University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I EXAMINATION 2009/2010

MAPH 40240

Physical Meteorology

Extern examiner: Professor Keith Shine

Head of School: Professor Micheal O Searcoid

Examiner: Dr. Rodrigo Caballero*

Time Allowed: 2 hours

Instructions for Candidates

Answer **any two (2)** of the following 3 questions. Each question carries 50 marks.
All symbols not explicitly defined here have the meaning normally given to them in the lecture notes.
A list of values of physical constants can be found on the last page.

Instructions for Invigilators

Non-programmable calculators may be used during this examination. Tephigram charts will be handed to each candidate as part of the examination material.

Question 1

(a) (*10 marks*) Consider a parcel that ascends dry adiabatically from sea level, where pressure is 1000 hPa, temperature is 30°C and relative humidity 25%. Use the tephigram chart in Figure 1 (page 5) to obtain the quantities listed below for the parcel when it is half-way to its lifting condensation level (i.e. when the pressure is $1000 - (1000 - p_{LCL})/2$): dewpoint temperature, mixing ratio, saturation mixing ratio, relative humidity, vapour pressure. To compute the vapour pressure, you will need to show that

$$e \approx p\frac{w}{\epsilon} \qquad (1)$$

where $\epsilon = m_v/m_d = 0.622$.

(b) (*6 marks*) Consider a sounding which has a CINE of 50 J kg^{-1}. Calculate the initial upwards speed that a parcel requires in order to just reach its level of free convection.

(c) (*12 marks*) In a well-mixed boundary layer 2 km deep, with constant potential temperature of 300 K, a parcel rises from near the surface with a potential temperature 1 K greater than that of the mixed layer. What is the upward speed of the parcel due to its CAPE at the top of the mixed layer? You do not need the tephigram chart to solve this problem; instead, note that the buoyancy force (per unit mass) on an air parcel can be written

$$g\frac{\rho_s - \rho_p}{\rho_p} \qquad (2)$$

where ρ_s is the ambient density while ρ_p is the parcel density, and re-write this expression in terms of potential temperature.

(d) (*10 marks*) In the days before refrigeration, it was common in Mediterranean countries to keep drinking water cool by storing it in a porous and well-ventilated vessel. What is the name given to the minimum temperature that the water can be cooled to using this method? Given an environment with a pressure of 1000 hPa, a temperature of 30°C and 25% relative humidity, use the tephigram in Figure 1 (page 5) to estimate this minimum temperature.

(e) (*12 marks*) On cold days, it is common to "see your breath", that is for a small cloud of condensation to form in exhaled air. Explain this phenomenon in simple qualitative terms. Using the plot of saturation vapour pressure in Figure 2 (page 6), estimate the maximum temperature at which you can see your breath if the ambient air is perfectly dry, given that exhaled air typically has a temperature of around 35°C and a relative humidity of 92%. Explain how you obtain your result, using a sketch if necessary.

Question 2

On the 15th of October 2009, the world's news media were gripped by the story of a runaway home-made helium-filled balloon in Colorado, USA, which travelled about 50 miles and reached heights of about 2000 m while believed to be carrying a 6-year old boy. When the balloon finally landed, no boy was found inside. A question that naturally comes to mind is: given the balloon's dimensions, was its observed flight actually possible while carrying the weight of a 6-year-old? We will attempt to answer this question using some rough estimates here.

(a) (*10 marks*) Consider a body of volume V and uniform density ρ_b immersed in a hydrostatic fluid of uniform density ρ_a. Show that the net force on the body is

$$F = gV(\rho_a - \rho_b) \tag{3}$$

(b) (*10 marks*) Judging by news pictures, the balloon was roughly disk-shaped, with a radius of about 2.5 m and a thickness of perhaps 2 m. Given these dimensions, could the balloon have lifted off the ground carrying a boy of mass, say, 20 kg in addition to the mass of the balloon's plastic skin, say another 10 kg? Assume an ambient temperature of 20°C and pressure of 1000 hPa. The balloon was filled with helium, which has a molecular weight of 4 AMU. Take the molecular weight of air to be 29 AMU. Assume that the helium in the balloon was at the same temperature and pressure as the surroundings.

(c) (*10 marks*) Show that in a hydrostatic, isothermal atmosphere, density decreases with height as

$$\rho(z) = \rho(0)e^{-z/H} \tag{4}$$

where $H = R_d T/g$ is the scale height.

(d) (*8 marks*) The Fort Collins area of Colorado, where the incident took place, has a surface elevation of about 1500 m. Give a rough (but mathematically motivated) estimate of the surface pressure there. How does this affect your conclusion to part (b)?

(e) (*12 marks*) Now estimate the maximum height the balloon could have reached. Make the same assumptions as in (b). Additionally, assume an isothermal atmosphere, and assume that the volume of the balloon does not change during ascent.

Question 3

(a) (*10 marks*) Explain the concept of *saturation vapour pressure.* Explain, in simple physical terms, how and why the saturation vapour pressure of water depends on temperature, curvature of the liquid/vapour interface, and purity of the liquid phase.

(b) (*10 marks*) Raoult's law can be written

$$e_s(T, r, f) \;=\; f\, e_s(T, r, 1). \qquad (5)$$

Show that for a droplet that has formed around a soluble aerosol of mass M_s, the water fraction f can be written

$$f \;=\; \frac{1}{1 + AM_s/r^3} \qquad (6)$$

where M_s is the mass of the aerosol and

$$A \;=\; \frac{3i}{4\pi\rho_w}\frac{m_v}{m_s} \qquad (7)$$

with i the number of ions per molecule of solute, m_s the molecular weight of the solute, m_v the molecular weight of water and ρ_w the density of liquid water. Compute the value of A for an aerosol composed of NaCl, which has $i = 2$ and $m_s = 58.44$. Take $m_v = 18$ and $\rho_w = 1000$ kg m^{-3}.

(c) (*12 marks*) Figure 3 on page 7 shows a solution to Kelvin's equation (the y-axis shows $e/e_{s\infty} \times 100$). Using this plot in combination with the results in (b), compute the value of M_s required to maintain a droplet of radius 0.1 μm in equilibrium in an environment where the humidity is saturated with respect to a flat surface of pure water. Is this equilibrium stable or unstable? Why?

(d) (*4 marks*) Figure 4 on page 8 shows the scattering efficiency of spherical droplets as a function of radius. Using this plot, estimate the mass scattering coefficient k_s for the droplet in (c) when it is illuminated by radiation of wavelength 0.5 μm. Is this radiation in the solar or terrestrial range? Recall that

$$Q_s \;=\; \frac{m}{\pi r^2} k_s \qquad (8)$$

where m is the droplet mass.

(e) (*14 marks*) Suppose a beam of 0.5 μm radiation passes normally through a 1 km layer of air containing a suspension of droplets all identical to that in (c) with a concentration of 100 cm^{-3}. Compute the transmissivity of the slab, neglecting all processes other than scattering by the droplets. Re-calculate the answer assuming a single-scatter albedo of 0.5.

Figure 1:

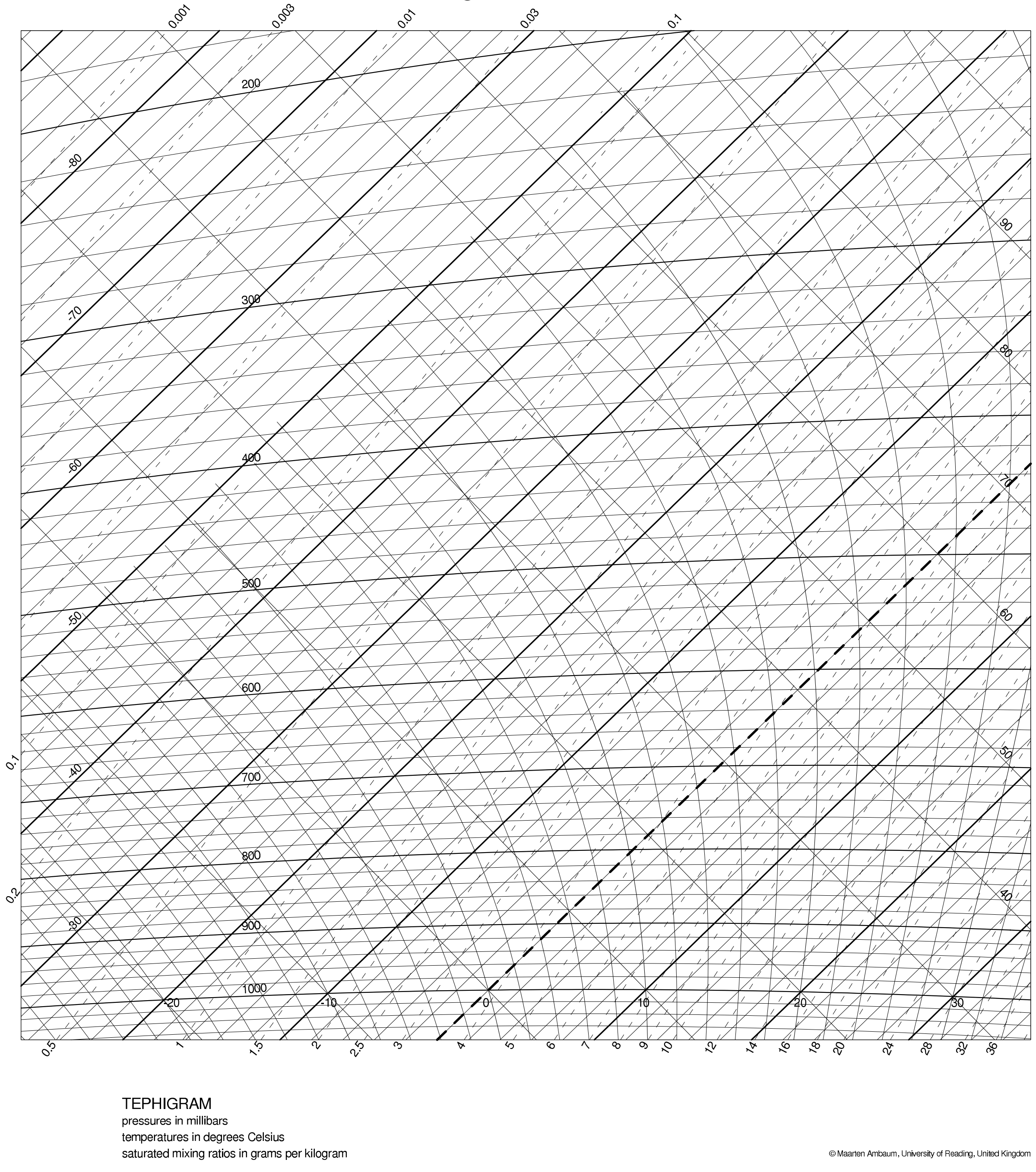
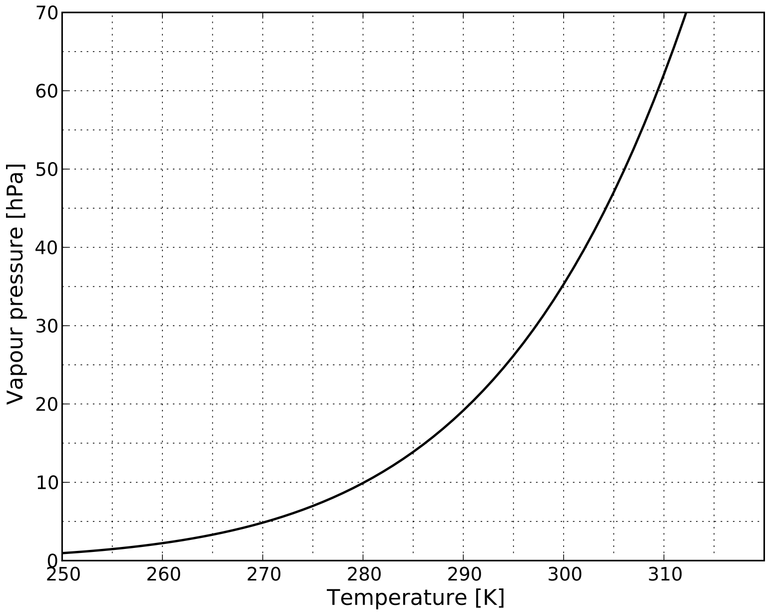
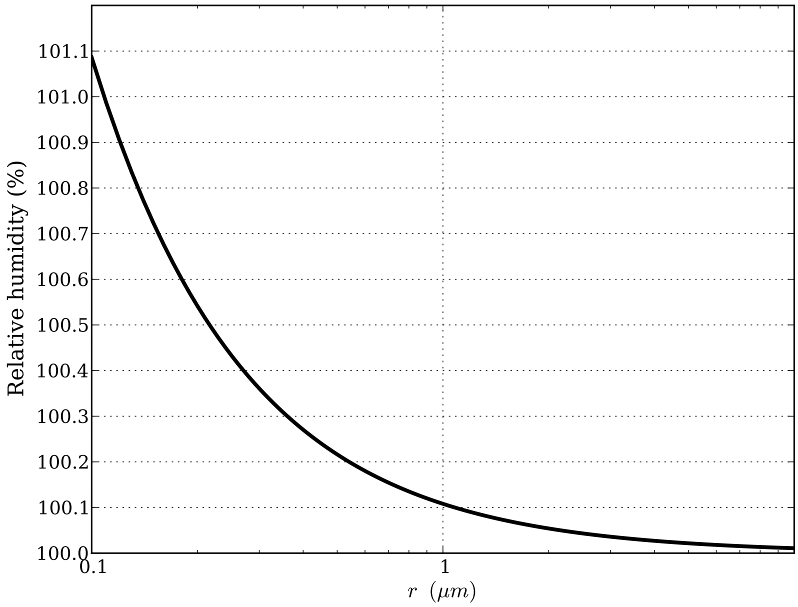

0.001
0.003
0.01
0.03
0.1
200
300
400
500
600
700
800
900
1000
-80
-70
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
70
80
90
0.1
0.2
0.5
1
1.5
2
2.5
3
4
5
6
7
8
9
10
12
14
16
18
20
24
28
32
36
TEPHIGRAM
pressures in millibars
temperatures in degrees Celsius
saturated mixing ratios in grams per kilogram
© Maarten Ambaum, University of Reading, United Kingdom

Figure 2:



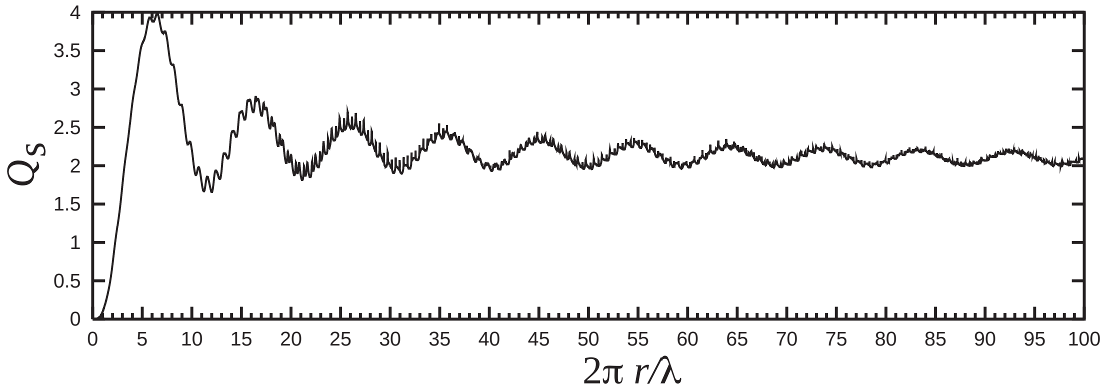
70
60
50
40
30
20
10
0
Vapour pressure [hPa]
250
260
270
280
290
300
310
Temperature [K]

Figure 3:



101.1
101.0
100.9
100.8
100.7
100.6
100.5
100.4
100.3
100.2
100.1
100.0
Relative humidity (%)
0.1
1
r (μm)

Figure 4:



Values of physical constants

Gravitational acceleration $g = 9.8$ m s^{-2}
Gas constant for dry air $R_d = 287$ J K^{-1} kg^{-1}
Specific heat capacity of dry air at constant pressure $c_{pd} = 1004$ J K^{-1} kg^{-1}
Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Boltzmann's constant $k = 1.38 \times 10^{-23}$ m^2 kg s^{-1} K^{-1}

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