University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER II EXAMINATION 2006/2007

MAPH 40260
Numerical Weather Prediction

Extern examiner: Prof Frank Hodnett

Head of School: Prof Séan Dineen

Examiner: Prof Peter Lynch*

Time Allowed: 3 hours

Instructions for Candidates

Answer **four** (4) questions.
Question 1 **must** be answered, and carries 40 marks.
Three additional questions, each carrying 20 marks, must be answered.
Please use separate answer book for each question.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1 [mandatory]

Trace the development of numerical weather prediction (NWP) through the twentieth century, addressing each of the following topics/questions.

(a) (*5 marks*) What are the main components of an operational NWP system?

(b) (*5 marks*) How has the S1 score of the operational 500 hPa forecasts at NMC/NCEP evolved over the past fifty years? Include a graphical indication.

(c) (*5 marks*) What are the main factors leading to improvements in the skill of operational NWP in recent decades?

(d) (*5 marks*) What key roles did the following scientists play in the development of NWP: Vilhelm Bjerknes, Max Margules, Lewis F. Richardson, Jule Charney.

(e) (*5 marks*) State briefly the principal causes of the failure of Richardson's forecast.

(f) (*5 marks*) What were the four crucial developments in the period 1920–1950 that made NWP feasible.

(g) (*5 marks*) List the main physical processes that are parameterised in modern NWP models.

(h) (*5 marks*) What were the key developments in data assimilation during the past twenty years?

Question 2

Consider the linear advection equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0,$$

where c is a constant advecting velocity.

(a) (*4 marks*) Write a finite difference equation (FDE) approximation to the PDE using the leap-frog scheme.

(b) (*4 marks*) Define the condition for the FDE to be *consistent* with the PDE. How is consistency related to the *local truncation error*? Define the condition of *convergence* of the FDE.

(c) (*4 marks*) Verify the consistency of the leapfrog scheme for the linear advection equation. What is the order of the local truncation error in terms of the space and time increments Δx and Δt?

(d) (*4 marks*) How is convergence normally established in practice? State the Lax-Richtmyer Theorem, relating stability and convergence.

(e) (*4 marks*) What is the relevance of the truncation error for NWP? What is the truncation error of a typical scheme used in operational NWP? What is the relevance of the numerical stability of the scheme?

Question 3

Consider the following two model equations

$$\frac{du}{dt} = -\kappa u\,, \qquad \text{the "friction equation"}$$

$$\frac{du}{dt} = i\omega u\,, \qquad \text{the "oscillation equation"}\,.$$

(a) (*8 marks*) Write the finite difference approximations to these two equations, using the Euler forward method. Investigate the *numerical stability* of the FDE in each case. What are the conditions for stability?

(b) (*8 marks*) Repeat the stability analysis, now using the leap-frog scheme for the friction equation and for the oscillation equation. What are the conditions for stability in this case?

(c) (*4 marks*) Write down (without a formal stability analysis) a finite difference scheme that is stable for the combined equation

$$\frac{du}{dt} = i\omega u - \kappa u\,.$$

Question 4

Consider the shallow water equations in the form

$$\begin{aligned}
\frac{\partial u}{\partial t} &= -\frac{\partial \Phi}{\partial x} + R_u \\
\frac{\partial v}{\partial t} &= -\frac{\partial \Phi}{\partial y} + R_v \\
\frac{\partial \Phi}{\partial t} &= -\bar{\Phi}\delta + R_\Phi\,.
\end{aligned}$$

where the gravity wave terms are written explicitly and the expressions R_u, R_v and R_Φ contain all the remaining terms.

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(a) (*6 marks*) Write a semi-implicit finite difference approximation to this system, using centered implicit approximations for the gravity wave terms. You may leave the spatial derivatives in continuous form.

(b) (*10 marks*) Derive a Helmholtz equation for Φ^{n+1}, the geopotential at the new time-level. Outline an algorithm for advancing the solution (u, v, Φ) in time.

(c) (*4 marks*) State the advantages and disadvantages of the semi-implicit scheme. Name some operational models that use a scheme of this sort.

Question 5

In three-dimensional variational assimilation (3D-Var), we define the analysis to be the state vector $\mathbf{x_a}$ that minimizes the *cost function*

$$J(\mathbf{x}) = \frac{1}{2}\left\{(\mathbf{x}-\mathbf{x_b})^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x_b}) + (\mathbf{y_o}-H(\mathbf{x}))^T\mathbf{R}^{-1}(\mathbf{y_o}-H(\mathbf{x}))\right\}$$

where all symbols have their conventional meanings.

(a) (*6 marks*) Derive an expression for the gradient of J with respect to the state vector $\mathbf{x}$.

(b) (*7 marks*) Setting the gradient to zero, show that the 3D-Var analysis may be written

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{W}[\mathbf{y_o} - H(\mathbf{x_b})]$$

where the *gain matrix* $\mathbf{W}$ is given by

$$\mathbf{W} = \left[\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}$$

(c) (*7 marks*) Show that $\mathbf{W}$ is equal to the gain matrix

$$\mathbf{W}_{\text{OI}} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}$$

that is obtained in optimal interpolation analysis.

Question 6

(a) (*6 marks*) Given a discrete time-series of values $\{x_n\}$, write the general expression for a non-recursivei digital filter applied to this series. Define the frequency response function of the filter.

(b) (*6 marks*) Describe one method of selecting the filter coefficients so as to realize a low-pass filter with a specified pass-band edge.

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(c) (*8 marks*) Describe the implementation of a digital filter initialization scheme based on a non-recursive filter. Itemize the key stages in the process. List the advantages of the DFI method compared to alternative methods.

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