UNIVERSITY COLLEGE DUBLIN

NATIONAL UNIVERSITY OF IRELAND, DUBLIN

An Colaiste Ollscoile Baile Atha Cliath

Ollscoil na hEireann, Baile Atha Cliath

WINTER EXAMINATIONS 2004

SCMXF0028
SCMXP0028

M.Sc. in METEOROLOGY
MAPH P310
DYNAMIC METEOROLOGY

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Instruction for Candidates: Candidates should attempt any 4 out of the 8 questions.
All questions carry equal marks.

Time: 3 hours

Notes for Invigilators: Four questions to be answered, separate answer books for each question.

Question1. Express the force balances in the isobaric form of the horizontal momentum equation in natural coordinates. Show that one of the types of balanced flow that can exist on an f-plane is inertial motion in which the particles move anticyclonically in circles. What is the period?

Question 2. Prove Kelvin's circulation theorem for a frictionless fluid

$$\frac{DC_a}{Dt} = -\oint \frac{1}{\rho} dp \quad \text{..................(2.1)}$$

where C_a is the absolute circulation about a closed chain of fluid particles. Show that for a barotropic fluid the absolute circulation is conserved following the motion.

Question 3. (a) Given the following governing equations for a shallow water model with a horizontal lower boundary and a free surface on a β - plane

$$\frac{Du}{Dt} = fv - \frac{\partial \Phi}{\partial x} \quad \text{...............(3.1)}$$

$$\frac{Dv}{Dt} = -fu - \frac{\partial \Phi}{\partial y} \quad \text{.................(3.2)}$$

$$\frac{D\Phi}{Dt} = -\Phi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad \text{..................(3.3)}$$

derive the vorticity equation

$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad \text{..................(3.4)}$$

(b) Use the above equations to derive the potential vorticity equation

$$\frac{D}{Dt}\left[\frac{\zeta + f}{\Phi}\right] = 0 \quad \text{................(3.5)}$$

Show that the linearized form of the potential vorticity equation for small perturbations about a state of rest on an f-plane ($f=f_0$) can be written

$$\frac{\partial}{\partial t}\left(f_0\zeta' - \frac{\Phi'}{\lambda^2}\right) = 0 .... (3.6)$$

where $\lambda = \overline{\Phi}^{1/2} / f_0, \Phi = \overline{\Phi} + \Phi'$.

Question 4. At an initial time t=0 the free surface height of a shallow water model with a flat bottom on an f-plane is given by

$$h = H + h_0, |x| \le L$$

$$h = H, |x| > L$$

where H and h_0 are positive constants, with $h_0 \ll H$. The initial velocity is zero everywhere.

Using (3.6), find the final geopotential of the free surface when the system has adjusted to geostrophic balance ($t=\infty$).
(Use symmetry and assume that the final geopotential field and geostrophic current are continuous.)

Qusetion 5. Using the linearized forms of (3.1), (3.2) and (3.3) for small amplitude perturbations of a shallow water model about a state of rest on an f-plane, find the phase speed of gravity-inertia waves propagating in the x-direction. Calculate the group velocity. Comment on the effects of rotation on the properties of the waves.

Qusetion 6. (a) Show that in a hydrostatic atmosphere the internal and gravitational potential energies are proportional and that the sum of these two forms of energy (i.e., the total potential energy - TPE) can be written

$$TPE = \left(\frac{c_p}{c_v}\right) E_I .... (6.1)$$

where c_p and c_v are the specific heats of air at constant pressure and constant volume, respectively, and E_I is the internal energy.
(b) Show that the TPE of a unit column of atmosphere of uniform potential temperature θ extending from the surface ($p=p_0$) to the top of the atmosphere ($p=0$) is

$$TPE = \frac{c_p}{g}\frac{p_0\theta}{\kappa + 1} .... (6.2)$$

where where $\kappa = R/c_p$ and g is the acceleration of gravity.
c) Consider two air masses of uniform potential temperatures θ_1 and θ_2 ($\theta_2 > \theta_1$) which are separated by a vertical partition. Each air mass occupies a horizontal area A and extends from

the surface to the top of the atmosphere. Show that the available potential energy for this system is given by

$$APE = \frac{c_p}{g}\frac{p_0}{\kappa + 1}\left(1 - \frac{1}{2^\kappa}\right)(\theta_2 - \theta_1)A \ldots\ldots\ldots\ldots\ldots\ldots (6.3)$$

Question 7. Starting from the momentum and continuity equations for the free surface shallow water model on an f-plane, derive the quasi-geostrophic forms of these equations. Hence show that the quasi-geostrophic potential vorticity equation for the model can be written

$$\frac{D_g}{Dt}\left[\nabla^2\psi - \frac{\psi}{\lambda^2}\right] = -\frac{f_0}{\Phi_0}\frac{D_g\Phi_s'}{Dt} \ldots\ldots\ldots\ldots\ldots\ldots (7.1)$$

where ψ is the geostrophic streamfunction, $\lambda=\sqrt{\Phi_0}/f_0$, Φ_0 is the unperturbed geopotential of the free surface for a resting basic state, Φ_s' is the geopotential of the orography (assumed small by comparison with Φ_0) and

$$\frac{D_g}{Dt} = \frac{\partial}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} \ldots\ldots\ldots\ldots\ldots\ldots (7.2)$$

Question 8. (a) Show that the thermodynamic equation for adiabatic motion in a compressible atmosphere

$$c_v\frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (8.1)$$

can be re-written in the form

$$\frac{1}{\gamma p}\frac{Dp}{Dt} = \frac{1}{\rho}\frac{D\rho}{Dt} \ldots\ldots\ldots\ldots\ldots\ldots (8.2)$$

where $\gamma=c_p/c_v$. [Use the relationship $R=c_p-c_v$ where R is the gas constant and (c_p, c_v) are the specific heats of air at constant pressure and constant volume, respectively.]

(b) Using (8.2) along with the remaining set of primitive equations (in height coordinates) on an infinite f-plane, show that the phase speed of a small-amplitude Lamb wave (horizontal acoustic wave) superimposed on a resting isothermal basic state characterized by $\overline{T}(z)=T_0$ is given by p.t.o.

$$c = \pm\left[\gamma R T_0 + \frac{f_0^{\,2}}{k^2}\right]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8.3)$$

[Assume the wave propagates in the x-direction with wavenumber k, that w=0 and ∂/∂y=0, and that the wave amplitudes are functions of z.]

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