University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER II EXAMINATION 2007/2008

MAPH 40260

Numerical Weather Prediction

Extern examiner: Prof Keith Shine

Head of School: Prof Séan Dineen

Examiner: Prof Peter Lynch*

Time Allowed: 3 hours

Instructions for Candidates

Answer **four (4)** questions.
Question 1 **must** be answered, and carries 40 marks.
Three additional questions, each carrying 20 marks, must be answered.
Please use separate answer book for each question.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1 [mandatory]

Trace the development of numerical weather prediction (NWP) through the twentieth century, addressing each of the following topics/questions.

(a) (*5 marks*) Briefly describe the main components of an operational numerical weather prediction (NWP) system?

(b) (*5 marks*) Describe, in outline, Margules' analysis of the continity equation, and his conclusions concerning weather prediction.

(c) (*5 marks*) Describe the key developments between Richardson's work and the ENIAC integrations that led to the feasibility of NWP.

(d) (*5 marks*) List the main types of observations used in atmospheric analysis, commenting on the importance of each.

(e) (*5 marks*) Give a brief account of the ENIAC integrations, including the equation used and the main outcome.

(f) (*5 marks*) What is the central message of Chaos Theory? Describe briefly its implications for weather prediction.

(g) (*5 marks*) Give an outline description of the ECMWF Ensemble Prediction System, including the main products.

(h) (*5 marks*) List the main physical processes that are parameterised in modern NWP models, with a bried description of each process.

Question 2

(a) (*8 marks*) Given two independent, unbiassed temperature observations T_1 and T_2 with (unknown) errors ε_1 and ε_2, and (known) variances σ_1^2 and σ_2^2, derive an expression for the best linear unbiassed estimate (BLUE) of the temperature. State all assumptions you make.

(b) (*3 marks*) Assuming $T_1 = 12°\text{C}$, $T_2 = 14°\text{C}$, $\sigma_1 = 1°\text{C}$ and $\sigma_2 = 3°\text{C}$, find the temperature that minimizes the least squares error, and also find its variance.

(c) (*6 marks*) Show that the temperature that minimizes the *cost function*

$$J(T) = \frac{1}{2}\left[\left(\frac{T - T_1}{\sigma_1}\right)^2 + \left(\frac{T - T_2}{\sigma_2}\right)^2\right]$$

yields the same estimate of T as the least squares estimate.

(d) (*3 marks*) Briefly discuss the consequences of this equivalence in the context of optimal interpolation (OI) analysis and variational assimilation.

Question 3

The one-dimensional advection equation may be written in either the Eulerian form or the Lagrangian form:

$$\frac{\partial Y}{\partial t} + u\frac{\partial Y}{\partial x} = 0 \qquad \text{or} \qquad \frac{dY}{dt} = 0\,.$$

(a) (*7 marks*) Describe the procedure used to develop a finite difference approximation to the Lagrangian form of the equation.

(b) (*8 marks*) Assuming that linear spatial interpolation is used to evaluate the solution at the departure point, show that the semi-Lagrangian scheme is stable irrespective of the time step.

(c) (*5 marks*) Discuss the consequences of unrestricted stability for operational numerical weather prediction. Does unconditional stability of the scheme enable an arbitrarily large time step to be used in practice, or are there other considerations limiting it?

Question 4

(a) (*5 marks*) Discuss the difficulties encountered when unbalanced initial data are used to make a numerical forecast. Explain how the process of initialization alleviates the difficulties.

(b) (*6 marks*) Briefly describe each of the following methods of dealing with spurious gravity-wave noise:

- Use of filtered equations
- Static initialization
- Dynamic initialization
- Variational initialization

(c) (*9 marks*) Assume that the system of model equations is written in the form

$$\dot{\mathbf{X}} + i\mathbf{L}\mathbf{X} + \mathbf{N}(\mathbf{X}) = 0\,,$$

where $\mathbf{X}$ is the state vector, $\mathbf{L}$ is a matrix and $\mathbf{N}$ is a nonlinear vector function. Show how this system can be written in normal mode form using the eigenvector matrix of $\mathbf{L}$. Describe the procedures of linear and nonlinear normal mode initialization for this system.

Question 5

(a) (*7 marks*) Write the six-point Crank-Nicholson or centered implicit finite difference equation (FDE) for the linear advection equation in one dimension.

(b) (*8 marks*) Assuming that the FDE has a solution of the form

$$Y_m^n = A\exp[ik(m\Delta x - Cn\Delta t)]\,,$$

show that the physical phase speed c and the computational phase speed C are related by the equation

$$C = \left(\frac{2}{k\Delta t}\right)\tan^{-1}\left[\left(\frac{c\Delta t}{2\Delta x}\right)\sin k\Delta x\right]$$

(c) (*5 marks*) Interpret this relationship in terms of propogation of wave components of the numerical solution and discuss its implications for the stability and accuracy of forecasts using the scheme.

Question 6

Consider the linear shallow water equations on an f-plane:

$$\begin{aligned}
\frac{\partial u}{\partial t} - fv + \frac{\partial}{\partial x}\left(\frac{p}{\rho_0}\right) &= 0 \\
\frac{\partial v}{\partial t} + fu + \frac{\partial}{\partial y}\left(\frac{p}{\rho_0}\right) &= 0 \\
\frac{\partial}{\partial t}\left(\frac{p}{\rho_0}\right) + gH\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) &= 0\,.
\end{aligned}$$

Assume that the density ρ_0 is constant.

(a) (*4 marks*) Introduce characteristic scales: Length (L), velocity (V), advective time (L/V), pressure variations (P) and assuming F, g and H are constant, indicate the scale of each term in the system (*e.g.*, $\partial u/\partial t \sim V^2/L$).

(b) (*5 marks*) Assuming typical mid-latitude synoptic values for the scales, evaluate the order of magnitude of each term. Show how the Rossby Number arises naturally in the comparison of terms of the momentum equations. Show how, to lowest order, the flow is geostrophically balanced.

(c) (*6 marks*) Show how the scaling must be modified to ensure balance in the continuity equation. Discuss briefly the relevance of the Dines compensation mechanism in this context.

(d) (*5 marks*) Show how a 10% error in the wind speed may result in a 100% error in the wind tendency. Discuss the implications of this for numerical weather prediction.

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