University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I EXAMINATION 2009/2010

# MAPH 40460

## Dynamic Meteorology

Extern examiner: Professor Keith Shine

Head of School: Professor Mícheál Ó Searcóid

Lecturer: Professor Ray Bates*

**Time Allowed: 3 hours**

**Instructions for Candidates**

Answer **four (4)** questions.
All questions carry equal marks.
Total: 100 marks.

**Instructions for Invigilators**

Non-programmable calculators may be used during this examination.

Question 1.

(a) Using the hydrostatic equation and the ideal gas law, show that the difference in geopotential between two isobaric surfaces with pressures $p_0$ and $p_1$ ($p_0 > p_1$) is

$$\Phi_1 - \Phi_0 = R\langle T\rangle \ln\left(\frac{p_0}{p_1}\right) \qquad (1.1)$$

where R is the gas constant and $\langle T\rangle$ is the mean temperature of the intervening layer.

(b) Using the hydrostatic equation and the definition of the geostrophic wind $\left(\vec{V}_g\right)$, show that the difference in $\vec{V}_g$ between levels $p_1$ and $p_0$ is

$$\vec{V}_g(1) - \vec{V}_g(0) = \frac{R}{f}\vec{k} \times \vec{\nabla}\langle T\rangle \ln\left(\frac{p_0}{p_1}\right) \qquad (1.2)$$

(c) Assuming $\langle T\rangle$ for the 1000-850 hPa layer in the neighbourhood of a point on the 45° latitude circle is given by

$$\langle T\rangle = 278K - \left(4\times 10^{-3}\frac{K}{km}\right)y \qquad (1.3)$$

where y is the distance northward from this point, use (1.1) and (1.2) to find the thickness of the layer and the geostrophic wind difference across the layer at the point in question. [R = 287 J/(kgK), g = 9.81m/s$^2$]

Question 2.

(a) The momentum equation in natural coordinates for steady horizontal flow parallel to circular geopotential contours on an f-plane (the gradient wind equation) is

$$\frac{V^2}{R} + f_0 V + \frac{\partial\Phi}{\partial n} = 0 \qquad (2.1)$$

where V is the wind speed, n is the distance normal to the flow direction to the left, and R is the radius of curvature (defined to be positive if the centre of curvature is in the direction of increasing n). If the quadratic (2.1) is solved for V, what conditions does the solution have to satisfy in order for it to be physical? Name the different types of physical solution that exist.

(b) Using (2.1), show that for a regular low the geostrophic wind, $V_g$, is larger than V, while for a regular high $V_g$ is smaller than V. (Confine attention to the Northern Hemisphere.)

(c) If the slope of the isobaric surfaces normal to the flow in a regular low is given by $\partial Z/\partial n$= -10m/100km, where Z is the geopotential height, find the geostrophic and the gradient winds for R=100 km and R=300km (take $f_0 = 10^{-4}s^{-1}$, g = 9.81 m/s$^2$).

Question 3.

(a) The potential vorticity equation for the shallow water model with a free surface on an f-plane is given by

$$\frac{D}{Dt}\left[\frac{\varsigma + f_0}{\Phi}\right] = 0 \qquad (3.1)$$

Show that for small perturbations about a state of rest, eq. (3.1) linearizes to

$$\frac{\partial}{\partial t}\left(\varsigma' - \frac{f_0}{\overline{\Phi}}\Phi'\right) = 0 \qquad (3.2)$$

(b) Using (3.2), show that the final geostrophically adjusted state corresponding to an initial state of rest with a height field of the form

$$h = H + h_0, |x| \le L$$
$$h = H, |x| > L$$

where $h_0$ << H, is given by

$$\Phi' = \begin{cases} gh_0\left[1 - \exp(-L/R)\cosh(x/R)\right], & x \le L \\ gh_0 \sinh\left(\dfrac{L}{R}\right)\exp(-x/R), & x > L \end{cases} \qquad (3.3)$$

with $R = \sqrt{\overline{\Phi}}/f_0$.

Note: because the height field is symmetrical about x = 0, it is necessary to solve only for x≥0.

Hint: in addition to the condition of symmetry, assume that both $\Phi'$ and $d\Phi'/dx$ in the adjusted state are continuous everywhere.

Question 4.

(a) The vorticity equation for the shallow water model with a rigid lid and a flat bottom on a β-plane can be written

$$\frac{D}{Dt}\left[\zeta + f\right] = 0 \qquad (4.1)$$

where ζ is the relative vorticity and f is the Coriolis parameter. Show that in terms of the streamfunction ψ eq. (4.1) can be written

$$\frac{\partial}{\partial t}\left(\nabla^2\psi\right) + J(\psi, \nabla^2\psi) + \beta\frac{\partial \psi}{\partial x} = 0 \qquad (4.2)$$

where $\nabla^2$ is the horizontal Laplacian and J is the Jacobian.

(b) Using (4.2) show that the frequency of 2D Rossby waves superimposed on a constant basic current $\bar{u}$ is

$$\nu = \bar{u}k - \frac{\beta k}{K^2} \qquad (4.3)$$

where $K^2 = k^2+l^2$ and (k,l) are the wavenumbers in (x,y).
Hint: assume $\psi = \bar{\psi}(y) + \psi'(x,y,t)$ and linearize.
If the x-wavelength is 9,000 km, the y-wavelength is infinite and $\beta = 1.6\text{x}10^{-11}\ \text{m}^{-1}\text{s}^{-1}$, what value of $\bar{u}$ is required to reduce the phase speed of the Rossby wave to zero?

(c) Calculate the group velocity for the waves described by (4.3).

Question 5.

(a) Write down the governing momentum and continuity equations for the shallow water model with a rigid horizontal lid, mean depth H, bottom topography of height $h_s(x,y)$ and fluid depth h(x,y) on a β-plane.
(b) Derive the quasi-geostrophic form of these equations.
Note: use constant-f geostrophy in defining the geostrophic current.
(c) Hence show that the quasi-geostrophic potential vorticity equation for this model can be written

$$\frac{D_g}{Dt}\left[\zeta_g + f + \frac{f_0}{H}h_s\right] = 0 \qquad (5.1)$$

Question 6

(a) In the Eady model of baroclinic instability, the solution for the perturbation streamfunction is given by

$$\psi' = A\left[\sinh \alpha z^* - \frac{c\alpha}{\Lambda}\cosh \alpha z^*\right]\cos ly \exp[ik(x-ct)] \qquad (6.1)$$

where the complex phase speed c (= $c_r + ic_i$) is given by

$$c = \frac{\Lambda H}{2}\left[1 \pm \frac{2}{\alpha H}\left\{\left(\frac{\alpha H}{2} - \tanh\frac{\alpha H}{2}\right)\left(\frac{\alpha H}{2} - \coth\frac{\alpha H}{2}\right)\right\}^{1/2}\right] \qquad (6.2)$$

In the above, Λ is the vertical wind shear, H the height of the lid and $\alpha = N(k^2+l^2)^{1/2}/f_0$, N being the buoyancy frequency and $f_0$ the (constant) Coriolis parameter.

For the case $l = 0$, show that in the limit $L_x / L_R \to \infty$ (where $L_x = 2\pi/k$ and $L_R = NH/f_0$), the complex phase speed for the unstable wave is given by

$$c = \frac{\Lambda H}{2}\left(1 + \frac{i}{\sqrt{3}}\right) \qquad (6.3)$$

[Make use of the series expansions
$\tanh x = x - x^3/3 + ....$
$\coth x = 1/x + x/3 - .....$ ]

(b) For the same case $l = 0$ and in the limit $L_x / L_R \to \infty$, show that the solution for $\psi'$ given by (6.1) reduces in the case of the unstable wave to

$$\psi' = AkL_R\left[\frac{z^*}{H} - \frac{1}{2}\left(1 + \frac{i}{\sqrt{3}}\right)\right]\exp[ik(x - ct)] \qquad (6.4)$$

[In deriving (6.4), make use of the series expansions
$\sinh x = x + x^3/3! + .....$
$\cosh x = 1 + x^2/2! + .....$]

(c) Does the perturbation streamfunction given by (6.4) slope eastward or westward with height? Qualitatively, what does the direction of vertical slope of the streamfunction tell us about the direction of the eddy heat transport?

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