University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER II EXAMINATION 2008/2009

MAPH 40260
Numerical Weather Prediction

Extern examiner: Prof Keith Shine

Head of School: Prof Mícheál Ó Searcóid

Examiner: Prof Peter Lynch*

Time Allowed: 3 hours

Instructions for Candidates

Answer **four (4)** questions.
Question 1 **must** be answered, and carries 40 marks.
Three additional questions, each carrying 20 marks, must be answered.
Please use separate answer book for each question.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1 [mandatory]

Trace the development of numerical weather prediction (NWP) through the twentieth century, addressing each of the following topics/questions.

(a) (*5 marks*) What are the main components of an operational NWP system. Briefly describe the function of each.

(b) (*5 marks*) What key roles did the following scientists play in the development of NWP: Vilhelm Bjerknes, Max Margules, Lewis F. Richardson, Jule Charney.

(c) (*5 marks*) Explain briefly the principal causes of the failure of Richardson's forecast.

(d) (*5 marks*) What were the four crucial technical and/or scientific developments in the period 1920–1950 that made NWP feasible.

(e) (*5 marks*) List the main categories of observations used in present-day NWP analyses, commenting on the importance of each.

(f) (*5 marks*) List five physical processes that are parameterized in modern NWP models, briefly describing each process.

(g) (*5 marks*) How has the S1 Score of the operational 500 hPa forecast from NMC/NCEP evolved over the past fifty years? Include a graphical illustration.

(h) (*5 marks*) What are the main factors that have led to improvements in the skill of operational NWP predictions during the past two decades.

Question 2

(a) (*10 marks*) Consider the oscillation equation

$$\frac{d\psi}{dt} = i\omega\psi \qquad (1)$$

and the following three-time-level finite difference approximation to it:

$$\frac{\phi^{(n+1)} - \phi^{(n-1)}}{2\Delta t} = i\omega \left[\frac{\phi^{(n+1)} + 2\phi^{(n)} + \phi^{(n-1)}}{4} \right] . \qquad (2)$$

Using the Taylor series method, show that the scheme (2) is second order accurate.

(b) (*10 marks*) Using the von Neumann method of stability analysis, show that the scheme (2) is unconditionally stable.

Question 3

(a) (*6 marks*) Consider the 1D advection equation $D\psi/Dt = 0$ for the case of a constant wind u (assume $u > 0$). The integral of this equation along the trajectory of a particle that arrives at the gridpoint $x_I (= I\Delta x)$ at time $(n + 1)\Delta t$, having departed from the point x_* at time $n\Delta t$, is given by

$$\psi_I^{(n+1)} = \psi_*^{(n)} . \tag{3}$$

Write down the semi-Lagrangian integration scheme based on (3) that uses linear interpolation from the two gridpoints surrounding the departure point to estimate the departure point value.

(b) (*8 marks*) Show that for this linear interpolation scheme the von Neumann amplification factor for the Fourier component in x of wavenumber k is

$$\lambda = [1 - \hat{\alpha}(1 - e^{-ik\Delta x})]e^{-ipk\Delta x} \tag{4}$$

where $\alpha = u\Delta t/\Delta x$, $p = [\alpha]$ (integer part of α) and $\hat{\alpha} = \alpha - p$. Hence show that the integration scheme is unconditionally stable.

(c) (*6 marks*) Using (4), show that the numerical phase speed of the advected Fourier component is

$$c^* = \frac{1}{k\Delta t}\left[pk\Delta x + \tan^{-1}\left(\frac{\hat{\alpha}\sin k\Delta x}{1 - \hat{\alpha}(1 - \cos k\Delta x)}\right)\right] . \tag{5}$$

Note: c^* is equivalent to ω^*/k, where ω^* is the numerical frequency.

Question 4

(a) (*6 marks*) Consider the linear advection equation in one dimension

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 .$$

Retaining the continuous representation in time, show that if a centered finite difference approximation to the x-derivative is used, the phase speed of the numerical solution is

$$C = \left(\frac{\sin k\Delta x}{k\Delta x}\right) c \tag{6}$$

for a solution of the form $U_m(t) = \exp[ik(m\Delta x - Ct)]$. What is the phase speed of the shortest wave?

(b) (*4 marks*) Show that if a spectral solution to (6) of the form

$$U(x,t) = \sum_k \exp[ik(m\Delta x - Ct)]$$

is sought, the phase speed is represented exactly: $C = c$.

(c) (*10 marks*) Write down a spectral approximation to Burgers' Equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}\,, \qquad -\pi < x \le \pi \tag{7}$$

by expanding the solution in harmonic components

$$u(x,t) = \sum_{n=-N}^{N} U_n \exp inx\,.$$

Show that, when the nonlinear terms are ignored, the n-th component decays exponentially as $\exp(-\nu n^2 t)$. Show that, through the nonlinear term, energy can move from any scale to any other scale.

Question 5

Consider the barotropic vorticity equation on a sphere

$$\frac{\partial \zeta}{\partial t} + \frac{2\Omega}{a^2}\frac{\partial \psi}{\partial \lambda} + \frac{1}{a^2}J(\psi,\zeta) = 0 \tag{8}$$

where the Jacobian term is defined as

$$J(\psi,\zeta) = \left[\frac{\partial \psi}{\partial \lambda}\frac{\partial \zeta}{\partial \mu} - \frac{\partial \psi}{\partial \mu}\frac{\partial \zeta}{\partial \lambda}\right]\,,$$

$\mu = \sin\phi$ and all other notation is standard.

(a) (*6 marks*) Assuming that the streamfunction may be expanded in spherical harmonics

$$\psi(\lambda,\mu,t) = \sum_{n=0}^{N}\sum_{m=-n}^{n} \psi_n^m(t) Y_n^m(\lambda,\mu)\,,$$

show that the vorticity $\zeta(\lambda,\mu,t)$ has a similar expansion, with

$$\zeta_n^m = -\frac{n(n+1)}{a^2}\psi_n^m\,.$$

(b) (*8 marks*) Give a *general argument* leading to the conclusion that the evolution of the spectral coefficients is governed by a set of ode's of the form

$$\frac{d\zeta_n^m}{dt} = -i\sigma_n^m \zeta_n^m + \tfrac{1}{2} i \sum_{k\ell rs} I_{nrs}^{mk\ell} \zeta_r^k \zeta_s^\ell$$

where $\sigma_n^m = -2\Omega m/n(n+1)$ is the Rossby-Haurwitz wave frequency (it is not necessary to derive explicit expressions for the coefficients $I_{nrs}^{mk\ell}$).

(c) (*6 marks*) Describe, in outline, the *transform method*, and give the reason why it is considered superior to the interaction coefficient method indicated in part (b) above.

Question 6

(a) (*6 marks*) Given a discrete time series of values $\{x_n\}$, write down the general expression for a *non-recursive* digital filter applied to this series. Define the frequency response of the filter.

(b) (*6 marks*) Describe one of the methods of selecting the filter coefficients so as to realize a low-pass filter with a specified pass-band edge.

(c) (*8 marks*) Describe the implementation of a digital filter initialization scheme based on a non-recursive filter. Itemize the key stages in the process. List some advantages of the DFI method compared to alternative initialization methods.

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