Show that

$$\frac{D_a\vec{i}}{Dt} = \vec{\Omega} \times \vec{i} \qquad (1.2)$$

c) Assuming that expressions similar to (1.2) hold for the total derivatives of **j** and **k** in the inertial frame, what is the resulting relationship between (D_aA/Dt) and (DA/Dt)?

Question 2.

a) Using the Lagrangian control volume method, derive the continuity equation

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \bullet \vec{U} = 0 \qquad (2.1)$$

in (x,y,z,t) coordinates, where ρ is the density and **U** is the 3D velocity vector.

b) Again using the Lagrangian control volume method, derive the continuity equation in isobaric coordinates for a hydrostatic atmosphere. Comment on the advantages of this equation.

Question 3.

a) State the assumptions used in constructing the shallow water model with a rigid horizontal upper lid, bottom topography $h_s(x,y)$ and fluid depth $h(x,y)$ on a β-plane.

b) Starting from the primitive equations of motion on a β-plane and using the above assumptions, show that the governing equations for the model in question are

$$\frac{Du}{Dt} = fv - \frac{\partial \Phi_T}{\partial x} \qquad (3.1)$$

$$\frac{Dv}{Dt} = -fu - \frac{\partial \Phi_T}{\partial y} \qquad (3.2)$$

$$\frac{Dh}{Dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \qquad (3.3)$$

where $\Phi_T = p_T/\rho$, p_T being the pressure at the upper lid and ρ being the density.

c) Using (3.1), (3.2) and (3.3) derive the potential vorticity equation

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$$\frac{D}{Dt}\left[\frac{\zeta + f}{h}\right] = 0 \qquad (3.4)$$

for this model (ζ = relative vorticity).

Question 4.

a) The vorticity equation for the shallow water model with a rigid lid and a flat bottom on a β-plane can be written

$$\frac{D}{Dt}[\zeta + f] = 0 \qquad (4.1)$$

where ζ is the relative vorticity and f is the Coriolis parameter. Show that in terms of the streamfunction ψ eq. (4.1) can be written

$$\frac{\partial}{\partial t}\left(\nabla^2\psi\right) + J(\psi, \nabla^2\psi) + \beta\frac{\partial\psi}{\partial x} = 0 \qquad (4.2)$$

where ∇^2 is the horizontal Laplacian and J is the Jacobian.

b) Using (4.2) show that the frequency of 2D Rossby waves superimposed on a basic current $\bar{u}$ is

$$\nu = \bar{u}k - \frac{\beta k}{K^2} \qquad (4.3)$$

where $K^2 = k^2 + l^2$ and (k,l) are the wavenumbers in (x,y).
Hint: assume $\psi = \bar{\psi} + \psi'(x, y, t)$ and linearize.

c) Calculate the group velocity for the waves described by (4.3).

Question 5.

a) The quasi-geostrophic potential vorticity equation for the free-surface shallow water model on a β-plane ($f = f_0 + \beta y$) is given by

$$\frac{D_g}{Dt}\left[\nabla^2\psi + f - \frac{\psi}{\lambda^2}\right] = -\frac{f_0}{\Phi_0}\frac{D_g\Phi'_s}{Dt} \qquad (5.1)$$

where D_g / Dt is the material derivative following the geostrophic motion, ψ is the geostrophic streamfunction, ∇^2 is the horizontal Laplacian, $\lambda = \sqrt{\Phi_0} / f_0$, Φ_0 is the mean geopotential of the free surface and Φ'_s is the geopotential of the orography (assumed small by comparison with Φ_0).

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Using the linearized perturbation form of (5.1) for a resting basic state with Φ'_s=0, derive the phase speed for a 1D Rossby wave propagating in the x-direction in this model.

b) Again using (5.1) but neglecting β, derive the solution for the perturbation streamfunction ψ' in the case of steady motion forced by a mean current $\bar{u}$ blowing over orography of the form

$$\Phi'_s = \mathrm{Re}\left[\hat{\Phi}_s \exp(ikx)\right] \qquad (5.2)$$

Hint: Since the forcing has spatial dependence of the form exp(ikx), the solution may also be assumed to have spatial dependence of this form.

Question 6.

a) Given the following governing equations for the two-layer model of baroclinic instability

$$\left(\frac{\partial}{\partial t}+U_1\frac{\partial}{\partial x}\right)\frac{\partial^2\psi_1'}{\partial x^2}+\beta\frac{\partial\psi_1'}{\partial x}=\frac{f_0}{\delta p}\omega_2' \qquad (6.1)$$

$$\left(\frac{\partial}{\partial t}+U_3\frac{\partial}{\partial x}\right)\frac{\partial^2\psi_3'}{\partial x^2}+\beta\frac{\partial\psi_3'}{\partial x}=-\frac{f_0}{\delta p}\omega_2' \qquad (6.2)$$

$$\left(\frac{\partial}{\partial t}+U_m\frac{\partial}{\partial x}\right)\left(\psi_1'-\psi_3'\right)-U_T\frac{\partial}{\partial x}\left(\psi_1'+\psi_3'\right)=\frac{\sigma\delta p}{f_0}\omega_2' \qquad (6.3)$$

where $U_m = (U_1+U_3)/2$ and $U_T=(U_1-U_3)/2$, show that the rate of change of the sum of the kinetic and available potential energies is

$$\frac{d}{dt}(K'+P')=4\lambda^2U_T\overline{\psi_T\frac{\partial\psi_m}{\partial x}} \qquad (6.4)$$

where $\psi_m=\left(\psi_1'+\psi_3'\right)/2$, $\psi_T=\left(\psi_1'-\psi_3'\right)/2$, $\lambda^2=(f_0)^2/\left[\sigma(\delta p)^2\right]$ and $\overline{(\ )}$ denotes an average over the wavelength of the disturbance.

b) Discuss the energetics of baroclinic waves qualitatively using a box diagram involving $\overline{P}$ (the mean available potential energy), P' and K'.

c) Discuss the physical mechanism of baroclinic instability in terms of the slope of particle trajectories ('the wedge of instability').

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